1.) is the primary mass (e.g., could be the sun).
2.) is the secondary mass (e.g., the earth).

→ To derive the distance to \( L_2 \).

\[
\begin{align*}
\text{force balance: } & F_c = F_{g,1} + F_{g,2}. \\
F_{g,1} &= \frac{G M_1 M}{(a+d)^2}, & F_{g,2} &= \frac{G M_2 M}{d^2}
\end{align*}
\]
1. \( \text{a)} \quad F_c = m a c = \frac{m v^2}{r} \)

\[ w^2 = \frac{v^2}{r^2}, \quad v = \sqrt{\frac{G M}{r}} \]

\[ \Rightarrow w^2 = \frac{G M}{r^3}, \quad r = a + d \]

\[ F_c = m (a + d) \frac{G M}{r^3} \]

\[ \Rightarrow \text{Force balance:} \quad F_c = F_{y,1} + F_{y,2} \]

\[ m (a + d) w^2 = \frac{G M_1 M_1}{(a + d)^3} + \frac{G M_2}{d^2} \]

\[ w^2 = \frac{G M_1}{(a + d)^3} + \frac{G M_2}{d^2(a + d)} \]

\[ \Rightarrow \text{Assume} \quad M_1 \gg M_2 \]

\[ w^2 \sim \frac{G M_1}{(a + d)^3} \]

\[(a + d)^3 \sim a^3 \text{ if } a \gg d \]

\[ \Rightarrow w^2 \sim \frac{G M_1}{a^3} \]
1. a) \[ (a+d)^2 = a^2 (1 + \frac{d}{a})^2 \]
\[ \approx a^2 (1 - 2d/a) \]

\[ \Rightarrow \text{Force equation becomes:} \]
\[ (a+d) \omega^2 = \frac{GM_1}{(a+d)^2} + \frac{GM_2}{d^2} \]

\[ \frac{GM_1}{a^3} (a+d) = \frac{GM_1}{a^2} (1 - 2d/a) + \frac{GM_2}{d^2} \]
\[ \frac{GM_1}{a^3} + \frac{GM_1 d}{a^3} = \frac{GM_1}{a^2} - 2 \frac{GM_1 d}{a^3} + \frac{GM_2}{d^2} \]
\[ 3 \frac{GM_1 d}{a^3} = \frac{GM_2}{d^2} \]
\[ d^3 = \frac{M_2 a^3}{3M_1} \]
\[ \therefore d = a \left( \frac{M_2}{3M_1} \right)^{1/3} \]
1.) b) \( d_{L_1} = \frac{r_{\text{Hill}}}{3M_1} \) \\
it is the same as distance to \( L_2 \) for this \\
level of approximation (we neglected higher order \( \) \\
terms in Binomial expansion) \\

1.) c) Hill radius tells us where \( F_{y_1} + F_{y_2} \) balance with \\
the centripetal force \( F_c \) \\

\( \Rightarrow \) within Hill radius satellites do not bound to the \\
secondary mass. \( \) (if low velocity) \\

\( \Rightarrow \) outside of Hill sphere \( F_{y_1} \) and \( F_c \) prevent the \\
satellite from being bound to the secondary mass. \\

2.) a) Ceres is a dwarf planet in the asteroid belt between \\
Mars and Jupiter \\

b) Largest stable orbit determined by Hill radius: \\
\( R_H = a \left( \frac{M_2}{3M_1} \right)^{\frac{1}{3}} \) \\
\( R_H = 2.76 [\text{AU}] \cdot \left( \frac{1.45 \times 10^{-8} [\text{M}_\odot]}{1 [\text{AU}]} \right) \cdot \left( \frac{9.65 \times 10^{20} [\text{kg}]}{3 \cdot (1.98 \times 10^{30} [\text{kg}])} \right)^{\frac{1}{3}} = 2.23 \times 10^7 [\text{m}] \)
2.) c) Does even orbital eccentricity affect the Hill radius?

→ YES! \( R_H \) is based on the assumption the orbit is circular.

→ \( R_H \) depends on distance to primary mass \( M_1 \)

→ So \( R_H \) will **DECREASE** as even goes towards pericentre and **INCREASE** towards apocentre.

3.) a) Astronomical object that gives smallest \( R_H \), most strongly limits the radial extent of satellites orbiting Saturn.

→ For example:

\[
R_{H, \text{sun}} = a_{\text{sat}} \cdot \left( \frac{M_{\text{sat}}}{3M_0} \right)^{1/3}
\]

\[
R_{H, \text{sun}} = 9.54 [\text{au}] \cdot \left( \frac{5.68 \times 10^{26}}{3 \times (1.9 \times 10^{30})} \right)^{1/3}
\]

\[
R_{H, \text{sun}} = 0.43 [\text{au}]
\]

→ \( R_{H, \text{Jupiter}} = (a_{\text{sat}} - a_{\text{Jup}}) \cdot \left( \frac{M_{\text{sat}}}{3M_{\text{Jup}}} \right)^{1/3} \)

\[
= 4.3 [\text{au}] \cdot \left( \frac{5.68 \times 10^{26}}{3 \times (1.9 \times 10^{27})} \right) = 2.0 \text{ au}
\]

→ \( R_{H, \text{sun}} < R_{H, \text{Jup}} \) the Sun is the stronger influencer.
3.) b) To find radial extent of satellites w.r.t. to the Sun. We do a similar calculation to c) a).

\[ R_{\text{sat}, \text{Eu}} = (a_{\text{sat}} - a_{\text{Eu}}) \cdot \left( \frac{M_{\text{Eu}}}{3M_{\text{sun}}} \right)^{1/3} \]

\[ = (9.54 \text{ [au]} - 0.035 \text{ [au]}) \cdot \left( \frac{2.31 \times 10^{21} \text{ [kg]}}{3 \times 1.98 \times 10^{30} \text{ [kg]}} \right)^{1/3} \]

\[ = 0.01424 \text{ [au]} \]

\[ R_{\text{sat}, \text{Eu}} = a_{\text{Eu}} \cdot \left( \frac{M_{\text{Eu}}}{3M_{\text{sat}}} \right)^{1/3} \]

\[ = 0.035 \text{ [au]} \cdot \left( \frac{2.31 \times 10^{21} \text{ [kg]}}{3 \times 5.68 \times 10^{26} \text{ [kg]}} \right)^{1/3} \]

\[ = 0.000631 \text{ [au]} \]

\( R_{\text{sat}, \text{Eu}} \) is smallest so Saturn has biggest influence on Venus.

4.) If \( \frac{R_{\text{Hill}}}{R} < 1 \) then the centre of the body is bound to the secondary mass, but other layers are not. The body could be broken apart by tidal forces.