Queen Mary University of London

Main Examination period 2020 – May/June – Semester B
Online Alternative Assessments

MTH5126: Statistics for Insurance

You should attempt ALL questions. Marks available are shown next to the questions.

Statistical tables
You may use any of the following: your own copy of statistical tables, online versions from the internet or appendices provided at the end of this assessment paper.

In completing this assessment, you may use books, notes, and the internet. You may use calculators and computers, but you should show your working for any calculations you do. You must not seek or obtain help from anyone else.

At the start of your work, please **copy out and sign** the following declaration:

> I declare that my submission is entirely my own, and I have not sought or obtained help from anyone else.

All work should be **handwritten**, and should **include your student number**.

You have **24 hours** in which to complete and submit this assessment. When you have finished your work:

- scan your work, convert it to a **single PDF file** and upload this using the upload tool on the QMplus page for the module;
- e-mail a copy to maths@qmul.ac.uk with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are not expected to spend a long time working on this assessment. We expect you to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the assessment period, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions
This module counts towards IFoA actuarial exemptions. For your submission to be eligible for IFoA exemptions, you must submit within the first **3 hours** of the assessment period. You may then submit a second version later in the assessment period if you wish, which will count only towards your degree. There are two separate upload tools on the QMplus page to enable you to submit a second version of your work.

Examiners: G. Ng, S. Liverani

Question 1 [14 marks].
There are only two companies operating in the space tourism industry: Helios plc and Jupiter plc. They share profits of £1 billion each year. Every year the companies need to decide between two business approaches: cautious and aggressive.

If both companies adopt the same approach in a given year, Helios captures $x\%$ of the total profits, where $x = 70 +$ the last digit of your student ID number. So for example:

If the last digit of your student ID number is 0, then $x = 70$;
If the last digit of your student ID number is 1, then $x = 71$;
...
If the last digit of your student ID number is 9, then $x = 79$.

If they adopt different approaches, then Jupiter captures 80% of the total profits if Helios is cautious and 60% of the total profits if Helios is aggressive. Neither company knows what the other company's approach will be before adopting its own approach.

(a) Explain why the above can be thought of as a zero-sum two-person game. **[2]**

(b) Helios plc decides to adopt a randomised strategy to setting its approach each year. Determine the optimal randomised strategy for Helios plc. **[8]**

(c) Under the strategy in (b), what is the expected profit for Helios plc? **[4]**

(a) Because total profits are fixed, whatever one company makes the other can be thought of as having lost, and vice-versa. [2]

(b) For x=70,
The matrix below shows the profits (in £m) for Helios plc. Alternatively, students may work in % instead of in monetary amounts. Other versions of the matrix are possible and will be accepted as long as they make sense.

Helios

		Cautious	**Aggressive**
Jupiter	**Cautious**	700	400
	Aggressive	200	700

[3]

1 mark for the entry on top right, 1 mark for the entry on bottom left, 1 mark for remaining entries

Suppose Helios randomly chooses the cautious approach with probability p, and the aggressive approach with probability $1-p$. In order to determine the optimal strategy, we need to equate Helios' payoffs when Jupiter is Cautious with Helios' payoffs when Jupiter is Aggressive:

$$700p+400(1-p) = 200p+700(1-p)$$

[2]

Solving for p,

$$300p+400 = 700-500p$$
$$\implies 800p = 300$$
$$\implies p = 3/8$$

[2]

So the optimal randomised strategy for Helios plc is to adopt the Cautious approach 3/8 of the time and the Aggressive approach 5/8 of the time. **[1]**

For $x = 71$,
The matrix remains the same except that entries 700 become 710.

$$710p+400(1-p) = 200p+710(1-p)$$
$$\implies 310p+400 = 710-510p$$
$$\implies 820p = 310$$
$$\implies p = 31/82$$

For $x = 72$,
The matrix remains the same except that entries 700 become 720.

