MTH6105 – Algorithmic Graph Theory **Spring 2024**

Assessed Coursework 1 **F. Fischer**

This assessment consists of three exercises, which carry equal weight and together contribute **10% of your mark for the module**. Please **upload your answers before the deadline**.

Any work you submit must be your own. You may discuss the exercises with other students, but you must write up your solution yourself. Copying a solution or submitting someone else's solution constitutes an assessment offence.

---

1. Let the *degree sequence* of a graph G be the sequence of length $|V(G)|$ that contains the degrees of the vertices of G in non-increasing order.

    (a) For each of the following sequences, either draw a simple graph whose degree sequence is equal to that sequence, or explain why such a graph does not exist: (i) $(4, 4, 4, 2, 2)$, (ii) $(4, 2, 2, 1, 1)$, (iii) $(3, 3, 3, 2, 1)$, (iv) $(4, 3, 3, 2, 1)$, (v) $(2, 2, 2, 1, 1)$.

    (b) Consider a simple graph with 9 vertices, such that the degree of each vertex is either 5 or 6. Prove that there are at least 5 vertices of degree 6 or at least 6 vertices of degree 5.

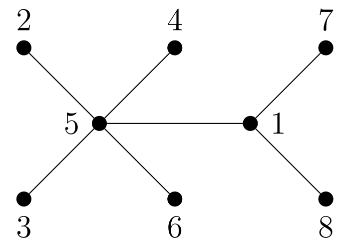
2. Let G be a graph and $e \in E(G)$. Let H be the graph with $V(H) = V(G)$ and $E(H) = E(G) \setminus \{e\}$. Then e is a *bridge* of G if H has a greater number of connected components than G.

    (a) Let G be the simple graph with $V(G) = \{u, v, w, x, y, z\}$ and $E(G) = \{uy, vx, vz, wx, xz\}$. For each $e \in E(G)$, state whether e is a bridge of G. Justify your answer.

    (b) Assume that G is connected and that e is a bridge of G with endpoints u and v. Show that H has exactly two connected components H_1 and H_2 with $u \in V(H_1)$ and $v \in V(H_2)$. To this end, you may want to consider an arbitrary vertex $w \in V(G)$ and use a u–w-path in G to construct a u–w-path or a v–w-path in H.

    (c) Show that e is a bridge of G if and only if it is *not* contained in a cycle of G.

3. (a) Determine the Prüfer code of the following tree.



    (b) Draw the trees with Prüfer codes (i) $(3, 2, 3, 2, 3)$, (ii) $(5, 5, 5, 5, 5)$, and (iii) $(6, 5, 4, 3, 2)$.