Next we look at 4-pt scattering in the φ^3 theory. The theory is defined by

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}} \]

where \( \mathcal{L}_0 \) is the free Lagrangian and \( \mathcal{L}_{\text{int}} \) is the interaction Lagrangian.

Again we look at the expansion of the S-matrix

\[ S = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^4 x_1 \ldots d^4 x_n T(\phi_i(x_1) \ldots \phi_i(x_n)) \]

Again we look at \( | i \rangle = \phi^4(p_1) \phi^4(p_2) | 0 \rangle \)

\( | f \rangle = \phi^4(p_3) \phi^4(p_4) | 0 \rangle \)

as initial and final states.

Then

\[ S_{\Phi^4} = \frac{1}{2!} \left( -\frac{i}{3!} \right)^2 \]

\[ \int d^4 x \, d^4 y \langle \Phi^4, p_4 | T(\phi^4(x) \phi^4(y)) | p_1, p_2 \rangle \]

This is what we need to compute.

We focus on terms of the form

\[ T(\phi^3(x) : \phi^3(y)) \text{ coeff} \times \phi^2(x) \phi^2(y) : \phi^2(x) \phi^2(y) \]

and out of : $\phi^2(x) \phi^2(y)$ : we need to pick the term with 2 as odd 2 as

Basically here are 3 ways to perform this:

```
2
\___\ 3
\   \ 
1 \_\_\ 4
```

"S-CHANNEL"

```
3
\___\ 
\   \ 
1 \_\_\ 2
```

"T-CHANNEL"

```
3
\___\ 4
\   \ 
1 \_\_\ 2
```

"U-CHANNEL"

+ diagrams with $x$ and $y$ swapped.

The numerical prefactor:

\[
T \left( : \phi(x) : \phi(y) : \right) = 3 \phi(x) \phi(y) : \phi(x) \phi(y) : + \]

+ terms with more or less contractions. Then

\[
\phi(x) \phi(y) = \left( \phi(x) + \phi(y) \right) \left( \phi(x) + \phi(y) \right)
\]

and need only $\phi^4$ term

\[
\phi^4 = \left( \phi(x) \phi(y) : + \phi(x) \phi(y) : + \phi(x) \phi(y) : + \phi(x) \phi(y) : \right)
\]

\[
\phi^2 \phi^2 = \left( \phi(x) \phi(y) : + 2 \phi(x) \phi(y) \phi(x) \phi(y) \right) + x \leftrightarrow y
\]

\[
\phi^2(x) \phi^2(y) \rightarrow \left[ \phi(x) \phi(y) : : + 2 \phi(x) \phi(y) \phi(x) \phi(y) \right] + x \leftrightarrow y
\]

+ terms which won't contribute
Let's look at various terms separately. We begin with

$$\langle 0 | a(x_3) a(x_4) \phi(x) \phi(y) a(p_1) a(p_2) | 0 \rangle =$$

and focus on

$$\phi(y) a(p_1) a(p_2) | 0 \rangle =$$

$$= \int dq_1 dq_2 e^{- i (q_1 + q_2) y} a(q_1) a(q_2) a(p_1) a(p_2) | 0 \rangle$$

which includes

$$N(q_1) \ N(q_2)$$

computed before

$$= (2 \pi)^6 \frac{1}{2 \omega(q_1) \ 2 \omega(q_2)} \cdot \frac{1}{N(q_1) \ N(q_2)}$$

$$\left[ \delta(q_1 - p_1) \delta(q_2 - p_2) + \delta(q_1 - p_2) \delta(q_2 - p_1) \right]$$

$$= -i (p_1 + p_2) y\Phi = 2 e^{- i (p_1 + p_2) y} \Rightarrow \Phi(y) a(p_1) a(p_2) | 0 \rangle = 2 e^{- i (p_1 + p_2) y} | 0 \rangle$$

