Actuarial Mathematics II
MTH5125

Mortality Profit and Death Strain

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Plan

- Profit for a policy
- Death Strain
- Expected Death Strain
- Actual Death Strain
- Mortality Profit

Profit for a policy

Profits and losses may arise from any element of the policy value basis.

- If the **interest** earned is greater than that assumed in the reserve, then the income will accumulate to more than the sum required to cover the cost of the benefits and the year-end reserve, giving an interest surplus.
- If the policyholder decides to **surrender** his or her policy (**to cease paying premiums**, and take some lump sum in respect of the future benefits already paid for) then the year-end outgo is not as assumed.
  - If the lump sum is less than the reserve there will be a surrender profit.
  - If no surrender benefit is paid, the profit will be equal to the reserve.

Profit

- If the experienced mortality is heavier than that assumed in the basis, then there will be a profit or loss from mortality, depending on the nature of the contract.
  - Where benefits are paid out on death, such as under a term inssurance, lighter mortality than assumed will give rise to a profit.
  - Where benefits are paid out on survival, such as under an annuity, then lighter mortality will give rise to a loss.

Mortality Profit - death strain

- Consider a policy issued t years ago to a life then aged x , with sum assured S payable at the end of the year of death.
- Also, assume that no survival benefit is due if the life survives to $t+1$.
  - we will extend these ideas to death benefits payable immediately on death, and to survival benefits, later.
- Let ${}_tV$ be the reserve at time t. The death strain at risk in the policy year t to $t+1$:

$$DSAR = \begin{cases} 0 & \text{if life survives to } t+1 \\ S - {}_{t+1}V & \text{if life dies in } [t, t+1) \end{cases}$$

- The maximum $S - {}_{t+1}V$ is the death strain at risk.
- The word strain is used loosely to mean a cost to the company.

Recursive formulae for policy values: death strain

$$\left({}_tV + P_t - e_t\right)(1 + i_t) = q_{x+t}\left(S + E_{t+1}\right) + p_{x+t}\ {}_{t+1}V$$

Available assets at $t+1$ = Required assets at $t+1$

Sum at Risk at time $t+1$ (Death Strain at Risk, Net Amount at Risk):

$$D_{t+1} \stackrel{\text{not}}{=} DS = S + E_{t+1} - {}_{t+1}V$$

The recursion formula becomes:

$$\left({}_tV + P_t - e_t\right)(1 + i_t) = {}_{t+1}V + q_{x+t}\ D_{t+1}$$

Recursive formulae for policy values: death strain

Special case: no expenses: $e_t = 0$ and $E_t = 0$

$$DS = S - {}_{t+1}V$$

DS is a random variable and:

$$\left({}_tV + P_t\right)\left(1 + i_t\right) = {}_{t+1}V + q_{x+t}\ D_{t+1}$$

Expected death strain

The expected amount of the death strain is called the expected death strain (EDS). This is the amount that the life insurance company expects to pay in addition to the year-end reserve for the policy. The probability of claiming in the policy year t to $t+1$ is q_{x+t} so that:

$$EDS = q_{x+t}\left(S - {}_{t+1}V\right)$$

Actual death strain

The actual death strain is simply the **observed** value at $t+1$ of the death strain random variable, that is:

$$ADS = \begin{cases} 0 & \text{if life survives to } t+1 \\ S - {}_{t+1}V & \text{if life dies in } [t, t+1) \end{cases}$$

The Mortality Profit

- The **Mortality Profit** is **Expected Death Strain – Actual Death Strain**
- The EDS is the amount the company expects to pay out, in addition to the year-end reserve for a policy.
- The ADS is the amount it actually pays out, in addition to the year-end reserve.
- If it actually pays out less than it expected to pay, there will be a profit.
- If the actual strain is greater than the expected strain, there will be a loss.

The Mortality Profit on a portfolio of policies

- We can consider the emergence of profits for a block of policies by comparing actual experience to expected experience.
- If all lives are the same age, and subject to the same mortality table:

$$Total\ DSAR = \sum_{all\ \ policies} (S - {}_{t+1}V)$$

$$\begin{aligned} Total\ EDS &= \sum_{all\ \ policies} \left[q_{x+t} (S - {}_{t+1}V) \right] \\ &= q_{x+t} \sum_{all\ \ policies} (S - {}_{t+1}V) \\ &= q_{x+t} DSAR \end{aligned}$$

$$Total\ ADS = \sum_{all\ \ claims} (S - {}_{t+1}V)$$

The Mortality Profit on a portfolio of policies

If the policies are identical, then:

- *Total EDS* = expected number of deaths $\times$ *DSAR*
- *Total ADS* = actual number of deaths $\times$ *DSAR*

In many situations the DSAR of each individual policy is not known, but the total DSAR is simply the total sum assured less the total year-end reserve, and the total EDS is $q_{x+t} \times DSAR$

The Mortality Profit: death benefits payable immediately

Where death benefits are payable immediately on death, in the calculation of the death strain we allow for interest between the time of payment and the end of the year of death. In this case, the death strain becomes:

$$DSAR = \begin{cases} 0 & \text{if life survives to } t+1 \\ S\,(1+i)^{1/2} - {}_{t+1}V & \text{if life dies in } [t, t+1) \end{cases}$$

The above formula assumes that death occurs half way through the year, on average.

The Mortality Profit: with survival benefits

- Suppose the contract provides for a benefit at the end of policy year t to $t+1$.
- By convention, the expected present value of this will have been included in ${}_{t}V$ but will fall outside the computation of ${}_{t+1}V$.
- So, the survival benefit needs to be allowed for as an additional payment.
- Let R be the benefit payable at the end of the policy year t to $t+1$ contingent on the survival of the policyholder.
- Assuming death benefits are paid at the end of the year of death, the recursive relationship between successive reserves is now:

$$\left({}_{t}V + P_t\right)\left(1 + i_t\right) = q_{x+t}\left(S + R\right) + p_{x+t}\ {}_{t+1}V$$

or

$$\left({}_{t}V + P_t\right)\left(1 + i_t\right) = {}_{t+1}V + q_{x+t}\left(S - {}_{t+1}V + R\right)$$

The Mortality Profit: with survival benefits

$$DSAR = \begin{cases} 0 & \text{if life survives to } t+1 \\ S - (R + {}_{t+1}V) & \text{if life dies in } [t, t+1) \end{cases}$$

$$EDS = q_{x+t}\left(S - (R + {}_{t+1}V)\right)$$

Mortality profit is still $EDS - ADS$

The Mortality Profit: with annuity benefit

- In the case of an annuity of R per annum, payable annually in arrears, with no death benefit, the DSAR is $-\left(R + {}_{t+1}V\right)$.
- Each death causes a negative strain or release of reserves.
- If the annuity benefit is paid annually in advance then DSAR is $-\ {}_{t+1}V$