Queen Mary
University of London

MSc Examination by Course Unit

Thursday 29th May 2014 2:30pm - 5:00pm

ASTM002 The Galaxy Duration: 2 hours 30 minutes

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Instructions:

Answer ALL questions from Section A. Answer ONLY TWO questions from Section B. Section A carries 50 marks, each question in section B carries 25 marks.

If you answer more questions than specified, only the first answers (up to the specified number) will be marked. Cross out any answers that you do not wish to be marked.

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Important note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from QMUL.

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EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM.

Examiners:
Dr. W. Sutherland, Prof. J. Emerson.

Useful Information

In this paper, π and e represent the standard mathematical constants.
G is the gravitational constant, with $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.
c is the velocity of light, with $c = 3.00 \times 10^8$ m s^{-1}.
The Boltzmann constant is $k_B = 1.38 \times 10^{-23}$ J K^{-1} .
One electron-volt (eV) $= 1.6 \times 10^{-19}$ J.
1 parsec (pc) $= 3.09 \times 10^{16}$ m.
1 astronomical unit (AU) $= 1.50 \times 10^{11}$ m.
The mass of the Sun is $M_\odot = 2.0 \times 10^{30}$ kg.
The distance of the Sun from the Galactic Centre is $R_0 = 8.0$ kpc.

Poisson's equation states that $\nabla^2\Phi = 4\pi G\rho$ at any point in a gravitational field, where Φ is the gravitational potential, G is the constant of gravitation, and ρ is the mass density at that point.

The Laplacian of a scalar function Φ in a spherical coordinate system (r, θ, ϕ) is

$$\nabla^2\Phi \;=\; \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\frac{\partial\Phi}{\partial r}\right) \;+\; \frac{1}{r^2\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial\Phi}{\partial\theta}\right) \;+\; \frac{1}{r^2\sin^2\theta}\,\frac{\partial^2\Phi}{\partial\phi^2} \;.$$

The Laplacian of a scalar function Φ in a cylindrical coordinate system (R, ϕ, z) is

$$\nabla^2\Phi \;=\; \frac{1}{R}\,\frac{\partial}{\partial R}\left(R\,\frac{\partial\Phi}{\partial R}\right) \;+\; \frac{1}{R^2}\,\frac{\partial^2\Phi}{\partial\phi^2} \;+\; \frac{\partial^2\Phi}{\partial z^2} \;.$$

The Jeans equations in a steady-state (i.e. time-independent), spherically-symmetric galaxy give the following result

$$\frac{\mathrm{d}}{\mathrm{d}r}\Big(n\,\langle v_r^2\rangle\Big) \;+\; \frac{n}{r}\,\Big[\,2\langle v_r^2\rangle \;-\; \langle v_\theta^2\rangle \;-\; \langle v_\phi^2\rangle\Big] \;=\; -\,n\,\frac{\mathrm{d}\Phi}{\mathrm{d}r} \;,$$

in spherical coordinates, where n is the number density of stars at a distance r from the centre, v_r, v_θ and v_ϕ are the components of the velocity in the r, θ and ϕ directions, and $\Phi(r)$ is the gravitational potential.

In the absence of cosmological effects, the apparent magnitude m of an astronomical object in a photometric band is related to its absolute magnitude M in that band and its distance D from the observer by

$$m - M \;=\; 5\log_{10}(D/\mathrm{pc}) \;-\; 5 \;+\; A \;,$$

where A is the extinction in the same band expressed in magnitudes.

SECTION A **Answer ALL questions in Section A**

Question A1

Discuss the main observed characteristics of spiral galaxies, refer to bars/bulges, colours, spectra, and gas and dust content; also discuss how these vary with Hubble type, from Sa to Sc or SBa to SBc.

[6 marks]

Question A2

The Tully-Fisher relation for spiral galaxies states that $L \simeq k v_{\mathrm{rot}}^4$, where L is luminosity, v_{rot} is the peak rotational velocity and k is a constant.
One spiral galaxy in the Virgo cluster is observed with $v_{\mathrm{rot}} = 180\,\mathrm{km\,s^{-1}}$ and apparent magnitude $m_V = 12.0$. Another galaxy, in the Coma cluster, is observed with $v_{\mathrm{rot}} = 240\,\mathrm{km\,s^{-1}}$ and $m_V = 14.45$. Estimate the ratio of distances between these two clusters.