Similarly

$$\langle 0 | a(x_3) a(x_4) \phi(x) = 2 e^{- i (p_3 + p_4) x} | 0 \rangle \Rightarrow \text{Hence}$$

$$\langle 0 | a(x_3) a(x_4) \phi(x) \phi(y) a(p_1) a(p_2) | 0 \rangle = 4 e^{- i (p_1 + p_2) y + i (p_3 + p_4) x}$$

since $$\langle 0 | 0 \rangle = 1$$
Next look at

\[ \langle p_3, p_4 | \Phi(x) \Phi(y) \Phi(x) \Phi(y) | p_1, p_2 \rangle = \]

\[ = 2 \int dq_1 \int dq_2 \int dp_1 \int dp_2 \ e^{i(p_1 x + p_2 y) + i(p_1 x + p_2 y) + i(p_3 x + p_4 y) + i(p_3 x + p_4 y) + i(p_1 x + p_2 y) + i(p_1 x + p_2 y) + i(p_3 x + p_4 y) + i(p_3 x + p_4 y)} \]

\[\langle 0 | a(p_3) a(p_4) a(k_1) a(k_2) a(q_1) a(q_2) a(p_1) a(p_2) | 0 \rangle \]

\[= \frac{(2\pi)^6}{2 \omega(p_3) \omega(p_4)} \frac{1}{(2\pi)^6} \frac{1}{2 \omega(q_1) \omega(q_2)} \]

\[\delta(k_1 - p_3) \delta(k_2 - p_4) + \delta(q_1 - p_1) \delta(q_2 - p_2) + \delta(k_1 - p_4) \delta(k_2 - p_3) + \delta(q_1 - p_2) \delta(q_2 - p_1)\]

\[i, = 2 \left[ e^\gamma + e^\gamma \right] \left[ e^\gamma + e^\gamma \right] \]

\[= 2 \left\{ e^\gamma + e^\gamma + \right. \]

\[\left. e^\gamma + e^\gamma \right\} \]

Add the x && y contributions we get
\[ \langle p_3 p_4 \mid \int \left( \phi(x) \phi(y) + 2 \phi(x) \phi(y) \phi(x) \phi(y) \right) \mid p_1 p_2 \rangle \]

\[ = 4 e^4 + x \leftrightarrow y \quad (A) \]

\[ = 4 e^4 + x \leftrightarrow y + 4 e^4 + x \leftrightarrow y \quad (B) \]

\[ = 4 e^4 + x \leftrightarrow y + 4 e^4 + x \leftrightarrow y \quad (C) \]

This is also multiplied by \(-i \Delta_F(x-y)\)

\[ \Delta_F(x-y) = \int \frac{d^4 \ell}{(2\pi)^4} \frac{i}{\ell^2 - m^2 + i\epsilon} \]

and further integrated over \(dx dy\)

Let's look at (A): its contribution to \(S\) is

\[ S^{(1)}_{\text{A}} = \frac{1}{2!} \left( -\frac{1}{3!} \right)^2 x \]

\[ x \cdot 4 \cdot \int dx dy \left[ e^{-i(p_1+p_2) y + i(p_3+p_4) x} + x \leftrightarrow y \right] \cdot \int \frac{d^4 \ell}{(2\pi)^4} \frac{i}{\ell^2 - m^2 + i\epsilon} e^{-i\Delta_F(x-y)} \]
\[= \text{all numerical factors plus cancel}
\]
\[
\begin{align*}
\frac{(-1)^2}{2!} \int \frac{d^4 l}{(2\pi)^4} \frac{x}{l^2 - m^2 + i\varepsilon} & \left[ e^{ig(l-p_1-p_2)+x[l(l+3)+\varepsilon]} \right] \\
+ x \leftrightarrow y
\end{align*}
\]

\[= \frac{(-1)^2}{2!} \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m^2 + i\varepsilon} \cdot \left[ \frac{4!}{(2\pi)^{4}} \frac{1}{8} (l-p_1-p_2) \frac{4!}{(2\pi)^{4}} \frac{1}{8} (l-p_3-p_4) \right] \]

\[\times 2 \quad \text{(from } x \leftrightarrow y)\]

\[= (-1)^2 (2\pi)^4 \delta \left( p_1 + p_2 - p_3 - p_4 \right) \cdot \frac{1}{(p_1+p_2)^2 - m^2 + i\varepsilon}\]

Note that
\[l = p_1 + p_2 = p_3 + p_4\]

\[\frac{1}{(p_1+p_2)^2 - m^2 + i\varepsilon}\]
Hence we have found

\[ S^{(2)}_{\mu} = \frac{(2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4)}{(i\epsilon)^2} \frac{1}{(p_1 + p_2)^2 - m^2 + i\epsilon} \]

which corresponds to

\[ \begin{array}{c}
\frac{2}{\epsilon} \left( \frac{2}{\epsilon} \right)^3 \left( \frac{2}{\epsilon} \right)^4 \\
\end{array} \]

\[ \frac{1}{\epsilon} \]

\[ l = p_1 + p_2 = p_3 + p_4 \]