$$\begin{aligned} 720p + 400(1-p) &= 200p + 720(1-p) \\ \implies 320p + 400 &= 720 - 520p \\ \implies 840p &= 320 \\ \implies p &= 32/84 = 8/21 \end{aligned}$$

For $x = 73$,
The matrix remains the same except that entries 700 become 730.

$$\begin{aligned} 730p + 400(1-p) &= 200p + 730(1-p) \\ \implies 330p + 400 &= 730 - 530p \\ \implies 860p &= 330 \\ \implies p &= 33/86 \end{aligned}$$

For $x = 74$,
The matrix remains the same except that entries 700 become 740.

$$\begin{aligned} 740p + 400(1-p) &= 200p + 740(1-p) \\ \implies 340p + 400 &= 740 - 540p \\ \implies 880p &= 340 \\ \implies p &= 34/88 = 17/44 \end{aligned}$$

For $x = 75$,
The matrix remains the same except that entries 700 become 750.

$$\begin{aligned} 750p + 400(1-p) &= 200p + 750(1-p) \\ \implies 350p + 400 &= 750 - 550p \\ \implies 900p &= 350 \\ \implies p &= 35/90 = 7/18 \end{aligned}$$

For $x = 76$,
The matrix remains the same except that entries 700 become 760.

$$\begin{aligned} 760p + 400(1-p) &= 200p + 760(1-p) \\ \implies 360p + 400 &= 760 - 560p \\ \implies 920p &= 360 \\ \implies p &= 36/92 = 9/23 \end{aligned}$$

For $x = 77$,

Continue to next page

The matrix remains the same except that entries 700 become 770.

$$770p + 400(1–p) = 200p + 770(1 - p)$$
$$\implies 370p + 400 = 770 - 570p$$
$$\implies 940p = 370$$
$$\implies p = 37/94$$

For x = 78,
The matrix remains the same except that entries 700 become 780.

$$780p + 400(1–p) = 200p + 780(1 - p)$$
$$\implies 380p + 400 = 780 - 580p$$
$$\implies 960p = 380$$
$$\implies p = 38/96 = 19/48$$

For x = 79,
The matrix remains the same except that entries 700 become 790.

$$790p + 400(1–p) = 200p + 790(1 - p)$$
$$\implies 390p + 400 = 790 - 590p$$
$$\implies 980p = 390$$
$$\implies p = 39/98$$

(c) For x=70,
If working with the matrix which shows profits for Helios,

$$700(3/8) + 400(5/8) = 512.5$$

[3]

If working with other versions of the matrix, students need to correctly interpret the matrix to arrive at the answer.

The expected profits for Helios plc is £512.5m.
one mark for correct monetary units, i.e., £m. **[1]**

For x=71,

$$710(31/82) + 400(51/82) = 517.195122$$

The expected profits for Helios plc is £517,195,122.

For x=72,

$$720(32/84) + 400(52/84) = 521.904762$$

The expected profits for Helios plc is £521,904,762.

For x=73,

$$730(33/86) + 400(53/86) = 526.627907$$

The expected profits for Helios plc is £526,627,907.

For x=74,

$$740(34/88) + 400(54/88) = 531.363636$$

The expected profits for Helios plc is £531,363,636.

For x=75,

$$750(35/90) + 400(55/90) = 536.111111$$

The expected profits for Helios plc is £536,111,111.

For x=76,

$$760(36/92) + 400(56/92) = 540.869565$$

The expected profits for Helios plc is £540,869,565.

For x=77,

$$770(37/94) + 400(57/94) = 545.638298$$

The expected profits for Helios plc is £545,638,298.

For x=78,

$$780(38/96) + 400(58/96) = 550.416667$$

The expected profits for Helios plc is £550,416,667.

For x=79,

$$790(39/98) + 400(59/98) = 555.204082$$

The expected profits for Helios plc is £555,204,082.

Question 2 [11 marks].

