Queen Mary
University of London

MSc/MSci Examination by Course Unit

Tuesday 19th May 2015 6:30 pm – 9:00 pm

SPA7010P / SPA7010U / ASTM002 The Galaxy Duration: 2 hours 30 minutes

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Instructions:

Answer ALL questions from Section A. Answer ONLY TWO questions from Section B. Section A carries 50 marks, each question in section B carries 25 marks.

If you answer more questions than specified, only the first answers (up to the specified number) will be marked. Cross out any answers that you do not wish to be marked.

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Important note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from QMUL.

Please check now to ensure you do not have any notes, mobile phones or unauthorised electronic devices on your person. If you have any, raise your hand and give them to an invigilator immediately. It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption is also an assessment offence.

EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM.

Examiners:
Dr. W. Sutherland
Prof. J. Emerson.

SECTION A **Answer ALL questions in Section A**

Question A1

Describe and contrast the observed properties of spiral and elliptical galaxies, including their morphologies, colours, spectra, gas and dust content, and stellar populations.

[6 marks]

Question A2

Explain the difference between collisional and collisionless processes in galaxy formation. In an encounter between two galaxies, explain which of these applies to the stars and gas respectively.

[4 marks]

Question A3

Explain the difference between "pressure support" and "rotational support" for a galaxy. Which of these dominate for spiral and elliptical galaxies respectively ?

[3 marks]

Question A4

State (without proof) the virial theorem for a self-gravitating stellar system, defining the terms used. What are the conditions required for its application?
A cluster of galaxies has an observed radius of $800\,\mathrm{kpc}$ and a velocity dispersion of $800\,\mathrm{km\,s^{-1}}$.
Use the virial theorem to estimate the total mass, giving your answer in solar masses.
(You may assume the potential energy of a uniform sphere of mass M and radius R is $U = -3GM^2/(5\,R)$).

[6 marks]

Question A5

An "integral of motion" is defined as any function of position and velocity which is conserved along a star's orbit.

a) Prove that specific energy $E_m = \frac{1}{2}v^2 + \Phi(\mathbf{x})$ is an integral of motion for any time-independent gravitational potential.

[2 marks]

b) Prove that the component of angular momentum around the z-axis, L_z, is an integral of motion for a time-independent potential which is axisymmetric around this axis.

[3 marks]

Question A6

Explain the meaning of the terms HI, HII and H_2 referring to hydrogen in the Galaxy, and describe the typical environments in which each of these are located.

[6 marks]

Question A7

In Galactic chemical evolution, the symbols X, Y, Z denote the mass fractions of hydrogen, helium and heavy elements respectively. Give typical values for these in the local interstellar medium. What processes respectively produced (a) most of the helium, and (b) most of the heavy elements ?

[4 marks]

Question A8

A star near the Galactic plane is observed to have apparent magnitudes in the blue and visual bands of $B = 12.49$, $V = 11.79$. Comparison of the star's spectrum with standard spectra indicates that the star has absolute magnitudes $M_B = 3.20$, $M_V = 2.70$ in these bands. Assuming that the reddening ratio for interstellar dust is $A_V/E(B-V) = 3.0$, estimate (i) the reddening of the star, (ii) the V-band extinction in magnitudes, and (iii) the distance to the star.

[4 marks]

Question A9

For a gravitational lens with perfect alignment, the angular Einstein ring radius is given by

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_S D_L}}\ \text{rad}\,,$$

where M is the lens mass, D_L is the distance from Earth to the lens, D_S is the distance from Earth to the source, and D_{LS} is the distance between lens and source.
Assuming a source star in the Large Magellanic Cloud at $D_S = 50\,\text{kpc}$ and a lens of mass $0.1\,M_\odot$ at distance $10\,\text{kpc}$, evaluate θ_E in arcseconds; comment on the implications for observations of microlensing.

[6 marks]

Question A10

Describe how observations of the Milky Way and Local Group galaxies can be used to constrain the location and nature of dark matter.

[6 marks]

SECTION B **Answer TWO questions from Section B**

Question B1

a) Define the terms *weak encounter* and *strong encounter* for two stars approaching each other in a large stellar system.

[2 marks]

b) In a weak encounter between two stars each of mass m with relative velocity v, the change in the velocity of one star in the reference frame of the other is given by

$$\delta v = \frac{2Gm}{bv},$$

where G is the constant of gravitation and b is the impact parameter.