We can then add a 4th FEYNMAN RULE:

4) FOR EACH PROPAGATOR, INCLUDE A FACTOR OF

\[ \frac{i}{\epsilon^2 - m^2 + i\epsilon} \]

WHERE \( \epsilon \) IS dictated by

MOMENTUM CONSERVATION AT EACH VERTEX

E.g. above \[ l = p_1 + p_2 = p_3 + p_4 \]

RMK

The \( x \leftrightarrow y \) term cancels the \( \frac{1}{2!} \) from the

Dyson series. This is a crucial feature which continues in any order of
perturbation theory.
CONTRIBUTIONS (B) AND (C):

These correspond to

\[
\begin{align*}
\text{(B): } & \quad \frac{(-i \lambda)^2 (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4)}{(p_3 - p_3)^2 - m^2 + i\varepsilon} \\
\text{(C): } & \quad \frac{(-i \lambda)^2 (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4)}{(p_3 - p_3)^2 - m^2 + i\varepsilon} 
\end{align*}
\]

The whole 4- \lambda^+ amplitude is

\[
S_{12 \rightarrow 34}^{(12)} = (-i \lambda)^2 (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) \cdot \left[ \frac{i}{(p_1 + p_2)^2 - m^2 + i\varepsilon} + \frac{i}{(p_2 - p_3)^2 - m^2 + i\varepsilon} + \frac{i}{(p_3 - p_3)^2 - m^2 + i\varepsilon} \right] 
\]

\[
= (-i \lambda)^2 (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) \cdot \left[ \frac{i}{S - m^2 + i\varepsilon} + \frac{i}{t - m^2 + i\varepsilon} + \frac{i}{u - m^2 + i\varepsilon} \right] 
\]
A quick glance at loops

In QCD theory we wish to look at a higher-order correction to the 4-PT scattering amplitude $12 \to 34$.

We saw that at leading order

$$S^{(4)}_{12} = -i \pi (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) =$$

$$= \times$$

At order $\alpha^2$ we have the following diagrams contributing:

Diagrams with a closed loop (as opposed to tree diagrams).
Let's look at the first diagram. Ignoring the purely numerical prefactor, we get

\[ p_1^2 \frac{\delta(l_1 + l_2 - p_1 - p_2)}{(2\pi)^4} \delta(l_1 + l_2 - p_3 - p_4) \]

\[ = (-i)^2 \frac{i}{p_1^2 - m^2 + i\varepsilon} \frac{i}{l_2^2 - m^2 + i\varepsilon} \]

\[ = (-i)^2 \frac{(2\pi)^4}{(2\pi)^4} \delta(p_1 + p_2 - p_3 - p_4) \]

\[ \int \frac{d^4 l}{(2\pi)^4} \frac{i}{l^2 - m^2 + i\varepsilon} \frac{i}{(l - p_1 p_2)^2 - m^2 + i\varepsilon} \]

As \( \varepsilon \to 0 \) the integral behaves as

\[ \int \frac{d^4 l}{l^4} \] which is a logarithmic divergence.

This is not going to be a catastrophe as the renormalisation programme will take care of it via an appropriate redefinition of fields.
For the moment, let's extract the superficial degree of divergence of a \( \text{L-loop diagram} \)

**Superficial Degree of Divergence, \( D \)**

For a diagram with \( \text{L} \) loops (2 scalar fields in \( d \) dimensions),

\[
D = d - \text{L} - 2 \mathfrak{f} + \mathfrak{f}
\]

\( \mathfrak{f} = \# \text{ internal legs (propagators)} \)

\( \) each carries a \( \frac{1}{\ell^2} \)

So

\[
D = dL - 2 \mathfrak{f}
\]

We wish to find a formula involving

the \# of vertices and \# of external legs.
Let \( V_N = \# \) vertices with \( N \) legs.

