MTH5125 Actuarial Mathematics II

May 2022 – Solutions

<u>Q1</u>

(a) The Gross Future Loss Random Variable is L_0 where

if the annual premium is G

$L_0 = 50120\, v^{K_{59}+1} - 0.976G\, \ddot{a}_{\overline{K_{59}+1}|}$

(b) we seek minimum G such that $\Pr(L_0 < 0) \geq 0.96$

Let G_n be premium that gives $L_0 = 0$ for $K_{59} = n$

$\Pr(L_0 < 0 \mid G = G_n) = \Pr(K_{59} > n)$ which we need to be ≥ 0.96

$\Pr(K_{59} > n) \geq 0.96$ if $l_{59+n} \geq 0.96\, l_{59}$

from AM92 Ultimate

$l_{59} = 9354.004$ so $0.96\, l_{59} = 8979.844$

$l_{63} = 9037$ $l_{64} = 8934$ therefore n=4

$G_4 = \frac{50120\, v^5}{0.976\ddot{a}_5}$

if i=0.04

G = £**9116.40**

(c) the equation of value is

PV benefits + PV expenses + PV profit = PV premiums

note need to mention profit here

if the new annual premium is G'

PV benefits = 50000 A_{59}

PV premiums = G' $\ddot{a}_{59}$

PV expenses = 120 A_{59} + 0.024G' $\ddot{a}_{59}$

PV profit = 0.02(50000) A_{59} = 1000 A_{59}

so the equation of value becomes

51120 A_{59} = 0.976G' $\ddot{a}_{59}$

from AM92 Ultimate

$\ddot{a}_{59} = 14.493$ and $A_{59} = 0.44258$

so $G' =$ **£1599.46**

(d) profits related to sum assured not good idea because

- link to very few of the cost drivers
- might incentivise sales people poorly
- increases risk of large losses if mortality experience adverse
- sum assured fixed for long term contracts

part a from lecture, parts b, c similar to seminar, part d unseen and higher order

IFoA syllabus 6.1, 6.2

<u>Q2</u>

(a) The equation of value is

PV benefits + PV expenses = PV premiums

for the 82000 sum assured policy let premium be G

$82000\, A_{62} = 0.965\, G\, \ddot{a}_{62}$

$A_{62} = 0.48458 \quad \ddot{a}_{62} = 13.401$

so G = £**3072.66**

for the 9500 sum assured policy let premium be H

$9500\, A_{62} = 0.965\, H\, \ddot{a}_{62}$

so H = £**355.98**

(b) we first need to calculate the reserves at 31/12/21

duration t=2 and age = 64

for the 82000 sum assured policy

${}_2V = 82000\, A_{64} - 0.965\,(3072.66)\, \ddot{a}_{64}$

$A_{64} = 0.51333 \quad \ddot{a}_{64} = 12.653$

${}_2V = 4575.44$

for the 9500 sum assured policy

${}_2V' = 9500\, A_{64} - 0.965\,(355.98)\, \ddot{a}_{64}$

${}_2V' = 530.07$

There are 15 x 82000 policies and 24 x 9500 policies

Therefore DSAR = 15x(82000 – 4574.44) + 24x(9500 – 530.07) = 1,376,647

EDS = q_{63} x DSAR = 0.011344 x 1376647 = 15,617

If total claims are 19,000 that can only be 2 deaths among 9500 policies

Therefore ADS = 19000 – 2x530.07 = 17,940

Mortality profit = EDS – ADS = **- £2323** a loss of £2323

(c) Mortality profit is negative, a loss

there have been 2 deaths from 39 policies = 5.1%

from AM 92 Ultimate q_{63} we expect 1.1%

however the loss would have been much worse if one of the claims had been from the larger sum assured policies

mortality experience worse than expected

mortality experience by sum assured level better than expected

part a, b similar to seminar but more challenging, part c unseen

IFoA syllabus 6.2

Q3

(a) the equation of value is

2000000 = PV joint life annuity

If the annuity amount per annum is J then

as ${}_tp_{x:y} = e^{-(\mu^R + \mu^B)t}$

where μ^R is the constant force of mortality for Rowdy and μ^B is the constant force of mortality for Bash