Suppose we have an exponentially distributed sampling distribution so that $X|\theta \sim exp(\theta)$. A random sample $x_1, x_2, \ldots, x_n$ is taken from this distribution.

(a) Write down the likelihood function for this sample. **[4]**

(b) The prior distribution for θ is $Gamma(\alpha, \beta)$ and the probability density function corresponding to this prior distribution for θ is

$$f(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta} \qquad \text{for } \theta > 0$$

Find the posterior distribution for θ. **[7]**

(a)

$$f(\underline{x}|\theta) = \prod_{i=1}^{n} \theta e^{-\theta x_i}$$

[2]

$$= \theta^n e^{-\theta \sum_{i=1}^{n} x_i}$$

[2]

(b) With our sample data, $\underline{x}$, the posterior distribution is:

$$f(\theta|\underline{x}) \propto f(\theta) f(\underline{x}|\theta)$$

[2]

$$\propto \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}\theta^n e^{-\theta \sum_{i=1}^{n} x_i}$$

[1]

$$\propto \theta^{\alpha+n-1}e^{-(\beta+n\bar{x})\theta}$$

[1]

This has the form of a gamma density. **[1]**

$$\theta|\underline{x} \sim Ga(\alpha+n, \beta+n\bar{x})$$

[2]

1 mark each for the two parameters of the Gamma distribution

Question 3 [13 marks].
A random sample $x_1, x_2, \ldots, x_n$ is taken from a distribution having the density function:

$$f(x) = \frac{k}{5} x^{-\frac{4}{5}} exp(-kx^{\frac{1}{5}}), \; x > 0.$$

(a) Find $L(k)$, the likelihood function for this sample. **[4]**

(b) A sample of n =20 values was collected and the following summary statistic was computed.

$$\sum_{i=1}^{20} x_i^{\frac{1}{5}} = 102.778$$

Determine the maximum likelihood estimate of k. **[9]**

(a)

$$L(k) = \prod_{i=1}^{n} \frac{k}{5} x_i^{-\frac{4}{5}} e^{-kx_i^{\frac{1}{5}}}$$

[2]

$$= k^n e^{-k\sum x_i^{\frac{1}{5}}} \times constant$$

[2]

(b) So the log-likelihood is:

$$logL = nlogk - k\sum x_i^{\frac{1}{5}} + constant$$

[3]

Differentiating with respect to k:

$$\frac{\partial}{\partial k} logL = \frac{n}{k} - \sum x_i^{\frac{1}{5}}$$

[2]

Equating this to zero and using information given in the question for n and $\sum x_i^{\frac{1}{5}}$, we find that:

$$\hat{k} = \frac{n}{\sum x_i^{\frac{1}{5}}} = \frac{20}{102.778} = 0.1946$$

[2]

Differentiating again:

$$\frac{\partial^2}{\partial k^2} logL = -\frac{n}{k^2} < 0$$

[1]

So $\hat{k} = 0.1946$ is the maximum likelihood estimate of k. **[1]**

Question 4 [17 marks].
The table below shows annual aggregate claim statistics for four risks over six years.

Annual aggregate claims for risk i, in year j are denoted by X_{ij}.

Risk, i	$\bar{X}_i = \frac{1}{6}\sum_{j=1}^{6} X_{ij}$	$s_i^2 = \frac{1}{5}\sum_{j=1}^{6}\left(X_{ij} - \bar{X}_i\right)^2$
1	46.8	1227.4
2	30.2	1161.4
3	74.5	1340.3
4	60.7	1414.7

(a) For the data above, calculate the value of Z, the credibility factor under the assumptions of Empirical Bayes Credibility Theory (EBCT) Model 1. [**9**]

(b) Comment on why the credibility factor is relatively low in this case. [**4**]

(c) Using your answer from part (a), calculate the credibility premium for each risk above. [**4**]

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(a) $E[s^2(\theta)]$ is estimated by the average of the sample variances:

$$E[s^2(\theta)] = \frac{1227.4 + 1161.4 + 1340.3 + 1414.7}{4} = 1285.95$$

[2]