A star moves through a spherical distribution of overall radius R containing N stars distributed uniformly in space. If the mean change in the square of the velocity is $\delta(v^2) = (\delta v)^2$ in a single weak encounter, show that the changes in v^2 caused by weak encounters with impact parameters in the range b to $b + \mathrm{d}b$ during a time t is

$$\Delta v^2 = \left(\frac{2Gm}{bv}\right)^2 \left(\frac{3bvtN\mathrm{d}b}{2R^3}\right),$$

and hence show that the total change in v^2 in a time t caused by weak encounters with all impact parameters is

$$\Delta v^2(t) = 6\left(\frac{Gm}{v}\right)^2 \frac{v\,t\,N}{R^3} \ln\left(\frac{b_{\mathrm{max}}}{b_{\mathrm{min}}}\right),$$

where b_{max} and b_{min} are the largest and smallest values of the impact parameter.

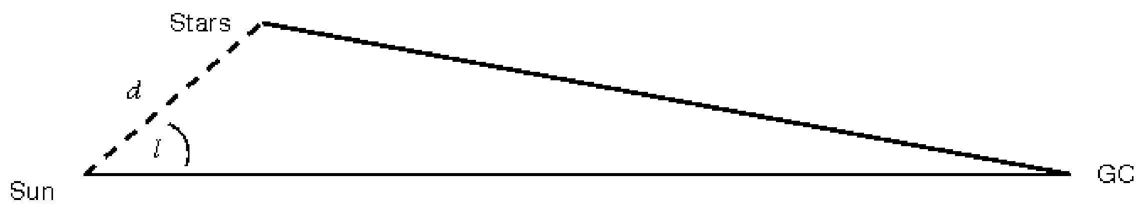
[7 marks]

c) From the result above, write down an expression for the relaxation time T_{relax}; hence show that for suitable choices of b_{min} and b_{max}, the ratio of the relaxation time to the crossing time, T_{cross}, is given approximately by

$$\frac{T_{\mathrm{relax}}}{T_{\mathrm{cross}}} \approx \frac{N}{6\ln N}.$$

(You may assume that in a stellar system of radius R containing N stars each of mass m, the typical velocity v is given by $v \approx \sqrt{GNm/R}$.)

[6 marks]



d) Consider observing a group of stars in the Galactic plane, at a common distance d from the Sun at Galactic longitude ℓ from the Galactic centre (see diagram above, where GC denotes the Galactic centre). Assume that stars in the Galactic disk follow approximately circular orbits around the Galactic centre with a rotation curve given by $v_\phi(R)$, where R is Galactocentric radius in cylindrical polar coordinates, and the stars orbit clockwise in the above diagram.

Assume that the angle at the Galactic centre above is given by $\theta \simeq d \sin \ell / R_0$; hence show that in Cartesian coordinates with the x-axis parallel to the Sun-GC line, the *relative* velocity vector between stars and Sun is given by

$$\langle \mathbf{v}_S \rangle - \mathbf{v}_\odot \simeq \begin{pmatrix} v_0 \, d \, (\sin \ell)/R_0 \\ -d \cos \ell \, (\mathrm{d}v_\phi/\mathrm{d}R)_{R_0} \\ 0 \end{pmatrix} .$$

where $v_0 \equiv v_\phi(R_0)$.
Hence, decomposing velocity components parallel and perpendicular to the line of sight, show that the mean observed radial velocities v_r and transverse velocities v_t of these stars are approximately (to first order in d/R_0) given by

$$\langle v_\mathrm{r} \rangle \simeq A \, d \, \sin 2\ell \qquad \text{and} \qquad \langle v_\mathrm{t} \rangle \simeq B \, d + A \, d \, \cos 2\ell$$

where A, B are Oort's constants (see Appendix).

[10 marks]

Question B2

a) The collisionless Boltzmann equation states that

$$\frac{\partial f}{\partial t} + \sum_{j=1}^{3}\left(\frac{dx_j}{dt}\frac{\partial f}{\partial x_j} + \frac{dv_j}{dt}\frac{\partial f}{\partial v_j}\right) = 0$$

where $f(\mathbf{x}, \mathbf{v}, t)$ is the distribution function, x_j and v_j are the j-components of position and velocity, and t is time.

Derive from this the second of the Jeans equations,

$$\frac{\partial(n\langle v_i\rangle)}{\partial t} + \sum_{j=1}^{3}\frac{\partial}{\partial x_j}\Big(n\langle v_i v_j\rangle\Big) = -\frac{\partial \Phi}{\partial x_i}\, n \ ,$$

where n is the number density of stars, $\langle v_i\rangle$ is the mean value of the v_i velocity component at a point, and $\Phi(\mathbf{x}, t)$ is the gravitational potential. (Explain your definitions, working and assumptions).

