Welcome to MTH6157 *Survival Models*

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Module introduction

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Office hours: Monday 2.00 – 3.00
Thursday 2.00 – 3.00

or please email to agree a time to meet on Teams

Module summary and objectives

Mortality and Survival are important variables in the management of insurance companies and in various healthcare and public policy settings

- Actuaries need to consider a number of ways of modelling survival
- These have included adapting models used elsewhere in statistics and specific actuarial methods
- We also need methods for testing these models and analysing published mortality data

Module summary and objectives

In this module we will learn about

- Building, using and comparing different mathematical models for survival
- Understanding the distributions, parameters and measures used in different methods
- Models in common use and the assumptions they are based on
- Evidence based decision-making
- Applying analytical skills to an unfamiliar problem

Teaching

Everything you will need for this module will be included in the timetabled sessions

4 hours per week

Monday 11 – 1 in Maths MB203

Thursday 11 – 1 in Engineering 216

Question about 2-hour classes

Teaching

These will be a mixture :

- teaching new material (lectures)
- discussion about real-world applications
- implementing the models in R
- going through problem questions

Assessment

Assessment for this module

70% Exam in January 2024

- **2-hour, handwritten exam on campus**
- **allowed up to 3 pages of notes**
- **lots of guidance to be given in week 12**

30% assessed coursework set in 2 parts

- **first part set in week 3 due in week 4 (modelling using R)**
- **2nd part set in week 8 due in week 9 (analysis using R)**

Institute and Faculty of Actuaries examinations (for Actuarial students only)

This module is one of five modules that combine to cover the material for the IFoA's CS2 exam:

- Survival Models (25%)
- Random Processes (25%)
- Time Series (20%) [Semester B]
- Statistics for Insurance (20%) [already completed]
- Intro to Machine Learning (10%)

For full details of how IFoA exemptions work see the page on QM Plus:

https://qmplus.qmul.ac.uk/mod/page/view.php?id=597978

Resources

The most important resources (and everything you will need to complete the module):

1. Your own lecture notes
2. Course materials posted on QM Plus
3. Practice questions and coursework
4. Additional online resources signposted
5. Independent study

Books

If you like to look at other resources to see how different people explain things try:

- **Macdonald, A.S., Richards, S.J. & Currie, I.D. (2018) *Modelling Mortality with Actuarial Applications*, Cambridge University Press (ISBN 9781107051386)**
- Kalbfleish, J.D. & Prentice, R.L. (2002) *The Statistical Analysis of Failure Time Data* (2nd ed.), Wiley
- Elandt-Johnson, R.C. & Johnson, N.L. (1999) *Survival Models and Data Analysis,* Wiley

There is one important BAJ paper on Survival Models:

- Macdonald, A. S. (1996) "An actuarial survey of statistical models for decrement and transition data" *British Actuarial Journal* vol.2, pp.129–155; 429–448; 703–726 [note paper is in 3 parts]

Topics in this *Survival Models* module

1. Mortality and Survival model concepts
2. Censoring and the Kaplan Meier Estimate
3. Proportional Hazard Models
4. Markov Processes and Multi State Models
5. Binomial and Poisson models
6. Exposed to risk and Census methods
7. Graduation – tests and methods
8. Mortality Projections

3 objectives for our lectures and tutorials

- Learn the mathematics of Survival Models
- Cover material in IFoA examination syllabus
- Find real-world applications to talk about

Survival and Mortality: What are we modelling and why?

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Why might we want to study survival and mortality?

- Life Assurance Actuary
- Medical Statistician
- Government Policy Advisor
- Climate Scientist

This week we will cover:

1. Actuarial notation (Actuarial Maths refresher)
2. General pattern of mortality
3. Factors affecting mortality
4. The actuarial concept of Selection
5. Life Assurance financial considerations

Mortality & survival rates and Life tables

actuarial mortality and survival notation

p_x

q_x

$_tp_x$

μ_{x+t}

l_x

d_x

Force of mortality μ_{x+t}

If T is future lifetime of a life age x then the force of mortality is defined as

$$\mu_x = \lim_{h \to 0} \frac{1}{h} P[T \leq x+h \mid T > x]$$

for very small h, ${}_hq_x \approx h.\mu_x$

As we go through this module, the force of mortality will be the most important of the actuarial quantities. This is because it is most often the best one to model when estimating survival.