The overall mean, $\bar{X}$, is estimated by the average of the sample means:

$$\bar{X} = \frac{46.8 + 30.2 + 74.5 + 60.7}{4} = 53.05$$

[1]

$$\begin{aligned} var[m(\theta)] &= \frac{1}{(N-1)}\sum_{i=1}^{N}(\bar{X}_i - \bar{X})^2 - \frac{1}{Nn}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2 \\ &= \frac{1}{4-1}\sum(X_i - \bar{X})^2 - \frac{1}{6}E(s^2(\theta)) \\ &= \frac{1}{3}\left[(46.8-53.05)^2 + (30.2-53.05)^2 + (74.5-53.05)^2 + (60.7-53.05)^2\right] \\ -\frac{1285.95}{6} \\ &= 359.94 - 214.33 \\ &= 145.6 \end{aligned}$$

[4]

The credibility factor is

$$Z = \frac{n}{n + \frac{E(s^2(\theta))}{var(m(\theta))}}$$

[1]

$$= \frac{6}{6 + \frac{1285.95}{145.6}} = 0.4045$$

[1]

(b) The variation within risks, measured by $E[s^2(\theta)]$, is much bigger relative to the variation between risks, measured by $var[m(\theta)]$.

So we put more weight on the information provided by the data set as a whole, and less on individual risks, resulting in a low credibility factor.

Any other sensible comment award two marks to a max of four. **[4]**

(c) The credibility premia are:
For risk 1: 0.4045*46.8 + 0.5955*53.05 = 50.5 **[1]**
For risk 2: 0.4045*30.2 + 0.5955*53.05 = 43.8 **[1]**
For risk 3: 0.4045*74.5 + 0.5955*53.05 = 61.7 **[1]**
For risk 4: 0.4045*60.7 + 0.5955*53.05 = 56.1 **[1]**

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Question 5 [22 marks].
Individual claim amounts from a portfolio of general insurance policies have a uniform distribution over the range (0, 200). Excess of loss reinsurance is arranged so that the expected amount paid by the insurer on any claim is £50.

(a) Show that the retention limit, M, in the reinsurance arrangement above is £58.58. **[11]**

(b) Aggregate annual claims from this same portfolio have a compound Poisson distribution with Poisson parameter 20. Calculate the variance of the aggregate annual claims paid by the insurer. You may quote without proof the result for the variance of a compound Poisson distribution with Poisson parameter λ. **[11]**

(a) The expected amount paid by the insurer on any claim is:

$$E(Y) = \int_0^M x\frac{1}{200}dx + \int_M^{200} M\frac{1}{200}dx = 50$$

[5]

2 marks for the first integration, 2 marks for the second integration, 1 mark for equating them to 50.

Solving this gives:

$$\left[\frac{x^2}{400}\right]_0^M + \left[\frac{Mx}{200}\right]_M^{200} = 50$$

[2]

$$\implies \frac{M^2}{400} + M - \frac{M^2}{200} = 50$$

$$\implies M^2 - 400M + 20,000 = 0$$

$$\implies M = 58.579 \text{ or } 341.42$$

[3]

Since claims are a maximum of £200, it must be the first value of M. **[1]**

(b) The variance of the aggregate annual claims, S, paid by the insurer is given by:

$$var(S) = \lambda E(Y^2) = 20E(Y^2)$$

[2]

where:

$$E(Y^2) = \int_0^M x^2 \frac{1}{200} dx + \int_M^{200} M^2 \frac{1}{200} dx$$

[4]

$$= \left[\frac{x^3}{600}\right]_0^M + \left[\frac{M^2 x}{200}\right]_M^{200}$$

[2]

$$= \frac{M^3}{600} + M^2 - \frac{M^3}{200} = 2,761.42$$

[2]

Hence:

$$var(S) = 20 \times 2,761.42 = 55,228.47$$

[1]

To avoid the issue of rounding brought over from earlier parts of this question, consider accepting students' answer rounded to the nearest whole number for this last part of the question.