[10 marks]

b) One of the Jeans equations in a cylindrical coordinate system (R, θ, z) centred on the Galaxy, with $z = 0$ in the plane, gives

$$\frac{\partial(n\langle v_z\rangle)}{\partial t} + \frac{\partial(n\langle v_R v_z\rangle)}{\partial R} + \frac{\partial(n\langle v_z^2\rangle)}{\partial z} + \frac{n\langle v_R v_z\rangle}{R} = -\,n\,\frac{\partial \Phi}{\partial z} \ ,$$

where n is the star number density, v_R and v_z are the velocity components in the R and z directions, $\Phi(R, z)$ is the Galactic gravitational potential and t is time. Assuming that the Galaxy is in a steady state, show that the surface mass density $\Sigma(z, R_0)$ within a distance z of the mid-plane of the Galactic disc at the solar radius R_0 is given by

$$\Sigma(z, R_0) \simeq \frac{-1}{2\pi G n}\frac{\partial}{\partial z}\left(n\langle v_z^2\rangle\right)$$

for stars in the direction of the Galactic poles. Explain the assumptions you make.

[8 marks]

c) A spherically-symmetric galaxy is dark-matter dominated and has a gravitational potential

$$\Phi(r) = -\frac{GM_{\text{tot}}}{r + a}$$

at a radial distance r from its centre, where a is a positive constant and M_{tot} is the total mass.

A population of stars is distributed within this potential, and the stars contribute negligibly to the total density. The system of stars has an isotropic velocity distribution with a velocity dispersion σ that is constant across the galaxy, has zero net rotation, and has number density n_0 at the centre. Assuming that the potential is constant over time, derive an expression for the number density $n(r)$ of stars as a function of radius r.

(You may quote a suitable Jeans equation from the Appendix below).

[7 marks]

Question B3

a) A galaxy is modelled using a spherically-symmetric gravitational potential of the form

$$\Phi(r) \;=\; -\,\frac{4\pi G k}{r}\,\ln\left(\frac{r+a}{a}\right)\;,$$

where r is the radial distance from the centre of the galaxy, a and k are constants and G is the constant of gravitation. Using Poisson's equation $\nabla^2\Phi = 4\pi G\rho$, show that the mass density ρ as a function of distance r implied by this potential is

$$\rho(r) \;=\; \frac{k}{r(r+a)^2}\;.$$

(You may quote a suitable equation from the Appendix below).

[8 marks]

b) The distance l from the Milky Way to M31 (the Andromeda galaxy) is assumed to satisfy the differential equation

$$\frac{d^2 l}{dt^2} = -\frac{GM}{l^2}$$

with the constant M being the total mass of the Local Group. Verify that

$$\begin{aligned} t &= \tau_0(\eta - \sin\eta)\;, \\ l &= \left(GM\tau_0^2\right)^{\frac{1}{3}}(1-\cos\eta) \end{aligned}$$

with τ_0 a constant, and η a parameter, is a solution of the above differential equation.

[6 marks]

c) Explain how one may use observable quantities in the above equations to estimate the total mass M of the Local Group.

[3 marks]

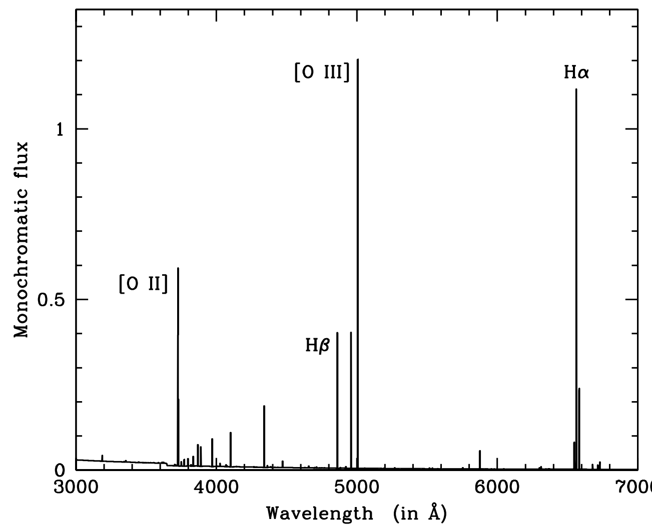
d) For the potential in part (a) above, find an expression for the circular velocity $v_c(r)$; and hence show that the logarithmic derivative of this is given by

$$\frac{d\ln v_c}{d\ln r} \equiv \frac{r}{v_c}\,\frac{dv_c}{dr} = \frac{\dfrac{r+a}{r}\ln\left(\dfrac{r+a}{a}\right) - \dfrac{r}{r+a} - 1}{2\left[1 - \dfrac{r+a}{r}\ln\left(\dfrac{r+a}{a}\right)\right]}\;.$$

By evaluating the latter at $r = a$ and $r = 5a$ respectively, show that the rotation curve is approximately flat between these radii.

[8 marks]

Question B4



a) The figure (above) shows the visible-wavelength spectrum of the Orion Nebula. Briefly explain the different emission lines labelled, and the physical mechanisms responsible for producing them.

[3 marks]

b) Explain, based on the energy levels of the hydrogen atom, why an HII region emits approximately one Balmer photon for each Lyman-continuum photon ($\lambda < 912$Å) emitted by hot stars inside it.