PV joint life annuity = J $\int_0^\infty v^t e^{-(\mu^R + \mu^B)t} dt$

= J $\int_0^\infty e^{-(0.14 + 0.11 - \ln(1.02))t} dt$

= $\frac{J}{0.25 + \ln 1.02}$

therefore J = 2000000 x 0.269803 = £539605 per annum

(b) the reversionary annuity has Rowdy as annuitant and Bash as counter-life

we seek 539605 $\bar{a}_{B|R}$ = 539605 ($\bar{a}_R$ - $\bar{a}_{B:R}$)

using the same methodology as before

$\bar{a}_{B:R} = \frac{1}{0.25 + \ln 1.02} = 3.706413$

$\bar{a}_R = \frac{1}{0.14 + \ln 1.02} = 6.257719$

so the cost of the reversionary annuity is 539605 x (6.257719 – 3.706413)

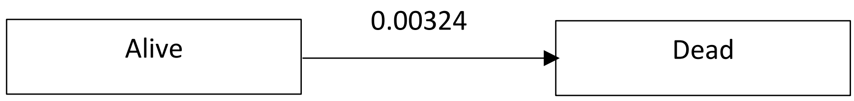
= **£1,376,698**

part a, b similar to seminar

IFoA syllabus 5.1.1, 5.1.2

<u>Q4</u>

(a) a two-state model



(b) a multi-state model

note that transitions from unemployed and ill back to active are not needed for this insurance product although students will not be penalised for including them (although no transition intensities for these are given in the question).

(c) the annuity to value future premiums is of the form

Premium amount x $\int_0^5 v^t \, {}_tp^{\,AA} \, dt$

where ${}_tp^{\,AA}$ is the probability of remaining in the Alive or Active state from time 0 to t

note important here that the upper bound of integral is 5 not ∞

In the 2 state model

$${}_tp^{\,AA} = e^{-0.00324t}$$

In the multi state model

$${}_tp^{\,AA} = e^{-(0.00324+0.00327+0.00892)t} = e^{-0.01543t}$$

As the death transition intensity is the same under both models, the present value of the benefit is the same in each. Therefore the only difference in premiums is the change in annuity value under the premium waiver terms.

if the interest rate is the same under the two models, the increase in premiums under the multi state model is given by the effect of the lower p probability across the bounds of integration

$$100\left(\frac{\exp(-(0.01543)5)-\exp(0)}{\exp(-(0.00324)5)-\exp(0)} - 1\right)\%$$

= 362% higher

part a from lecture, parts b and c unseen and part c challenging and higher order

IFoA syllabus 5.1.3

<u>Q5</u>

(a) we first develop the unit fund

	Year 1	Year 2	Year 3
Fund c/f	0	2054.64	4219.20
Premium	2000	2000	2000
Allocated	1980	1980	1980
Bid Offer	-29.70	-29.70	-29.70
Return	146.27	300.37	462.71
Fee	-41.93	-86.11	-132.64
Fund y/e	2054.64	4219.2	6499.57

and then the projected revenue accounts

	Year 1	Year 2	Year 3
Unallocated	20.00	20.00	20.00
Bid Offer	29.70	29.70	29.70
Expenses	-39.00	-39.00	-39.00
Interest	0.21	0.21	0.21
Fee	41.93	86.11	132.64
Surplus	52.84	97.02	143.55

(b) the original NPV is the year end surplus from the projected revenue accounts discounted at the risk discount rate and allowing for survival

profit vector = (52.84, 97.02, 143.33)

from AM92 Ultimate p_{60} = 1 – 0.008022 = 0.991978 and p_{61} = 1 – 0.009009 = 0.990991

profit signature = (52.84 x 1, 97.02 x 0.991978, 143.33 x 0.991978 x 0.990991)

profit signature = (52.84, 96.24, 141.12)

therefore at i^d = 0.06

NPV = $\frac{52.84}{1.06^1} + \frac{96.24}{1.06^2} + \frac{141.12}{1.06^{\,3}}$ = £253.99

therefore the maximum reduction in NPV is 25.40

as the guaranteed death benefit is in year 1 and will only affect the year 1 projected revenue account surplus – the maximum cost of death benefit for this change in NPV is

$25.40 \times 1.06 = 26.92$

$q_{60} = 0.008022$

Therefore the maximum guaranteed death benefit is

Year 1 end unit value + 26.92/0.008022

= 2054.64 + 3356.27 = £**5410.91**

part a similar seminar, part b uses ideas from seminar but format unseen

IFoA syllabus 6.4, 6.5