Question 6 [13 marks].
Peace-Of-Mind Insurance Ltd assumes that individual claims arising during each year from a particular type of annual insurance policy follow a normal distribution. Claims are assumed to arise independently and this insurer assesses its solvency position at the end of each year.

Peace-Of-Mind has an initial surplus of £100,000 and expects to sell 100 policies at the beginning of the coming year in respect of identical risks. The insurer incurs expenses of $0.2P$ at the time of writing each policy, where P is the annual premium for an individual policy. In the coming year, it is expected that each policy will be sold for an annual premium of £5,000.

In your calculations in this question, you can ignore interest.

(a) Show that:

$$U(1) = 500,000 - S(1)$$

where
$U(1)$ is the insurer's surplus at the end of the coming year, and
$S(1)$ is the aggregate claims at the end of the coming year. **[5]**

(b) The distribution of $S(1)$ is given as $N(\mu, \sigma^2)$, where $\mu = 350,000$ and $\sigma = 100,000$. Hence, calculate the probability that Peace-Of-Mind will prove to be insolvent at the end of the coming year. **[8]**

(a) Using the information given in the question we can see that the insurer's surplus at the end of the coming year will be:

$$U(1) = \text{initial surplus} + \text{ premiums} - \text{expenses} - \text{claims}$$

$$= 100,000 + 100 \times 5,000 - 100 \times 0.2 \times 5,000 - S(1)$$

[4]

$$= 500,000 - S(1)$$

[1]

(b) Students need to note that being insolvent is when surplus is negative:

$$P(U(1) < 0)$$

[2]

$$= P(S(1) > 500,000)$$

[2]

$$= P(\frac{S(1) - 350,000}{100,000} > \frac{500,000 - 350,000}{100,000})$$

[1]

$$= P(N(0,1) > 1.5)$$

[1]

$$= 1 - \Phi(1.5)$$

[1]

$$= 1 - 0.9332 = 0.067$$

[1]

So the required probability is 6.7%.

Question 7 [10 marks].
A speed camera used on a London motorway incorrectly gives a positive reading for drivers who are not over the legal limit one time in one hundred and an incorrect negative reading for drivers who are over the limit one time in ten. A positive reading means the driver will receive a ticket for speeding.

If one driver in ten is actually over the limit on a particular day, what is the probability that a driver who receives a ticket for speeding is in fact over the legal limit? [**10**]

The question involves the following events:
D: The driver is speeding.
S: The driver is not speeding.
F: The driver fails the speed test.
P: The driver passes the speed test.

[2]

We are told that:
$P(F|S) = 0.01$ **[1]**
$P(P|D) = 0.1$ **[1]**
and
$P(D) = 0.1$ **[1]**
The probabilities for the complementary events are:

$P(P|S) = 0.99$
$P(F|D) = 0.9$
and
$P(S) = 0.9$ **[1]**

Using Bayes' formula, the probability that a driver who receives a ticket is in fact over the legal limit is:
$P(D|F) = \frac{P(F|D)P(D)}{P(F|D)P(D)+P(F|S)P(S)} = \frac{0.9\times0.1}{0.9\times0.1+0.01\times0.9} = 0.9090$ **[4]**

End of Paper – An appendix of 3 pages follows.