Important relationships

$$ {}_tq_x = \int_0^t {}_sp_x \, \mu_{x+s} \, ds $$

$$ {}_tp_x = \exp\left(-\int_0^t \mu_{x+s} \, ds \right) $$

If we are going to model μ_x we need these relationships to find the survival and mortality probabilities that are generally used in actuarial calculations

We will derive these formulae properly in the next couple of lectures

General pattern of mortality

l_x

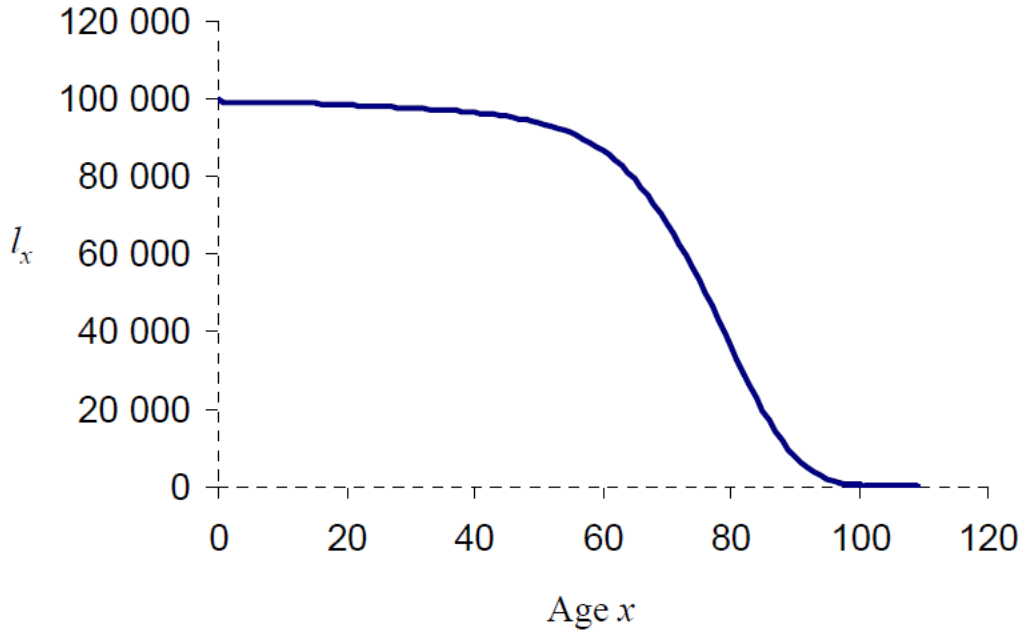

Figure 4

l_x (ELT15 (Males) Mortality Table)

d_x

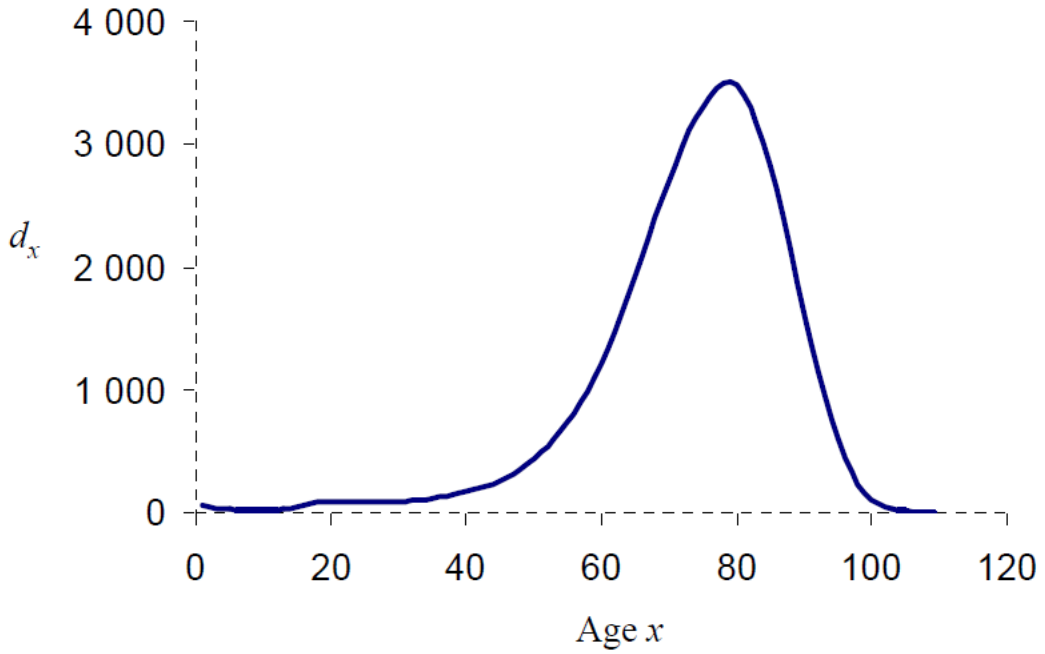

Figure 5

d_x (ELT15 (Males) Mortality Table)