Then

\[
N V_N = E + 2 I
\]

with \( I = \# \) int. legs
\( E = \# \) ext. legs

E.g. \[ \begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{array} \]

\[
4 \times 2 = 4 + 2 \times 2
\]

Finally, if there are \( n \) total

\( \sqrt{\text{vertices}} \) in the diagram, then

\[
L = I - V + 1
\]

This is because the

\# of INDEPENDENT internal vertices = \# loops

= \# internal links - \# delta functions

and \# delta functions = \( V - 1 \)

\( \uparrow \)

Overall momentum conservation.
We then have

\[
\begin{align*}
D &= dL - 2I \\
NV_N &= E + 2I \\
L &= I - V + 1
\end{align*}
\]

Write

\[ D = D(V_N, E) \]

(assume \( V = V_N \) to simply so only \( N - p + \) vertices \( \Rightarrow \))

\[ I = \frac{1}{2}(NV_N - E) \]

\[
L = \frac{1}{2}(NV_N - E) - V_N + 1
\]

\[ = \frac{N-2}{2}V_N - \frac{E}{2} + 1 \]

\[ \Rightarrow \]

\[ D = d\left(\frac{N-2}{2}V_N - \frac{E}{2} + 1\right) - NV_N + E \]

or
Set \( d = 4 \) \( \Rightarrow \)

\[
D = 2(N-2)V_N - 2E + 4 - NV_N + E = 0 \]

\[
D = 4 - E + (N-4)V_N
\]

Note this:

In \( \phi^4 \) theory

\( N = 4 \) and

\[
D_{\phi^4} = 4 - E \quad \text{Key feature: independent of } V_N \]

\( \text{(does not grow with } V_N \text{ as it could have} \)

\( D > 0 \) is divergent \( \Rightarrow \)

only occurs for \( E = 2, 4 \)

\( \text{limited # of cases} \)

The key to renormalizability ----

\( \text{NOTE: } N > 4 \text{ D grows with } V_N \Rightarrow \text{BAD} \)

\( N < 4 \text{ D always with } V_N \Rightarrow \text{GOOD}. \)
AN EXAMPLE OF SCATTERING AMPLITUDE IN YUKAWA THEORY

(Poskin p. 116)

We take the theory of a fermion $f$ coupled to a
real scalar $\phi$

\[ L = L_0 + L_I \]
\[ L_0 = \bar{\psi}(i \gamma \cdot p - m) \psi + \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 \]
\[ L_I = - g \bar{\psi} \psi \phi \]

\[ \text{known as "Yukawa coupling".} \]

As an example we compute to lowest order the
scattering amplitude for the process

\[ e^-(p_1, n_1) + \bar{e}^+(p_2, n_2) \rightarrow e^-(p_3, n_3) + \bar{e}^+(p_4, n_4) \]

The relevant term from the Dyson expansion is

\[ (-g)^2 \int d^4x d^4y \quad T \left( \bar{\psi} \psi \phi(x) \bar{\psi} \psi \phi(y) \right) \]

and we need to focus on the term

\[ (-g)^2 \int d^4x d^4y \quad \delta^4(x-y) : (\bar{\psi} \psi \phi(x) \bar{\psi} \psi \phi(y)) : \]

Thus we compute
\[ S_{pi} = \frac{(-i\hbar)^2}{2} \int \int d^4 x \, d^4 y \, \Delta_F(x - y) \]

\[ \langle 0 | a_{n_3}^+(p_3) a_{n_4}^+(p_4) : (\bar{\psi}\psi)(x) \, (\bar{\psi}\psi)(y) : a_{n_1}^+(p_1) a_{n_2}^+(p_2) | 0 \rangle \]

Next we decompose \( \Psi = \Psi^{(+)} + \Psi^{(-)} \)

\[ \Psi^{(+)} \quad \Psi^{(-)} \]

and

\[ \Psi = \Psi^{(+) +} + \Psi^{(-)} \]

so that in order to get a nonzero result we need \( \Psi^{(+)} \Psi^{(+) +} \Psi^{(-)} \Psi^{(-)} \to \)

\[ (-1)^j (-1)^j (-1)^j (-1)^j \]

\[ \Psi^{(+) (x)} \Psi^{(+) (y)} \Psi^{(-) (y)} \Psi^{(-) (y)} \]

where the \( \otimes \) sign comes from the orthogonality nature of the fields.

