Actuarial Mathematics II
MTH5125

Cash flow analysis: profit testing

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Plan

- Based on Chapter 13 in DHW
- Profit Testing
- Profit testing - Reserves
- Profit vector
- Profit signature
- Profit measures

Cash flow analysis

Cash flow analysis for traditional *life insurance* contracts is also called **profit testing**.

Traditional actuarial analysis focuses on determining the EPV of a cash flow series, usually under a constant interest rate assumption.

This emphasis on the EPV was important in an era of manual computation, but with powerful computers available actuaries can do better.

Cash flow analysis uses cash flow projections to model risk

It provides actuaries with a better understanding of:

- liabilities and
- relationship between the liabilities and the corresponding assets.

- First we will consider only those cash flows generated by the policy,
- Then we introduce reserves to complete the cash flow analysis.

The net cash flow for a policy - example

A life office offers a 10-year term insurance issued to a life aged 60. The sum insured, denoted S, is $\$100,000$, payable at the end of the year of death. Level annual premiums, denoted P, of amount $\$1,500$ are payable throughout the term.

Profit testing

We want to analyze the cash flows from this policy at discrete intervals throughout its term.

We use a time interval of one year for this example, taking time 0 to be the moment when the policy is issued.

The purpose of a profit test is to identify the profit which the insurer can claim from the contract at the end of each time period

Profit testing

To do this, the insurer needs to make assumptions (profit test basis) about:

- the expenses which will be incurred,
- the survival model for the policyholder,
- the rate of interest to be earned on cash flows within each time period before the profit is released
- other items such as an assessment of the probability that the policyholder surrenders the policy.

Profit testing

For this example, we use the following profit test basis.

- Interest: 5.5% per year effective on all cash flows.
- Initial expenses: \$400 plus 20% of the first premium.
- Renewal expenses: 3.5% of premiums.
- Survival model: $q_{60+t} = 0.01 + 0.001t$ for $t = 0, 1, ..., 9$.

Profit testing

The initial expenses represent the **acquisition costs** for the policy - paid by the insurer when the policy is issued, at $t = 0$.

For each year that the policy is still in force, cash flows contributing to the surplus emerging at the end of that year are the premium less any renewal expense, interest earned on this amount and the expected cost of a claim at the end of the year.

The calculations of the emerging surplus, are called the net cash flows for the policy

Profit testing

At $t = 0$ the only entry is the total initial expenses for the policy,$\$(400 + 0.2P)$.

- assumed to occur and be paid at time 0, so no interest accrues on them.

For the first policy year ($t = 1$) there is a premium payable at time 0, but no expenses since these are included in the row for $t = 0$.

- interest is earned at 5.5%: $1,500 \times 0.055 = 82.5$
- expected death claims, payable at time 1, are $q_{60}S = 0.01 \times 100,000 = 1,000$.
- Hence the emerging surplus, or net cash flow, at time 1 is $1500 + 82.5 - 1000 = 582.5$.

For subsequent policy years, the net cash flows are calculated assuming the policy is still in force at the start of the year.

Profit testing

$t = 2$

- interest is earned at 5.5%:
  $(1,500 - 0.035 \times 1500) \times 0.055 = 79.61$
- expected death claims, payable at time 1, are
  $q_{60}S = 0.01 \times 100,000 \times 2 = 1,100.$
- Hence the emerging surplus, or net cash flow, at time 1 is
  $1500 + 79.61 - 1,100 = 427.11.$

Profit testing

Time t	Premium at $t-1$	Expenses E_t	Interest	Expected death claims	Surplus emerging at t
0		700.00			−700.00
1	1500	0.00	82.50	1000	582.50
2	1500	52.50	79.61	1100	427.11
3	1500	52.50	79.61	1200	327.11
4	1500	52.50	79.61	1300	227.11
5	1500	52.50	79.61	1400	127.11
6	1500	52.50	79.61	1500	27.11
7	1500	52.50	79.61	1600	−72.89
8	1500	52.50	79.61	1700	−172.89
9	1500	52.50	79.61	1800	−272.89
10	1500	52.50	79.61	1900	−372.89

Profit testing

At $t = 7$:

$$
\begin{aligned}
& 1,500 - 0.035 \times 1,500 \\
& +0.055 \times (1,500 - 0.035 \times 1500) \\
& -100000 \times (0.01 + 6 \times 0.001) \\
= \; & -72.89
\end{aligned}
$$

Reserves

Typical feature of net cash flows: several of the net cash flows in later years are negative.

- level premium is sufficient to pay the renewal expenses and expected death claims in the early years, but,
- with an increasing probability of death, level premiums not sufficient in the later years.

Policy values!

- the insurer needed to set aside assets to cover negative expected future cash flows.

So far we used the term reserves loosely for policy values
Now, the reserve is the actual amount of money held by the insurer to meet future liabilities.

Reserves

The reserve may be equal to the policy value, or may be some different amount.

It should not be less than the policy value, but may be greater than the policy value to allow for uncertainty or adverse experience.

Note that the negative cash flow at time 0 does not require a reserve since it will have been paid as soon as the policy was issued.

Reserves

The amount of the reserves is determined by a process separate from the profit test and is based on a set of assumptions

the reserve basis, which may be different from the profit test basis likely to be more conservative than the profit test basis.

