Actuarial Mathematics II
MTH5125

Pension Models

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Plan

- Types of Pension plans
- The Salary Scale
- Pensions as Multiple State Models
- Pensions as Multiple Decrement Models
- Service Table

Pension Plans

The two major categories of employer sponsored pension plans:
The defined contribution pension plan:

- It specifies how much the employer will contribute, as a percentage of salary, into a plan.
- The employee may also contribute,and the employer's contribution may be related to the employee's contribution (for example, the employer may agree to match the employee's contribution up to some maximum).
- The contributions are accumulated in a notional account, which is available to the employee when he or she leaves the company.
- The contributions may be set to meet a target benefit level, but the actual retirement income may be well below or above the target, depending on the investment experience.

Pension Plans

The defined benefit plan:

- It specifies a level of benefit, usually in relation to salary near retirement (final salary plans), or to salary throughout employment (career average salary plans).
- The contributions, from the employer and, possibly, employee are accumulated to meet the benefit.
- If the investment or demographic experience is adverse, the contributions can be increased;
- If experience is favorable, the contributions may be reduced.
- The pension plan actuary monitors the plan funding on a regular basis to assess whether the contributions need to be changed.

Benefits on retirement due to age

Benefit linked to length of service (number of years) up to retirement age

- normal retirement age (NRA) may be specified
- or might be given as an age range in which retirement on full benefits allowed

Service may be actual years of scheme membership or curtate number of years

Benefits on retirement due to age

Typical pension (per annum from retirement) 1/80th of pensionable salary for each year of pensionable service

- pensionable salary might be annualized salary at retirement - "final salary"
- or average annual salary in period of years (often 3 or 5) before retirement
- or career average salary, the average annualized salary over scheme membership

Additional lump sum benefit on retirement also common (e.g. 3x pension)

Benefits on retirement due to illness

When member needs to retire early due to ill health it is usual for a pension to be payable from that date

- based on service to the date of early retirement and salary at that time
- upon evidence of ill health
- there may be some discretion for trustees to increase benefits

Death in Service

- Lump sum payable on death before retirement for active scheme members
- Often 2x 3x or 4x salary at date of death

Pension Schemes Contributions

Usually contributions are made by both employer and employee - generally expressed as % of salary

- Employee % contributions are usually fixed
- Employer then pays balance of cost which is reviewed every 3 years
- If there is a shortfall of assets compared to liabilities in a pension scheme, the employer may need to make additional lump sum contributions

Pension Benefits on Withdrawal

On leaving the pension scheme prior to retirement, members will be entitled to either:

- A deferred pension payable from retirement age based on service to the date of leaving, or
- A return of employee contributions

Pension Funds

- Increasingly common for pensions to be DC not DB
- Members (and employer) pay contributions into a separately identifiable investment fund and the accumulated value of that fund is used to pay for benefits on retirement (e.g. by purchasing an annuity at that time)
- The pension scheme is then essentially a pooled savings vehicle for members (with tax benefits) with few if any contingent benefits

Replacement Ratio/Rate

The benefit under DB plan and the target under DC plan are set as an replacement ratio or replacement rate:

$$R = \frac{\text{pension income in the year after retirement}}{\text{salary in the year before retirement}}$$

- The target for R depends also on other post-retirement benefit - for example government benefit
- A total replacement ratio including government benefit and other savings of around 70% is often assumed to allow retirees to maintain their pre-retirement lifestyle.

The salary scale function

- The contributions and the benefits are related to salaries, so we need to model progression of salaries through an individual employment.
- Rate of salary function $\{\bar{s}_y\}_{y \geq x_0}$ where x_0 is some suitable initial age.
- For $y > x \geq x_0$ we define $\frac{\bar{s}_y}{\bar{s}_x}$ as the ratio of the annual rate of salary age y to the annual rate of salary at age x
- In the book there are example with $\bar{s}_y = 1.04^{y-20}$