Thus we need to compute

\[ \langle 0 | a_{n_3}^+(p_3) a_{n_4}^+(p_4) : (\bar{\psi}\psi)(x) \, (\bar{\psi}\psi)(y) : a_{n_1}^+(p_1) a_{n_2}^+(p_2) | 0 \rangle \]

\[ \sum_{s_1 \ldots s_4} \int \frac{d^3 q_1}{(2\pi)^3 \sqrt{2E(q_1)}} \int \frac{d^3 q_2}{(2\pi)^3 \sqrt{2E(q_2)}} \int \frac{d^3 k_1}{(2\pi)^3 \sqrt{2E(k_1)}} \int \frac{d^3 k_2}{(2\pi)^3 \sqrt{2E(k_2)}} \]

\[ i(q_1 x + q_2 y) - i(k_1 x + k_2 y) \]

\[ \epsilon \left( \frac{1}{s_{34}^3 (q_4 - k_1)(s_{34}^3 (q_2 - k_2))} \right) \]

From the previous inside the :
Then we have

\[ a_{s_1} (k_1) a_{s_2} (k_2) a_{n_1} (p_1) a_{n_2} (p_2) |0\rangle = \]

\[ = a_{s_4} (k_1) \left( a_{s_2} (k_2) a_{n_1} (p_1) \right) a_{n_2} (p_2) |0\rangle + \]

\[ - a_{s_4} (k_1) a_{n_1} (p_1) a_{s_2} (k_2) a_{n_2} (p_2) |0\rangle = \]

\[ = (2\pi)^3 (2\pi)^3 \left[ \delta_{s_2 s_4} \delta (k_2 - p_1) \delta_{s_1 s_2} \delta (k_1 - p_2) + \right] \]

\[ - \delta_{s_4 s_1} \delta (k_1 - p_1) \delta_{s_2 s_4} \delta (k_2 - p_2) \]

Thus we will get 4 terms (by multiplying each with + and - signs).

We'll get

\[ (2\pi)^6 \cdot (2\pi)^6 \]

\[ \left[ \delta_{s_2 s_4} \delta (k_2 - p_1) \delta_{s_1 s_2} \delta (k_1 - p_2) - \delta_{s_1 s_4} \delta (k_1 - p_1) \delta_{s_2 s_4} \delta (k_2 - p_2) \right] \]

\[ \left[ \delta_{s_4 s_3} \delta (q_2 - p_3) \delta_{s_3 s_4} \delta (q_1 - p_4) - \delta_{s_3 s_4} \delta (q_1 - p_3) \delta_{s_4 s_3} \delta (q_2 - p_4) \right] \]
EXPONENTIALS, SPINOR FACTORS AND SIGNS:

\[ (-i \kappa_3 - q_1) x - i \kappa_2 - q_2 \cdot y - (u_{S_3} (q_1) u_{S_1} (k_1)) (u_{S_4} (q_2) u_{S_2} (k_2)) \]

\[ A \times A' \rightarrow e^{i \phi_1 (p_2 - p_4) x - i \phi_2 (p_1 - p_3) y} \]

\[ B \times A' \rightarrow e^{i \phi_3 (p_2 - p_4) x - i \phi_4 (p_1 - p_3) y} \]

\[ A \times B' \rightarrow \text{Same as } B \times A' \text{ with } x \leftrightarrow y \]

\[ B \times B' \rightarrow \text{Same as } A \times A' \text{ with } x \leftrightarrow y \]

The \( x \leftrightarrow y \text{ term is going to cancel the } 2! \) from Dyson's formula \[\text{[Note: this cancellation always occurs!]}\]

Thus, we get \[\downarrow \text{ from } x \leftrightarrow y \text{ term} \]

\[ - \frac{(-1)^2}{\sqrt{2 \pi}} \prod_{i=1}^{2} \frac{1}{\sqrt{2E(p_i)}} \text{\ upper \ cancelled \ by \ the \ S \ functions} \]

\[- (\bar{u}_{\eta_4} (p_4) u_{\eta_2} (p_2))(\bar{u}_{\eta_3} (p_3) u_{\eta_1} (p_1)) \int dxdy e^{i \Delta F (x-y)} \]

\[- (\bar{u}_{\eta_3} (p_3) u_{\eta_2} (p_2))(\bar{u}_{\eta_4} (p_4) u_{\eta_1} (p_1)) \int dxdy e^{i \Delta F (x-y)} \]
Finally, let's compute

\[ \int d^4 x \, d^4 y \, e^{i \Delta(x-y)} = \]

\[ = \int d^4 x \int d^4 y \, e^{i \frac{i \cdot (x-y)}{e}} \left( \frac{1}{e^2 - m^2 + i \varepsilon} \right)^{Q_1 x + i Q_2 y} \]

\[ = \int d^4 l \int d^4 x \int d^4 y \, e^{i \frac{i \cdot (l+Q_1)x + i \cdot (Q_2 - l)y}{e^2 - m^2 + i \varepsilon}} = \]

\[ = (2\pi)^4 \delta^{(4)}(Q_1 - Q_2) \frac{i}{Q_1^2 - m^2 + i \varepsilon} \]

