Complex Networks (MTH6142)
Formative Assignment 4

- **1. Degree distribution of random graphs**
  A random graph ensemble $\mathbb{G}(N,p)$ with $p = \frac{c}{N-1}$ has a binomial degree distribution

$$P_B(k) = \left(\begin{array}{c} N-1 \\ k \end{array} \right) p^k (1-p)^{N-1-k}, \tag{1}$$

that in the limit of $N \gg 1$ can be approximated by a Poisson distribution $P_P(k)$ given by

$$P_P(k) = \frac{1}{k!} c^k e^{-c}. \tag{2}$$

(a) Calculate the generating function

$$G_B(x) = \sum_{k=0}^{N-1} P_B(k) x^k \tag{3}$$

for the binomial degree distribution $P_B(k)$ given by Eq. (1).

(b) Using the properties of the generating functions, evaluate the first moment $\langle k \rangle$ and the second moment $\langle k(k-1) \rangle$ of the degree distribution $P_B(k)$ given by Eq. (1).

(c) Calculate the generating function

$$G_P(x) = \sum_{k=0}^{\infty} P_P(k) x^k \tag{4}$$

for the Poisson degree distribution $P_P(k)$ given by Eq. (2).

(d) Using the properties of the generating functions, evaluate the first moment $\langle k \rangle$ and the second moment $\langle k(k-1) \rangle$ of the degree distribution $P_P(k)$ given by Eq. (2).

(e) Show that the first $\langle k \rangle$ and second moment $\langle k(k-1) \rangle$ of the binomial distribution $P_B(k)$ obtained in (b) are the same as the first $\langle k \rangle$ and second $\langle k(k-1) \rangle$ moments of the Poisson distribution $P_P(k)$ obtained in (d), as long as $p = \frac{c}{N-1}$ with c constant and $N \to \infty$.

- **2. A given random network**
  Consider a random network in the ensemble $\mathbb{G}(N,p)$ with $N = 4 \times 10^6$ nodes and a linking probability $p = 10^{-4}$.

(a) Calculate the average degree $\langle k \rangle$ of this network.

(b) Calculate the standard deviation σ_P using the approximated degree distribution given by Eq. (2).

(c) Assume that you observe a node with degree 2×10^3. How many standard deviations is this observation from the mean? Is this an expected observation or is this an unexpected observation?