1. a) The gravitational acceleration on Earth due to the Sun.

\[ g_0 = -\frac{GM_0}{r^2} \]

Gravitational acceleration of Earth due to the Moon.

\[ g_c = -\frac{GM_c}{r^2} \]

We can examine the ratio of these two accelerations.

\[ \frac{g_0}{g_c} = \left(\frac{GM_0}{r_0^2}\right) \left(\frac{r_0}{GM_c}\right) \]

\[ = \left(\frac{M_0}{M_c}\right) \left(\frac{r_c}{r_0}\right)^2 \]

\[ = \left(\frac{1.99\times10^{30}\text{ kg}}{7.35\times10^{22}\text{ kg}}\right) \left(\frac{3.84\times10^8\text{ m}}{1.495\times10^11\text{ m}}\right)^2 \]

\[ = 177 \]

The gravitational acceleration of Earth due to the Sun is 177 times larger than that of the Moon.
1. b) Again we can compute a ratio of tidal accelerations.

\[ a_{\text{tidal}} = \frac{GM}{r^3} \]

- Mass of tidal perturber
- Distance to tidal perturber
- At the surface of the Earth

\[ \Delta r = R^+ \quad (\Delta r \text{ region where acceleration is different}) \]

**Tidal acceleration due to the Sun**

\[ a_{\text{T}0} = \frac{GM_0}{a_0^3} \]

**Tidal acceleration due to the moon**

\[ a_{\text{Ta}} = \frac{GM_e}{a_e^3} \]

The ratio

\[ \frac{a_{\text{T}0}}{a_{\text{Ta}}} = \left( \frac{GM_0}{a_0^3} \right) \left( \frac{a_e^3}{GM_e} \right) = \left( \frac{M_0}{M_e} \right) \left( \frac{a_e}{a_0} \right)^3 \]

\[ = \left( \frac{1.99 \times 10^{30} \text{ kg}}{7.35 \times 10^{22} \text{ kg}} \right) \left( \frac{3.84 \times 10^8 \text{ m}}{1.495 \times 10^{11} \text{ m}} \right)^3 = 0.462 \]

So the tidal acceleration of the moon on the Earth is a little more than 2\( \times \) stronger than the tidal acceleration of the Sun.
1(b)

The moon's tidal acceleration on the Earth is about 2x greater than the Sun's.

*Note you can get the same answer/result by comparing the heights of the equilibrium tides.
$R_2 = R_{\text{Hill}}$

$$R_{\text{Hill}} = a \left( \frac{M_2}{3\,M_1} \right)^{1/3}$$

For material to be bound to $M_2$, it must reside inside the Hill radius. This is the mathematical requirement for stability.

$$R_2 = a \left( \frac{M_2}{3\,M_1} \right)^{1/3}$$

$a_{\text{Roche}} = R_2 \left( \frac{3\,M_1}{M_2} \right)^{1/3}$

using $u = \frac{4\pi}{3} \rho R^3$

For spheres

$a_{\text{Roche}} = R_2 \left\{ 3 \cdot \left( \frac{4\pi}{3} \rho_1 R_1^3 \right) \left( \frac{3}{4\pi \rho_2 R_2^3} \right) \right\}^{1/3}$

$a_{\text{Roche}} = R_1 \left( \frac{3\rho_1}{\rho_2} \right)^{1/3}$
CONTINUED...

This differs from derivation of rocite radius using the total acceleration at \( r \):

\[
\text{rocite} = R_1 \left( \frac{2 \rho_1}{\rho_2} \right)^{1/3}
\]

Why? The Hill sphere also takes into account the orbital motion about the primary.

Other factors that may affect the location where an object is presently disrupted include:

- An elongated, non-spherical shape (e.g., non-spherical shapes are not well approximated as point masses)

- Physical strength of material resisting flow deformation
2. CONTINUED

NOTE: BECAUSE OF THESE COMPLICATING FACTORS THERE ARE SEVERAL DEFINITIONS/DERIVATIONS OF THE ROCHE RADIUS OF THE FORM:

\[ r_{\text{roche}} = C \left( \frac{p_1}{p_2} \right)^{1/3} R, \]

EACH WITH DIFFERENT VALUES OF \( C \) BASED ON THE PHYSICAL FACTORS (E.G. ORBIT, STRENGTH, SHAPE,...) AND APPROXIMATIONS CONSIDERED.