This factor will disappear.

\[ S_{fi} = \langle \bar{u}_{n_4}(\vec{p}_4) u_{n_2}(\vec{p}_2) \bar{u}_{n_3}(\vec{p}_3) u_{n_1}(\vec{p}_1) \rangle \frac{i}{(p_1 - p_3)^2 - m^2 + i \varepsilon} + \]

\[ \left[ \left( \bar{u}_{n_4}(\vec{p}_4) u_{n_2}(\vec{p}_2) \bar{u}_{n_3}(\vec{p}_3) u_{n_1}(\vec{p}_1) \right) \frac{i}{(p_2 - p_3)^2 - m^2 + i \varepsilon} \right] \]
Pictorially, what we have found can be drawn as the following 2 Feynman diagrams:

\[ \begin{align*}
\bar{u}_3(p_3) & \quad \bar{u}_1(p_1) \\
1 & \quad 3 \\
\downarrow & \quad \downarrow \\
p_1-p_3 & \quad p_4-p_4
\end{align*} \]

\[ \begin{align*}
\bar{u}_{n+1}(p_4) \quad u_{n+1}(p_1) \\
1 & \quad 4 \\
\downarrow & \quad \downarrow \\
p_1-p_4 & \quad p_3-p_2
\end{align*} \]

- Only two channels this time!
- And a relative minus sign!

Notice an important \( \otimes \) sign in between!

We could obtain one from the other by exchanging \( 3 \leftrightarrow 4 \) (the final electrons) and including the \( \otimes \) sign. This is a completely general feature following from the anticommutative nature of spinor fields — and it’s the manifestation of Pauli’s exclusion principle!

Note in QED, the process \( e^- e^- \rightarrow e^- e^- \) is known as “Møller scattering.”
A FINAL NOTE:

The factor \( \prod_{i=1}^{n} \frac{1}{\sqrt{2E(p_i)}} \) in fact should not be there. This is because in an

normal situation for fermions, we have

\[
\{ a_2(p), a_3^+(\bar{q}) \} = (2\pi)^3 \delta^3(p-\bar{q}) \delta_{\bar{q}2} \text{ which is not}
\]

relativistic invariant. A properly defined

one-particle state should have been

\[
|\vec{p}, 2\rangle = \sqrt{2E(p)} a_2(\vec{p}) |0\rangle
\]

such that the

overlap

\[
\langle \vec{q}, 1 | \vec{p}, 2 \rangle = 2E(\vec{p}) (2\pi)^3 \delta^3(\vec{p}-\vec{q}) \delta_{\vec{q}2}
\]

is now relativistic invariant. [See also Peskin (3.106)]

Thus, redefining our states in this way we

introduce a factor of

\[
\prod_{i=1}^{n} \sqrt{2E(p_i)}
\]

which precisely cancels the one we found.

The true final result is then

\[
S_{pi} = - (1 - \frac{\gamma}{2}) (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)
\]

\[
\left[ \left( \bar{u}_4(p_4) u_2(p_2) \right) \left( \bar{u}_3(p_3) u_1(p_1) \right) \frac{1}{(p_1-p_3)^2 - m^2 + i\epsilon} + \frac{1}{(p_2-p_3)^2 - m^2 + i\epsilon} \right] - \left( \bar{u}_3(p_3) u_2(p_2) \right) \left( \bar{u}_4(p_4) u_1(p_1) \right)
\]
We can then figure out what are the new Feynman rules in this case:

* Vertex $\rightarrow -ig$
* $\psi | \vec{p}, \gamma \rangle \rightarrow \frac{\psi}{\gamma} \rightarrow m^{-} \bar{u}_\gamma (\vec{p})$
* $\langle \bar{\psi} | \bar{\gamma} \rightarrow \frac{\bar{\psi}}{\bar{\gamma}} \rightarrow m^{-} \bar{u}_\gamma (\vec{p})$

Had we used positive we would have got the corresponding rules (see Peskin p 118).

* Important rule for fermions: include a relative $\circ$ sign between the 2 diagrams because of the Fermi statistics.