

Economic Dynamics in Discrete Time

Jianjun Miao✓

The MIT Press
Cambridge, Massachusetts
London, England

Contents

Preface	xvii
Acknowledgments	xxiii
I Dynamical Systems	1
1 Deterministic Difference Equations	3
1.1 Scalar First-Order Linear Equations	3
1.2 Lag Operators	8
1.3 Scalar Second-Order Linear Equations	8
1.4 First-Order Linear Systems	11
1.4.1 <i>Nonsingular System</i>	12
1.4.2 <i>Singular System</i>	17
1.5 Phase Diagrams	20
1.6 Nonlinear Systems	21
1.7 Numerical Solutions Using Dynare	24
1.8 Exercises	30
2 Stochastic Difference Equations	33
2.1 First-Order Linear Systems	33
2.2 Scalar Linear Rational Expectations Models	35
2.2.1 <i>Lag Operators</i>	35
2.2.2 <i>Method of Undetermined Coefficients</i>	38
2.3 Multivariate Linear Rational Expectations Models	38
2.3.1 <i>Blanchard–Kahn Method</i>	39
2.3.2 <i>Klein Method</i>	41
2.3.3 <i>Sims Method</i>	43
2.4 Nonlinear Rational Expectations Models	48
2.5 Numerical Solutions Using Dynare	53
2.6 Exercises	62

3	Markov Processes	63
3.1	Markov Chains	64
3.1.1	<i>Classification of States</i>	67
3.1.2	<i>Stationary Distribution</i>	70
3.1.3	<i>Countable-State Markov Chains</i>	73
3.2	General Markov Processes	75
3.3	Convergence	78
3.3.1	<i>Strong Convergence</i>	79
3.3.2	<i>Weak Convergence</i>	81
3.4	Exercises	85
4	Ergodic Theory and Stationary Processes	87
4.1	Ergodic Theorem	87
4.2	Application to Stationary Processes	91
4.3	Application to Stationary Markov Processes	95
4.4	Exercises	98
II	Dynamic Optimization	99
5	Markov Decision Process Model	101
5.1	Model Setup	101
5.2	Examples	107
5.2.1	<i>Discrete Choice</i>	107
5.2.2	<i>Optimal Stopping</i>	108
5.2.3	<i>Bandit Model</i>	111
5.2.4	<i>Optimal Control</i>	114
5.3	Exercises	115
6	Finite-Horizon Dynamic Programming	117
6.1	A Motivating Example	117
6.2	Measurability Problem	120
6.3	Principle of Optimality	121
6.4	Optimal Control	128
6.5	Maximum Principle	134
6.6	Applications	138
6.6.1	<i>Secretary Problem</i>	138
6.6.2	<i>A Consumption–Saving Problem</i>	139
6.7	Exercises	141

7	Infinite-Horizon Dynamic Programming	143
7.1	Principle of Optimality	143
7.2	Bounded Rewards	152
7.3	Optimal Control	154
7.3.1	<i>Bounded Rewards</i>	154
7.3.2	<i>Unbounded Rewards</i>	157
7.4	The Maximum Principle and Transversality Conditions	162
7.5	Euler Equations and Transversality Condition	165
7.6	Exercises	170
8	Applications	173
8.1	Option Exercise	173
8.2	Discrete Choice	176
8.3	Consumption and Saving	178
8.3.1	<i>Deterministic Income</i>	180
8.3.2	<i>Stochastic Income</i>	187
8.4	Consumption/Portfolio Choice	195
8.5	Inventory	197
8.5.1	<i>Finite-Horizon Problem</i>	199
8.5.2	<i>Infinite-Horizon Problem</i>	203
8.6	Investment	208
8.6.1	<i>Neoclassical Theory</i>	208
8.6.2	<i>Q Theory</i>	209
8.6.3	<i>Augmented Adjustment Costs</i>	211
8.7	Exercises	217
9	Linear-Quadratic Models	221
9.1	Controlled Linear State-Space System	221
9.2	Finite-Horizon Problems	223
9.3	Infinite-Horizon Limits	226
9.3.1	<i>Value Function Iteration</i>	229
9.3.2	<i>Policy Improvement Algorithm</i>	230
9.3.3	<i>Lagrange Method</i>	230
9.4	Optimal Policy under Commitment	231
9.5	Optimal Discretionary Policy	237
9.6	Robust Control	241
9.6.1	<i>Belief Distortions and Entropy</i>	241
9.6.2	<i>Two Robust Control Problems</i>	243
9.6.3	<i>Recursive Formulation</i>	244