[5 marks]

Question A3

Define the terms “crossing time” and “relaxation time” for a stellar system.
In a large stellar system of N stars, the relaxation time T_{rel} and crossing time T_{cr} are approximately related by

$$\frac{T_{\mathrm{rel}}}{T_{\mathrm{cr}}} \approx \frac{N}{12 \ln N} \ .$$

For (a) a typical galaxy and (b) a typical globular cluster, quote approximate values for N, the system radius and typical velocity; hence derive T_{cr} and T_{rel} in each case, and state whether two-body encounters are important.
(Note: you may quote the relation $1\,\mathrm{km\,s^{-1}} \approx 10^{-6}\,\mathrm{pc\,year^{-1}}$).

[6 marks]

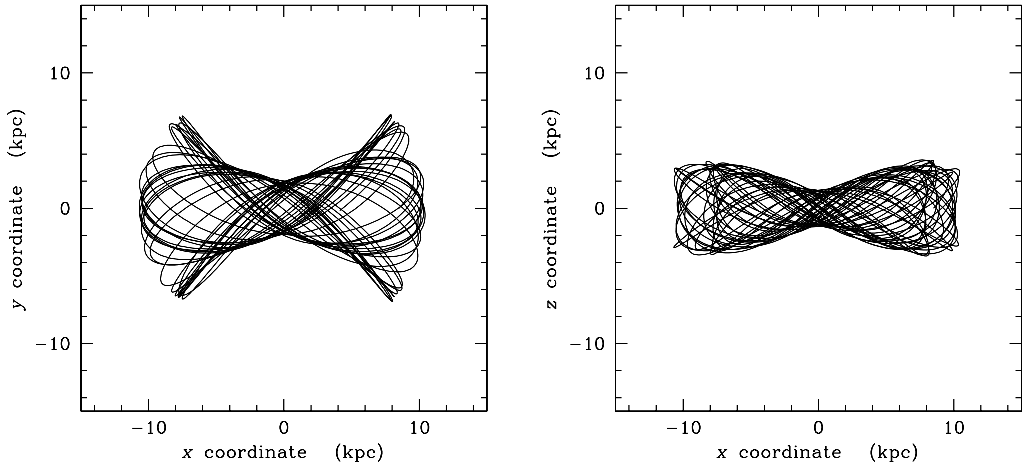
Question A4

The distribution function $f(\mathbf{x}, \mathbf{v}, t)$ describes the density of stars in 6-dimensional phase-space of position $\mathbf{x}$ and velocity $\mathbf{v}$, and time t.
Write down (without proof) expressions for the number density of stars $n(\mathbf{x}, t)$, and the mean velocity component in the $i-$direction, $\langle v_i \rangle$, in terms of f.

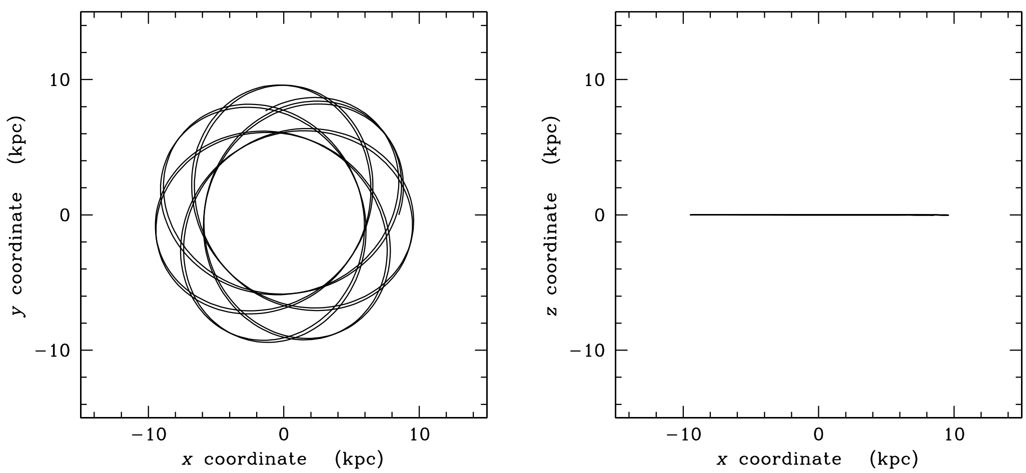
[4 marks]

Question A5

Potential A:



Potential B:



The diagrams above show the orbit of a star in two gravitational potentials, A (upper) and B (lower): each orbit is shown projected in the $x-y$ plane (left) and the $x-z$ plane (right).
What do you conclude about each of the potentials A and B: are they (i) spherical,
(ii) flattened (oblate), or (iii) triaxial? Justify your answer on the basis of the character of each orbit.

