Actuarial Mathematics II
MTH5125

Practice Set 4: Joint Life

Dr. Melania Nica

Spring Term

Exercise 1

A simple pension arrangement offers a pension of £9,000 per annum payable monthly in arrears from age 65 plus a widow or widower's pension of 50% of the original pension amount again payable monthly in arrears. This pension is to be funded by level annual-in-advance contributions up to age 65. Robbie is currently age 55 and his wife Sara is exactly five years younger. If the present value of Robbie's current pension savings is £23,000 how much should he now contribute per annum into this pension arrangement assuming 4% per annum interest and mortality using AM92 Ultimate and $\ddot{a}_{65:60} = 8.235$?

Exercise 1

The Equation of Value:

$$23,000 + EPV\ contributions = EPV\ pension + EPV\ widow's\ pension$$

$$EPV\ pension = 9,000 \times v^{10} \times {}_{10}p_{55} \times a_{65}^{(12)}$$

noting that the pension annuity starts in 10 years time so we need 10 year survival probability and v^{10}

From the AM92 table

$\ddot{a}_{65} = 12.276$ so $a_{65} = 12.276 - 1 = 11.276$ and

$a_{65}^{(12)} \approx a_{65} + \frac{11}{24} = 11.73433$

${}_{10}p_{55} = \frac{l_{65}}{l_{55}} = \frac{8821.2612}{9557.8179} = 0.922937$

$v^{10} = (1.04)^{10} = 0.675564$

$$EPV\ pension = 65,847.51$$

Exercise 1

$$\begin{aligned} EPV\ widow's\ pension &= 4,500 \times v^{10} \times {}_{10}p_{55:50} \times a^{(12)}_{65|60} \\ &= 4,500 \times v^{10} \times {}_{10}p_{55:50} \times \left(a^{(12)}_{60} - a^{(12)}_{65:60}\right) \\ &= 15,827.16 \end{aligned}$$

Note:
$a^{(12)}_{60} - a^{(12)}_{65:60} = a_{60} + \frac{11}{24} - a_{65:60} - \frac{11}{24} = a_{60} - a_{65:60} =$
$(\ddot{a}_{60} - 1) - (\ddot{a}_{65:60} - 1) = 14.134 - 8.235$
${}_{10}p_{55:50} = \frac{l_{65}\,l_{60}}{l_{55}\,l_{50}} = 0.882563$

Exercise 1

Let the annual contribution amount be C then

$$\begin{aligned} EPV\ contributions &= C \times \ddot{a}_{55:\overline{10}|} \\ &= 8.218877C \end{aligned}$$

$\ddot{a}_{55:\overline{10}|} = \ddot{a}_{55} - v^{10} \times {}_{10}p_{55} \times \ddot{a}_{65}$
${}_{10}p_{55} = \frac{l65}{l55} = \frac{8821.2612}{9557.8179} = 0.922937$
$\ddot{a}_{55} = 15.873$, $\ddot{a}_{65} = 12.276$

Exercise 1

The Equation of Value becomes:

$$23000 + 8.218877C = 65847.51 + 15827.16$$

$$C = 7,139.01 \ p.a.$$

Exercise 2

Five years go Xander took £23,000 of his savings and bought a life annuity paying a fixed amount annually in arrears. The insurance company that sold this annuity product used AM92 Ultimate mortality at 4% per annum interest, paid sales commission of 1.5% of the total purchase amount and allowed for £50 per annum of renewal expenses.

Now Xander, currently age 65, wishes that he had used the money to buy a reversionary annuity instead with himself as counter life and Yolanda (who is 2 years younger) as annuitant. The insurance company says that they will make the switch by applying the prospective gross premium reserve on the existing annuity policy, calculated on the original premium basis, minus a switching fee of £125 to provide an annually in arrears reversionary annuity which assumes 4% interest, 2% per annum of renewal expenses and no sales commission, using the Pensioner Mortality Table (PM92C20 and PF92C20).Calculate the amount of the reversionary annuity per annum.

First, we need to calculate the amount per annum for the initial annuity to Xander at age 60.
The equation of value:

$$EPV\ annuity\ benefit + EPV\ expenses = 23000$$

Let B be the amount of annuity per annum:

$$(B + 50)a_{60} = 0.985 \times 23000$$

From AM92 Ultimate table, $a_{60} = \ddot{a}_{60} - 1 = 14.134 - 1 = 13.134$ gives

$$B = 1,674.91\ p.a.$$

Exercise 2

We now need the reserve at $t = 5$ for this initial policy:

$$_5V = (1674.91 + 50)a_{65} = (1674.91 + 50)(\ddot{a}_{65} - 1) = 19,450.09$$

From AM92 table: $\ddot{a}_{65} = 12.276$
The amount available to buy the reversionary annuity is
$19,450.09 - 125$ (switching fee)$=$ $\$19325.09$

Exercise 2

The second equation of value for the new reversionary annuity with Xander as counter life and Yolanda as annuitant
The equation of value:

$$EPV\ \ reversionary\ \ annuity + EPV\ \ expenses = 19325.09$$

Let R be the amount of the reversionary annuity per annum, hence:

$$\begin{aligned} 1.02 \times R \times a_{65|63} &= 19325.09 \\ 1.02 \times R \times \left(\underbrace{a_{63}}_{Yolanda} - \underbrace{a_{65:63}}_{Xander:Yolanda} \right) &= 19325.09 \end{aligned}$$

Exercise 2

$$
\begin{aligned}
R &= \frac{19325.09}{1.02 \times (15.606 - 12.282)} \\
&= 5,699.809 \ \ p.a.
\end{aligned}
$$

Exercise 3

Former rock legends Calvin and Star both aged 50 have started to collaborate and their songs and new concerts have brought them good earnings. Star is worried about what will happen if Calvin dies and is willing to pay 15% of this year's earnings for some contingent life assurance. A life assurance company offers Star two options:

A reversionary annuity of £50, 000 per annum payable annually in arrears with Calvin as the counter life.

A contingent whole life assurance of £ 750, 000 payable at the end of the year if Calvin dies before Star.

If they evaluates these options using the pensioner mortality table (PM92) mortality table at 4% interest which option should they choose?

Exercise 3

Star should select the benefit with the higher expected present value
For the contingent assurance:

$$\begin{aligned} EPV\ \ benefit &= 750,000 \times A_{\overset{1}{50:50}} = 750,000\left(\frac{1}{2}A_{50:50}\right) \\ &= 119,884.6 \end{aligned}$$

using a simplifying assumption/approximation: symmetry (same mortality assumption applies to both lives).
Star has access only to pensioner mortality table for (male and female) From PM92 table: $\ddot{a}_{50:50} = 17.688$
$A_{50:50} = 1 - d \times \ddot{a}_{50:50} = 1 - \frac{0.04}{1.04} \times 17.688 = 0.319698$

Exercise 3

For the reversionary annuity:
If you think Star is Male: $\ddot{a}_{50} = 18.843$

$$\begin{aligned} EPV\ benefit &= 50,000 \times a_{50|50} \\ &= 50,000(a_{50} - a_{50:50}) \\ &= 50,000(\ddot{a}_{50} - \ddot{a}_{50:50}) \\ &= 57,750 \end{aligned}$$

$EPV\ contingent\ assurance > EPV\ reversionary\ annuity$
therefore Star should select the contingent assurance
If you think instead that Star is Female: $\ddot{a}_{50} = 19.539$