Reserve basis:

- Interest: 4% per year effective on all cash flows.
- Survival model: $q_{60+t} = 0.011 + 0.001t$ for $t = 0, 1, ..., 9$.

Reserves

Then the reserve required at the start of the $(t+1)$th year, i.e. at time t, is:

$$ {}_tV = 100,000A^1_{60+t:\overline{10-t|}} - P^{'}\ddot{a}_{60+t:\overline{10-t|}} $$

with the net premium:

$$ P^{'} = \frac{100,000A^1_{60:\overline{10|}}}{\ddot{a}_{60:\overline{10|}}} = 1447.63 $$

All functions are calculated using the reserve basis

Reserves

We can calculate the following

t	${}_tV$	t	${}_tV$
0	0.00	5	1219.94
1	410.05	6	1193.37
2	740.88	7	1064.74
3	988.90	8	827.76
4	1150.10	9	475.45

- Need to determine the reserves or the amounts that the insurer needs to assign from its assets to support the policy.
- Then we include the cost of assigning these reserves in our profit test

Reserves

- At time t, the cost at the end of the year from $t-1$ to t of setting up this reserve is ${}_tVp_{60+t-1}$
- E_t the renewal expenses and E_0 the initial expenses
- I_t denotes the interest earned in the year from $t-1$ to t: $i\left(P + {}_{t-1}V - E_t\right)$

Reserves

t	${}_{t-1}V$	P	E_t	I_t	$S\,q_{60+t-1}$	${}_tV\,p_{60+t-1}$	Pr_t
0			700.0				−700.00
1	0.00	1500	0.0	82.50	1000	405.59	176.55
2	410.05	1500	52.50	102.17	1100	732.73	126.99
3	740.88	1500	52.50	120.36	1200	977.04	131.70
4	988.90	1500	52.50	134.00	1300	1135.15	135.26
5	1150.10	1500	52.50	142.87	1400	1202.86	137.61
6	1219.94	1500	52.50	146.71	1500	1175.47	138.68
7	1193.37	1500	52.50	145.25	1600	1047.70	138.41
8	1064.74	1500	52.50	138.17	1700	813.69	136.72
9	827.76	1500	52.50	125.14	1800	466.89	133.52
10	475.45	1500	52.50	105.76	1900	0.00	128.71

For example:

$$Pr_7 = P + {}_6V - E_7 + i\left(P + {}_6V - E_7\right) - Sq_{66} - {}_7Vp_{66}$$

Reserves: Profit vector

Profit at t :

$$Pr_t = (1+i)\left(P + {}_{t-1}V - E_t\right) - Sq_{x+t-1} - {}_tVp_{x+t-1}$$

Alternative way of writing the profit:

$$\mathrm{Pr}_t = (1+i)\left(P - E_t\right) + \Delta V_t - Sq_{x+t-1}$$

where $\Delta V_t = (1+i)\ {}_{t-1}V - {}_tVp_{x+t-1}$ is the change in reserve in year t.

The vector $Pr = (Pr_0, ..., Pr_t)'$ is called the profit vector for the contract.

Reserves: Profit signature

Multiplying Pr_t by $(t-1)\, p_x$ gives a vector each of whose elements is the expected profit at the end of each year given only that the contract was in force at age x (in our example 60).
$\Pi_0 = Pr_0$;
$\Pi_t = (t-1)\, p_{60} Pr_t$ for $t = 1, 2, ..., 10$.

The vector: $(\Pi_0, \Pi_1, ..., \Pi_{10})'$ is the profit signature

The profit signature is the key to assessing the profitability of the contract.

Profit measures: Internal Rate of Return

Once we have projected the cash flows, we need to assess whether the emerging profit is adequate.

The internal rate of return (IRR) is the interest rate j such that the present value of the expected cash flows is zero. Given a profit signature $(\Pi_0, \Pi_1, ..., \Pi_n)'$ for n year contract the internal rate of return is j where:

$$\sum_{t=0}^{n} \Pi_t v_j^t = 0$$

- The insurer may set a minimum hurdle rate (or risk discount rate) for the internal rate of return, so that the contract is deemed adequately profitable if the IRR exceeds the hurdle rate.
- Solution of the equation above might not exist.

Profit measures: Net Present Value

We can use the risk discount rate to calculate the expected present value of future profit (EPVFP), also called the net present value (NPV) of the contract. Let r be the risk discount rate.

The **NPV** is the present value, at rate r, of the projected profit signature cash flows, so that:

$$\sum_{t=0}^{n} \Pi_t v_r^t = 0$$

Profit measures: Profit Margin

The **profit margin** is the NPV expressed as a proportion of the EPV of the premiums, evaluated at the risk discount rate.

For a contract with level premiums of P per year payable $m$thly throughout an n year contract issued to a life aged x, the profit margin is

$$Profit\ Margin = \frac{NPV}{P\ddot{a}^{(m)}_{x:\overline{n}|}}$$

For our example the profit margin using a risk discount rate of 10% is $\frac{NPV}{P\ddot{a}_{60:\overline{10}|}} = \frac{124.48}{96884} = 1.29\%$

Profit measures: Discounted Payback Period (DPP)

Discounted payback period (DPP) - the break-even period.

Using the risk discount rate ,r, and is the smallest value of m such that $\sum_{t=0}^{m} \Pi_t v_r^t \geq 0$

The DPP represents the time until the insurer starts to make a profit on the contract.