[4 marks]

Question A6

Describe the main properties and physical importance of dust in the interstellar medium: refer to its influence on observations, how it is observationally detected, and its influence on star formation.

[5 marks]

Question A7

List any four assumptions of the Simple Model of galactic chemical evolution.

[4 marks]

Question A8

A star near the Galactic plane has apparent magnitudes in the blue and visual bands of $B = 18.87$, $V = 17.47$. A spectrum of the star indicates that it is a K-dwarf with absolute magnitude $M_V = 7.20$ and unreddened colour index $(B-V)_0 = 1.10$.
Given that the reddening ratio for interstellar dust is $A_V/E(B-V) = 3.0$, estimate (i) the reddening of the star, (ii) the V-band extinction in magnitudes, and (iii) the distance to the star in parsecs.

[4 marks]

Question A9

Draw a sketch of the light-curve (flux vs time) for a typical gravitational microlensing event.
The Einstein ring radius for gravitational lensing is given by

$$r_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}D_L}{D_S}} \quad ,$$

where M is the lens mass, D_L is the distance from Earth to the lens, D_S is the distance from Earth to the source, and D_{LS} is the distance between lens and source.
Assuming a lens mass $M = 0.1\,M_\odot$, a source in the Galactic bulge at $D_S = 8\,\text{kpc}$, and the lens half-way to the source, calculate the Einstein radius: express your answer in astronomical units.

[6 marks]

Question A10

Describe the main observations indicating the presence of dark matter in the Milky Way and the Local Group.

[6 marks]

SECTION B **Answer TWO questions from Section B**

Question B1

An isolated system of N stars is bound by their own self-gravity. The ith star has a mass m_i, position vector $\mathbf{x}_i$ and velocity $\dot{\mathbf{x}}_i \equiv d\mathbf{x}_i/dt$ where t is time, and the origin is the centre of mass of the system. The total moment of inertia I of the system is defined as

$$I \equiv \sum_{i=1}^{N} m_i \mathbf{x}_i \cdot \mathbf{x}_i \qquad ,$$

where as usual $\mathbf{x}_i \cdot \mathbf{x}_i$ denotes scalar product.

a) Show that

$$\frac{1}{2}\frac{d^2 I}{dt^2} = \sum_{i=1}^{N} m_i \dot{\mathbf{x}}_i \cdot \dot{\mathbf{x}}_i + \sum_{i=1}^{N} m_i \mathbf{x}_i \cdot \ddot{\mathbf{x}}_i \qquad .$$

[4 marks]

b) Give an expression for T, the total kinetic energy of the system.

[2 marks]

c) Give an expression for the gravitational force on star i due to star j in terms of vectors $\mathbf{x}_i$, $\mathbf{x}_j$. Hence write down an expression for the acceleration $\ddot{\mathbf{x}}_i$ of star i as a sum over $j \neq i$.

[4 marks]

d) Hence, prove that

$$\sum_{i=1}^{N} m_i \mathbf{x}_i \cdot \ddot{\mathbf{x}}_i = -\frac{1}{2} \sum_{i,j,(i\neq j)} \frac{G m_i m_j}{|\mathbf{x}_i - \mathbf{x}_j|} \qquad ;$$

thus deduce the virial theorem,

$$2\langle T \rangle + \langle U \rangle = 0 \quad ,$$

where U is the total gravitational potential energy.