Statistics – Common Distributions

Discrete Distributions

Distribution	Density	Range of Variates	Mean	Variance
Uniform	$\frac{1}{N}$	$N = 1,2,\ldots$ $x = 1,2,\ldots,N$	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$
Bernoulli	$p^x(1-p)^{1-x}$	$0 \le p \le 1, x = 0,1$	p	$p(1-p)$
Binomial	$\binom{n}{x} p^x(1-p)^{n-x}$	$0 \le p \le 1, n = 1,2,\ldots$ $x = 0,1,\ldots n$	np	$np(1-p)$
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$	$\lambda > 0, x = 0,1,2,\ldots$	λ	λ
Geometric	$p(1-p)^x$	$0 < p \le 1, x = 0,1,2,\ldots$	$\frac{(1-p)}{p}$	$\frac{(1-p)}{p^2}$

Continuous Distributions

Distribution	Density	Range of Variates	Mean	Variance
Uniform	$\frac{1}{b-a}$	$-\infty < a < b < \infty$ $a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal $N(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}}\exp[\frac{-(x-\mu)^2}{2\sigma^2}]$	$-\infty < \mu < \infty$ $\sigma > 0, -\infty < x < \infty$	μ	σ^2
Lognormal (μ,σ^2)	$\frac{1}{x\sqrt{2\pi\sigma^2}}\exp[\frac{-(logx-\mu)^2}{2\sigma^2}]$	$-\infty < \mu < \infty$ $\sigma > 0, -\infty < x < \infty$	$e^{(\mu+\frac{1}{2}\sigma^2)}$	$e^{(2\mu+\sigma^2)}(e^{\sigma^2}-1)$
Exponential	$\lambda e^{-\lambda x}$	$\lambda > 0, x \ge 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma (α,λ)	$\frac{\lambda^\alpha x^{\alpha-1}e^{-\lambda x}}{\Gamma(\alpha)}$	$\lambda > 0, \alpha > 0, x > 0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
Weibull (c,γ)	$c\gamma x^{\gamma-1}e^{-cx^\gamma}$	$c > 0, \gamma > 0, x > 0$	$c^{-\frac{1}{\gamma}}\Gamma(1+\gamma^{-1})$	$c^{-\frac{2}{\gamma}}[\Gamma(1+2\frac{1}{\gamma})$ $-\Gamma^2(1+\frac{1}{\gamma})]$
Pareto(α,λ)	$\frac{\alpha\lambda^\alpha}{(\lambda+x)^{\alpha+1}}$	$\alpha > 0, \lambda > 0, x > 0$	$\frac{\lambda}{(\alpha-1)}$	$\frac{\lambda^2\alpha}{(\alpha-1)^2(\alpha-2)}$
Burr(α,λ,γ)	$\frac{\alpha\gamma\lambda^\alpha x^{\gamma-1}}{(\lambda+x^\gamma)^{\alpha+1}}$	$\alpha > 0, \lambda > 0, \gamma > 0, x > 0$	Not required	Not required

Useful Formulae

EBCT Model 1

$$E[m(\theta)] = \bar{X}$$

$$E[s^2(\theta)] = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2$$

$$var[m(\theta)] = \frac{1}{(N-1)}\sum_{i=1}^{N}(\bar{X}_i-\bar{X})^2 - \frac{1}{Nn}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2$$

EBCT Model 2

$$E[m(\theta)] = \bar{X}$$

$$E[s^2(\theta)] = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X}_i)^2$$

$$var[m(\theta)] = \frac{1}{P^*}\left[\frac{1}{Nn-1}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X})^2 - \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X}_i)^2\right]$$

Intermediate calculations

$\sum_{j=1}^{n} P_{ij} = \bar{P}_i$ $\quad \sum_{i=1}^{N} \bar{P}_i = \bar{P}$ $\quad \frac{1}{(Nn-1)}\sum_{i=1}^{N}\bar{P}_i\left(1-\frac{\bar{P}_i}{\bar{P}}\right) = P^*$

$\sum_{j=1}^{n}\frac{P_{ij}X_{ij}}{\bar{P}_i} = \bar{X}_i$ $\quad \sum_{i=1}^{N}\sum_{j=1}^{n}\frac{P_{ij}X_{ij}}{\bar{P}} = \bar{X}$

Standard normal distribution: values of the cumulative distribution function

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt, \quad \Phi(-x) = 1 - \Phi(x)$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

End of Appendix.