TYPICALLY \( C \approx 1.5 - 3.0 \) IN ALL THESE APPROXIMATIONS.
3. a) i) \[ H_1 = \frac{5}{2} \left( \frac{M_2}{M_1} \right) \left( \frac{R_1}{r} \right)^3 R_1 \]

At pericentre \( q = r = a (1 - e) \)

For Mercury

\[ r = q = 0.3871 \text{ au} \left( 1 - 0.2056 \right) \]

\[ = 0.3076 \text{ au} \]

\[ M_0 \text{ mass of Sun} \]

\[ H = \frac{5}{2} \left( \frac{1.99 \times 10^{30} \text{ kg}}{3.30 \times 10^{23} \text{ kg}} \right) \left( \frac{2.439 \times 10^{3} \text{ m}}{0.3076 \text{ au}} \right)^4 \]

\[ \text{mass of Mercury} \]

\[ \text{sun-mercury distance} \]

\[ H = 5.47 \text{ m} \]

- Height of EQ tide on Mercury at perihelion due to Sun.

ii) At apocentre the only thing that changes is the Sun-Mercury distance

\[ r = a (1 + e) = 0.3871 \text{ au} (1 + 0.2056) \]

\[ = 0.467 \text{ au} \]

Using the same expression for the tide we get

\[ H_{apo} = 1.57 \text{ m} \]
3 a) So as Mercury goes from pericentre @ 0.31 AU to apocentre @ 0.467 AU, the height of the tide in Mercury decreases from 5.47 m → 1.57 m at apocentre.

As Mercury continues around its orbit, the height of the tide oscillates from a maximum at pericentre to a minimum near apocentre.

3 b) This change in tidal height is doing work on the shape of the planet (or satellite)

If the planet is a non-elastic oscillator, then it dissipates mechanical energy converting it into heat, cracking of the surface, or into less organised forms of energy. Thus the surface may have been tidally heated and resurfaced or have large 'cracks' or faults.
4. a) THE ROCHE RADIUS

\[ R_{\text{roche}} = \left( \frac{3M_2}{\rho_2} \right)^{\frac{1}{3}} R_1 \]

\[ \rho_2 = \frac{M_2}{4\pi R_2^3} = 3910 \text{ kg/m}^3 \]

\[ R_{\text{roche}} = \left[ \frac{3}{(3910 \text{ kg/m}^3)} \right]^{\frac{1}{3}} R_2 \]

\[ = 1.83 R_2 = 6.21 \times 10^6 \text{ m} \]

By this definition Phobos orbits at about a \( \approx 1.5 \) Roche.

b.)

\[ H_1 = \frac{5}{2} \left( \frac{M_2}{M_1} \right) \left( \frac{R_1}{r} \right)^3 R_1 \]

\[ \frac{H_{\text{Phobos}}}{R_{\text{Phobos}}} = \frac{5}{2} \left( \frac{6.41 \times 10^{23} \text{ kg}}{1.08 \times 10^{10} \text{ kg}} \right) \left( \frac{1.12 \times 10^4 \text{ m}}{9.38 \times 10^3 \text{ m}} \right)^3 \]

\[ = 0.25 \]

\[ H_{\text{Phobos}} = 2.83 \times 10^3 \text{ m} \text{ or } 2.83 \text{ km} \]
4 b) Phobos is ellipsoidal in shape:

\[ A > B > C \]

The long axis of the ellipsoid:

\[ A = R + H \]

\[ (R + H)_{\text{Phobos}} = 2.83 \text{ km} + 11.2 \text{ km} \]

\[ = 14 \text{ km} \]

From this computation, this is similar to the observed long axis of Phobos of

\[ A \approx 13.4 \text{ km} \]

But a bit less.

This implies that perhaps Phobos is held together by physical strength of the material, that resists non deformation.
4. c) \( r_{\text{mu}} = a_{\text{phobos}} \left( \frac{M_{\text{phobos}}}{3 \, M} \right)^{1/3} \)

\[
\begin{align*}
\Gamma_{\text{mu}} &= \left( 9.38 \times 10^6 \text{ m} \right) \left( \frac{1.08 \times 10^{16} \text{ kg}}{3 \left( 6.41 \times 10^{23} \text{ kg} \right)} \right)^{1/3} \\
&= 1.67 \times 10^4 \text{ m} = 16.7 \text{ km} \\
\frac{\Gamma_{\text{mu}}}{r_{\text{phobos}}} &= 1.44
\end{align*}
\]

d.) The eccentricity of Phobos' orbit causes the distance to Mars to change during the orbit and the height of the tide to also oscillate as Phobos goes from apocentre to pericentre and back.
5. \[ \rho_{\text{clumps}} = 250 \text{ kg/m}^3 \]
\[ \rho_{\text{saturn}} = 687 \text{ kg/m}^3 \]
\[ r_{\text{saturn}} = 6.033 \times 10^7 \text{ m} \]
\[ a_{\text{clump}} = 1.4022 \times 10^8 \text{ m} \]

For these clumps to be disrupted, they would need to orbit inside the Roche radius. To not be disrupted they would need to orbit above the Roche radius.

\[ a_{\text{roche}} = \left( \frac{3 \rho_{\text{saturn}}}{\rho_{\text{clumps}}} \right)^{1/3} r_{\text{saturn}} \]
\[ = \left( \frac{3 \left( 687 \text{ kg/m}^3 \right)}{250 \text{ kg/m}^3} \right)^{1/3} \left( 6.033 \times 10^7 \text{ m} \right) \]
\[ a_{\text{roche}} = 1.22 \times 10^8 \text{ m} \]

So these clumps orbit above \( a_{\text{roche}} \), the Roche radius, for their density and should be stable against tidal disruption by Saturn.