[6 marks]

e) The dark matter halo of our Galaxy has been modelled by a spherical halo with density profile

$$\rho(r) = \frac{\rho_0 \, a^2}{r^2 + a^2}$$

where r is Galactocentric radius, ρ_0 is the halo central density and $a = 5\,\mathrm{kpc}$ is the core radius. Observations indicate that the circular velocity of the Sun around the Galaxy is approximately $220\,\mathrm{km\,s^{-1}}$, and the dark halo contributes approximately half of the total mass inside the Sun's orbit.
Hence estimate the local density of dark matter ρ at $r = R_0 = 8\,\mathrm{kpc}$, in units of $M_\odot\,\mathrm{pc}^{-3}$.

(You may quote the standard integral $\int \frac{x^2}{x^2+a^2}\,dx = x - a\tan^{-1}\frac{x}{a} + \mathrm{const}$.)

[9 marks]

Question B2

a) The continuity equation for the distribution function f of stars in the six-parameter phase space $(x_1, x_2, x_3, v_1, v_2, v_3)$ of position $\mathbf{x}$ and velocity $\mathbf{v}$ states that

$$\frac{\partial f}{\partial t} + \sum_{j=1}^{3} \left(\frac{\partial}{\partial x_j} \left(f \frac{dx_j}{dt} \right) + \frac{\partial}{\partial v_j} \left(f \frac{dv_j}{dt} \right) \right) = 0 \ ,$$

where t is time.

Derive the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \sum_{j=1}^{3} \left(\frac{dx_j}{dt} \frac{\partial f}{\partial x_j} + \frac{dv_j}{dt} \frac{\partial f}{\partial v_j} \right) = 0$$

from the continuity equation, justifying your assumptions.

[5 marks]

b) Describe the advantages of the Jeans equations relative to the collisionless Boltzmann equation for describing the distribution of stars in observed galaxies.

[3 marks]

c) Derive the first of the Jeans equations,

$$\frac{\partial n}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} (n \langle v_i \rangle) = 0 \, ,$$

from the collisionless Boltzmann equation, where n is the number density of stars and $\langle v_i \rangle$ is the mean value of the v_i velocity component at a point. (Explain your working and assumptions).

[8 marks]

d) The velocity dispersion tensor σ_{ij} is defined by

$$\sigma_{ij}^2 = \frac{1}{n} \int \Big(v_i - \langle v_i \rangle \Big) \Big(v_j - \langle v_j \rangle \Big) \, f \, \mathrm{d}^3 \mathbf{v}$$

where v_1, v_2 and v_3 are the components of the velocity vector $\mathbf{v}$, n is the number density of stars in space, f is the distribution function, and i and $j = 1$, 2 and 3. Prove that

$$\sigma_{ij}^2 = \langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle \quad .$$

[5 marks]

e) The third Jeans equation may be written as

$$n \frac{\partial \langle v_i \rangle}{\partial t} + n \sum_{j=1}^{3} \langle v_j \rangle \frac{\partial \langle v_i \rangle}{\partial x_j} = -n \frac{\partial \Phi}{\partial x_i} - \sum_{j=1}^{3} \frac{\partial}{\partial x_j} \left(n \sigma_{ij}^2 \right)$$

Give a physical interpretation of (a) the sum of the left-hand side, (b) the first term on the right-hand side, and (c) the second term on the right-hand side.

[4 marks]

Question B3

a) Define the symbols HI, HII and H_2 referring to hydrogen in the interstellar medium. For each of these species, describe (a) the typical environments in which they are found; (b) the main processes by which they are observed, including the physical emission/absorption processes and the key wavelengths.

[6 marks]

b) Describe the main processes determining the relative abundance of each state of hydrogen above.

[3 marks]

c) A warm atomic gas cloud has a temperature of $5000K$. Explain why it does not emit a significant amount of Balmer line emission, unless it contains hot stars.

[2 marks]

d) In a region of the Galaxy, the total mass of stars is M_{stars}, the total mass of interstellar gas is M_{gas}, and the mass of heavy elements in the interstellar medium is M_{metals}, while the metallicity of the gas is Z. The changes in these quantities in a small time interval are δM_{stars}, δM_{gas}, δM_{metals} and δZ respectively.
For the Simple Model of galactic chemical enrichment, derive the expression

$$\delta Z \;=\; \frac{\delta M_{\text{metals}}}{M_{\text{gas}}} \;-\; Z\,\frac{\delta M_{\text{gas}}}{M_{\text{gas}}} \;.$$