The salary scale

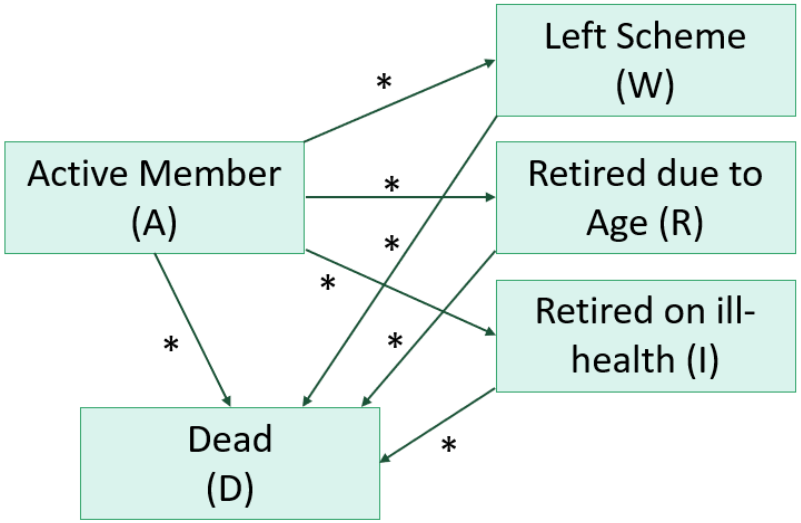
In practice it is common to use a salary scale $\{s_y\}_{y \geq x_0}$ rather than a function

$$\frac{s_y}{s_x} = \frac{\int_0^1 \bar{s}_{y+t}\, dt}{\int_0^1 \bar{s}_{x+t}\, dt}$$

$$\frac{s_y}{s_x} = \frac{\text{salary received in year of age } y \text{ to } y+1}{\text{salary received in year of age } x \text{ to } x+1}$$

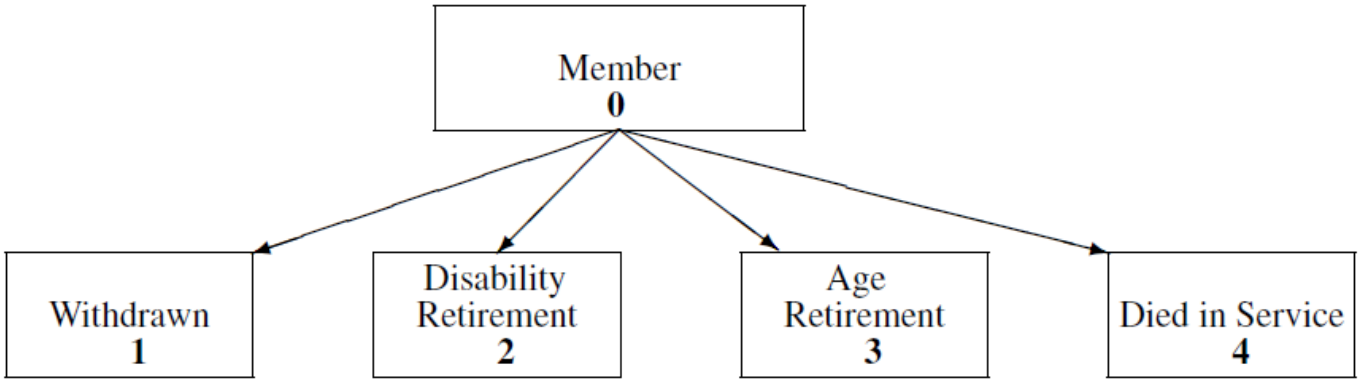
- better/most common used as salaries increase as a result of promotional forces and inflation.

Pension schemes as multiple state models



* transition intensities

Pension schemes as multiple decrement model for active members



Pension schemes: Service tables

Service table is similar to the life table which summarizes a survival model

We start at some minimum integer entry age, x_0, by defining an arbitrary radix, $l_{x0} = 1,000,000$.

Using the multiple decrement model above, we then define for integer ages $x_0 + k$ $(k = 0, 1, ...)$

$$\begin{aligned}
w_{x_0+k} &= l_{x_0}\ {}_kp_{x_0}^{00}\ p_{x_0+k}^{01} \\
i_{x_0+k} &= l_{x_0}\ {}_kp_{x_0}^{00}\ p_{x_0+k}^{02} \\
r_{x_0+k} &= l_{x_0}\ {}_kp_{x_0}^{00}\ p_{x_0+k}^{03} \\
d_{x_0+k} &= l_{x_0}\ {}_kp_{x_0}^{00}\ p_{x_0+k}^{04} \\
l_{x_0+k} &= l_{x_0}\ {}_kp_{x_0}^{00}
\end{aligned}$$

Note that:

$$l_x = l_{x-1} - w_{x-1} - i_{x-1} - r_{x-1} - d_{x-1}$$