MTH5129 Probability & Statistics II

Coursework 6

1. Prove that, if $X_1, X_2, ..., X_n$ is a sequence of independent random variables with $E(X_i) = \mu_j$, $\mathrm{Var}(X_j) = \sigma_j^2$ then

$$\mathrm{Var}\left(\sum_{j=1}^{n} X_j\right) = \sum_{j=1}^{n} \sigma_j^2.$$

2. Suppose that I roll a (fair) die repeatedly. Let S_n be the total number of 5's or 6's that I observe after throwing the die n times. What is the

$$\lim_{n\to\infty} P(0.3n < S_n < 0.4n)?$$

3. $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with mean μ and variance σ^2. How large a sample must be taken in order that you can be 95% certain that the sample mean

$$\bar{X_n} = \frac{1}{n} \sum_{n}^{i=1} X_i \quad \text{is within } 0.1\sigma \text{ of } \mu?$$

4. Suppose that I measure the heights of 100 people in London. A person's height has mean 160cm and standard deviation 15cm. Find the (approximate) probability that the mean height of these 100 people I measure is over 163cm. Assume each person's height is independent from the others'. Express your answer in terms of the Φ function.

5. [Gambler's ruin problem] Suppose that we are gambling repetitively on a game with probability of losing £1 in each gamble 0.55 and winning £1 with probability 0.45. Starting from an initial capital of £20. Show that the probability we have not gone bankrupt after 1000 games is (approximately) at most 0.0057.