[4 marks]

e) If δM_{metals} and δM_{stars} above are related by
$\delta M_{\text{metals}} \;=\; -\,Z\,\delta M_{\text{stars}} \;+\; p\,\delta M_{\text{stars}}$, where p is the yield of heavy elements, show that

$$\delta Z \;=\; -\,p\,\frac{\delta M_{gas}}{M_{gas}} \quad .$$

[2 marks]

f) Suppose the Simple Model is modified to allow metal-free gas to accrete into the system (but nothing leaves), so M_{tot} can increase. Show that δZ is now given by

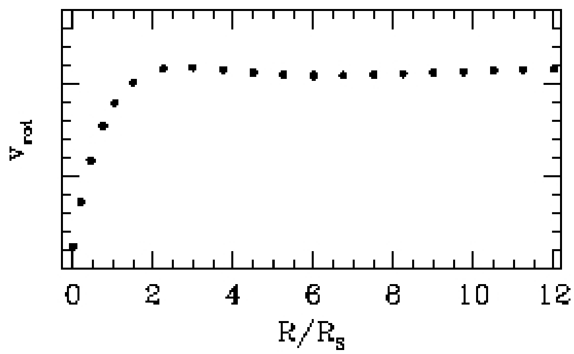
$$\delta Z = \frac{(p - Z)\delta M_{tot} - p\,\delta M_{gas}}{M_{gas}} \;.$$

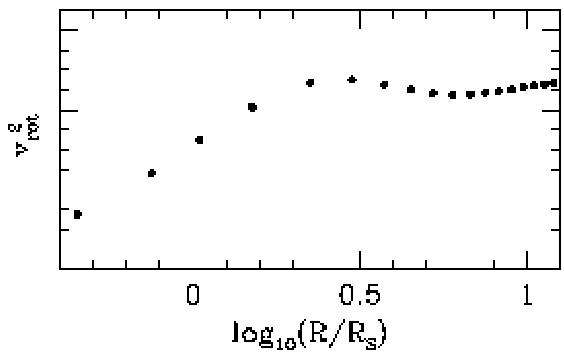
In the simple case where the gas accretion rate equals the rate at which mass is locked up into stars and stellar remnants, show that Z asymptotes to p. Explain physically why this is expected.

[8 marks]

Question B4

a) The graphs below show the observed rotation curve of one spiral galaxy. The left-hand graph plots the circular velocity v_{rot} against R/R_S, where R is the radial distance from the centre and R_S is the exponential scale length of the galaxy's disc. The right-hand graph plots v_{rot}^2 against $\log_{10}(R/R_S)$.





Comment on the main features of the rotation curve, and interpret this in terms of components of the galaxy (e.g. a bulge, a disc and/or a dark matter halo ?) Explain your reasoning.

[4 marks]

b) Which observational method was probably used to measure the rotation curve in the Figure above ? Explain your reasoning.

[3 marks]

c) In gravitational lensing, the physical Einstein ring radius is given by

$$r_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}\,D_L}{D_S}} \quad ,$$

where M is the lens mass, D_L is the distance from Earth to the lens, D_S is the distance from Earth to the source, and D_{LS} is the distance between lens and source.

The optical depth τ to microlensing is defined as the mean number of lenses within 1 r_E of the line of sight to a background source star.

Show that the optical depth τ through a distribution of microlenses of mass M along a line of sight to a given source is given by

$$\tau \;=\; \frac{4\pi G}{c^2 D_S}\int_0^{D_S} D_L\, D_{LS}\,\rho(D_L)\,\mathrm{d}D_L \qquad ,$$

where $\rho(D_L)$ is the mean mass density of lenses at distance D_L.

[9 marks]

d) The microlensing optical depth τ above is independent of lens mass M (for given mass density of lenses). However, real microlensing searches towards the Large Magellanic Cloud at $D_S = 50\,\mathrm{kpc}$ are only sensitive to lens masses in the range $10^{-6}M_\odot < M < 50M_\odot$. By finding an expression for the relevant lengths and timescales involved as a function of $\mu \equiv M/M_\odot$, and evaluating this for the two limits above, explain qualitatively why these limits occur. Suggest possible ways of extending the limits to smaller and larger lens masses.

[9 marks]

End of Paper