[4 marks]

c) Describe the general properties of dust in the interstellar medium, its effects on observations and star formation, and the methods used to observe it.

[5 marks]

d) List any four of the five assumptions behind the Simple Model of galactic chemical evolution.

[4 marks]

e) In the Simple Model of galactic chemical evolution, the change δZ in the heavy element mass fraction Z is given by

$$\delta Z \;=\; -\,p\,\frac{\delta M_{\mathsf{gas}}}{M_{\mathsf{gas}}} \quad .$$

where p is a parameter called the yield and M_{gas} is the mass of gas. Derive an expression for Z as a function of stellar mass $M_{\mathsf{stars}}(t)$ at time t, and thence show that the mean metallicity of a population of long-lived stars is given by

$$\langle Z \rangle \;=\; p\left(\frac{M_{\mathsf{gas}}(0)}{M_{\mathsf{stars}}} - 1\right)\ln\left(1 - \frac{M_{\mathsf{stars}}}{M_{\mathsf{gas}}(0)}\right) \;+\; p \;.$$

(You may quote the standard integral

$$\int \ln(1 - x/a)\,\mathsf{d}x \;=\; (x-a)\ln(1-x/a) - x + \text{constant} \qquad).$$

[7 marks]

f) Describe the “G-dwarf problem”, and discuss some possible modifications to the Simple Model which may help to resolve it.

[2 marks]

End of Paper - An Appendix of 1 page follows

Appendix - Useful Information

In this paper, π and e represent the standard mathematical constants.
G is the gravitational constant, with $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.
c is the velocity of light, with $c = 3.00 \times 10^8$ m s^{-1}.
1 parsec (pc) $= 3.09 \times 10^{16}$ m.
1 astronomical unit (AU) $= 1.50 \times 10^{11}$ m.
The mass of the Sun is $M_\odot = 2.0 \times 10^{30}$ kg.
The distance of the Sun from the Galactic Centre is $R_0 = 8.0$ kpc.

Poisson's equation states that $\nabla^2\Phi = 4\pi G\rho$ at any point in a gravitational field, where Φ is the gravitational potential, G is the constant of gravitation, and ρ is the mass density at that point.

The Laplacian of a scalar function Φ in a spherical coordinate system (r, θ, ϕ) is

$$\nabla^2\Phi \;=\; \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\frac{\partial\Phi}{\partial r}\right) \;+\; \frac{1}{r^2\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial\Phi}{\partial\theta}\right) \;+\; \frac{1}{r^2\sin^2\theta}\,\frac{\partial^2\Phi}{\partial\phi^2}\ .$$

The Laplacian of a scalar function Φ in a cylindrical coordinate system (R, ϕ, z) is

$$\nabla^2\Phi \;=\; \frac{1}{R}\,\frac{\partial}{\partial R}\left(R\,\frac{\partial\Phi}{\partial R}\right) \;+\; \frac{1}{R^2}\,\frac{\partial^2\Phi}{\partial\phi^2} \;+\; \frac{\partial^2\Phi}{\partial z^2}\ .$$

The Jeans equations in a steady-state (i.e. time-independent), spherically-symmetric galaxy give the following result

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(n\,\langle v_r^2\rangle\right) \;+\; \frac{n}{r}\,\left[\,2\langle v_r^2\rangle \;-\; \langle v_\theta^2\rangle \;-\; \langle v_\phi^2\rangle\,\right] \;=\; -\,n\,\frac{\mathrm{d}\Phi}{\mathrm{d}r}\ ,$$

in spherical coordinates, where n is the number density of stars at a distance r from the centre, v_r, v_θ and v_ϕ are the components of the velocity in the r, θ and ϕ directions, and $\Phi(r)$ is the gravitational potential.

In the absence of cosmological effects, the apparent magnitude m of an astronomical object in a photometric band is related to its absolute magnitude M in that band and its distance D from the observer by

$$m - M \;=\; 5\log_{10}(D/\mathrm{pc}) \;-\; 5 \;+\; A\ ,$$

where A is the extinction in the same band expressed in magnitudes.

Oort's constants within the Galaxy are defined as

$$A \;\equiv\; \frac{1}{2}\left(\frac{\langle v_\phi\rangle}{R} - \frac{\partial\langle v_\phi\rangle}{\partial R}\right) \qquad \text{and} \qquad B \;\equiv\; -\,\frac{1}{2}\left(\frac{\langle v_\phi\rangle}{R} + \frac{\partial\langle v_\phi\rangle}{\partial R}\right)\ ,$$

where $\langle v_\phi\rangle$ is the mean tangential velocity in the Galactic disc, and R is the distance from the Galactic Centre, and the above are evaluated at $R = R_0$.

End of Appendix