q_x

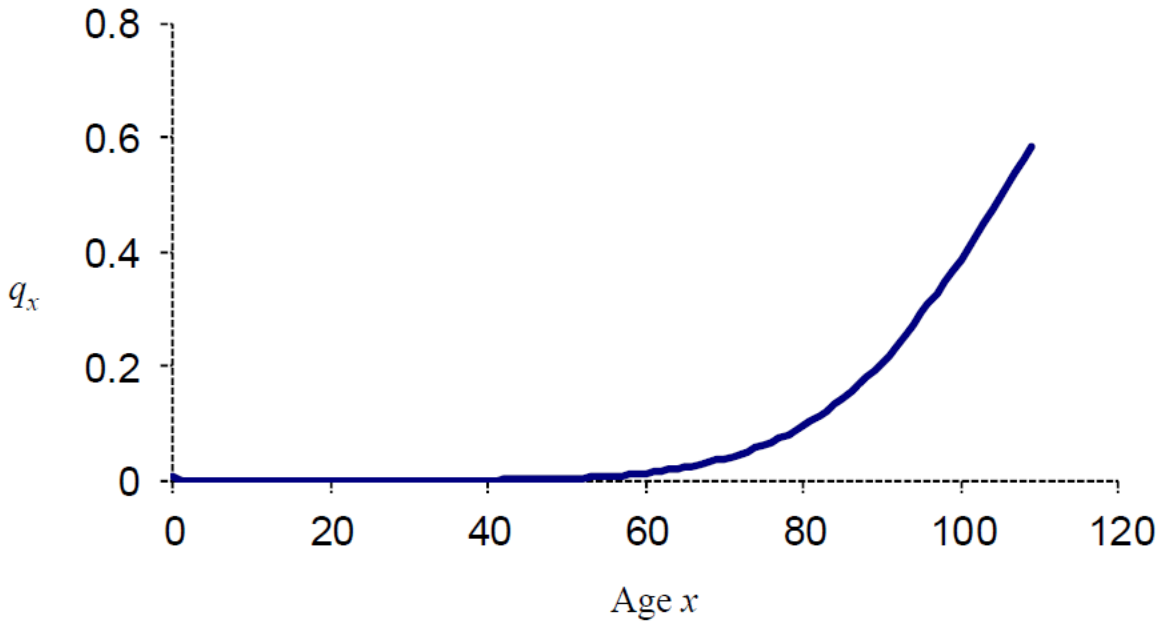

Figure 2

q_x (ELT15 (Males) Mortality Table)

q_x [log scale]

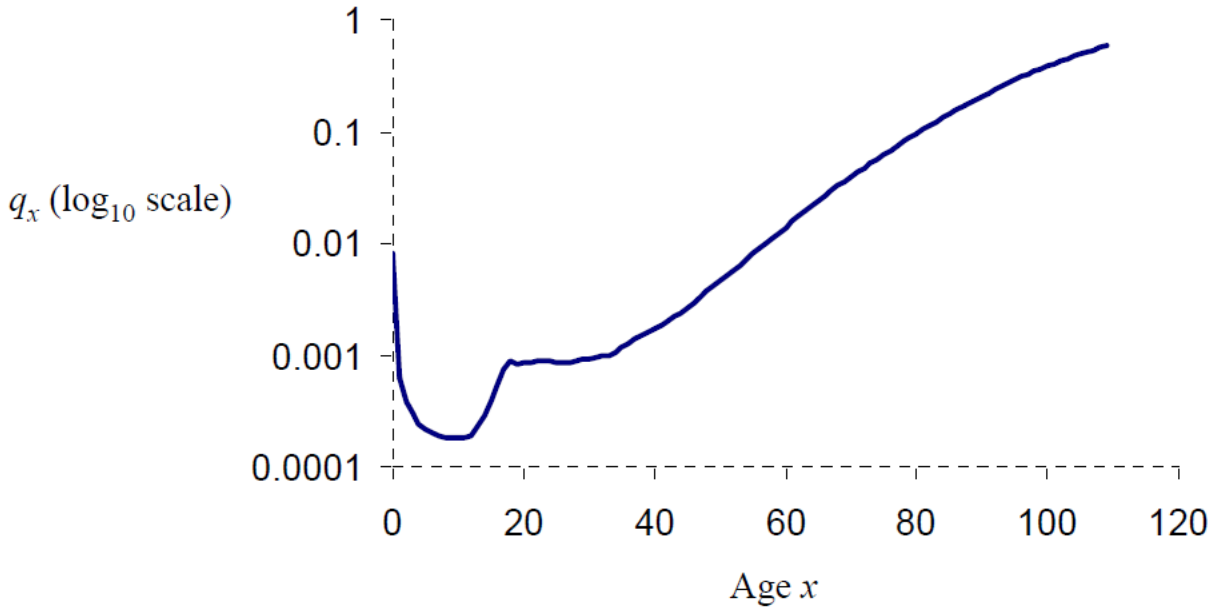

Figure 3

q_x on $\log_{10}$ scale (ELT15 (Males) Mortality Table)