9.6.4	<i>Linear-Quadratic Model with Gaussian Disturbances</i>	245
9.6.5	<i>Relative Entropy and Normal Distributions</i>	247
9.6.6	<i>Modified Certainty Equivalence Principle</i>	248
9.7	Exercises	248
10	Control under Partial Information	251
10.1	Filters	251
10.1.1	<i>Kalman Filter</i>	251
10.1.2	<i>Hidden Markov Chain</i>	258
10.1.3	<i>Hidden Markov-Switching Model</i>	260
10.2	Control Problems	261
10.3	Linear-Quadratic Control	263
10.4	Exercises	265
11	Numerical Methods	267
11.1	Numerical Integration	267
11.1.1	<i>Gaussian Quadrature</i>	267
11.1.2	<i>Multidimensional Quadrature</i>	269
11.2	Discretizing AR(1) Processes	270
11.2.1	<i>Tauchen (1986) Method</i>	270
11.2.2	<i>Tauchen-Hussey (1991) Method</i>	271
11.2.3	<i>Simulating a Markov Chain</i>	272
11.3	Interpolation	273
11.3.1	<i>Orthogonal Polynomials</i>	274
11.3.2	<i>Splines</i>	278
11.3.3	<i>Multidimensional Approximation</i>	280
11.4	Perturbation Methods	283
11.5	Projection Methods	285
11.6	Numerical Dynamic Programming	290
11.6.1	<i>Discrete Approximation Methods</i>	291
11.6.2	<i>Smooth Approximation Methods</i>	293
11.7	Exercises	296
12	Structural Estimation	297
12.1	Generalized Method of Moments	297
12.1.1	<i>Estimation</i>	298
12.1.2	<i>Asymptotic Properties</i>	300
12.1.3	<i>Weighting Matrix and Covariance Matrix Estimation</i>	302
12.1.4	<i>Overidentifying Restrictions</i>	303
12.1.5	<i>Implementation</i>	304
12.1.6	<i>Relation to Other Estimation Methods</i>	305

12.2	Maximum Likelihood	305
12.2.1	<i>Estimation</i>	305
12.2.2	<i>Asymptotic Properties</i>	306
12.2.3	<i>Hypothesis Testing</i>	307
12.3	Simulation-Based Methods	308
12.3.1	<i>Simulated Method of Moments</i>	309
12.3.2	<i>Simulated Maximum Likelihood</i>	311
12.3.3	<i>Indirect Inference</i>	311
12.4	Exercises	315
III	Equilibrium Analysis	317
13	Complete Markets Exchange Economies	319
13.1	Uncertainty, Preferences, and Endowments	319
13.2	Pareto Optimum	320
13.3	Time 0 Trading	321
13.3.1	<i>Equilibrium Computation</i>	323
13.3.2	<i>Two Welfare Theorems</i>	323
13.3.3	<i>Asset Pricing</i>	325
13.4	Sequential Trading	326
13.4.1	<i>Investment Opportunities</i>	326
13.4.2	<i>Ponzi Scheme and Portfolio Constraints</i>	327
13.4.3	<i>Radner Equilibrium</i>	328
13.4.4	<i>Arbitrage and State Prices</i>	329
13.4.5	<i>Complete Markets</i>	331
13.4.6	<i>Equilibrium with Transversality Condition</i>	332
13.4.7	<i>Natural Debt Limit</i>	334
13.5	Equivalence of Equilibria	334
13.6	Asset Price Bubbles	337
13.7	Recursive Formulation	342
13.8	Asset Pricing	344
13.8.1	<i>Capital Asset-Pricing Model</i>	345
13.8.2	<i>Factor-Pricing Model</i>	345
13.8.3	<i>Consumption-Based Capital Asset-Pricing Model</i>	346
13.9	Exercises	348
14	Neoclassical Growth Models	353
14.1	Deterministic Models	353
14.1.1	<i>A Basic Ramsey Model</i>	353
14.1.2	<i>Incorporating Fiscal Policy</i>	362

14.2	A Basic RBC Model	365
14.2.1	<i>Steady State</i>	366
14.2.2	<i>Calibration</i>	367
14.2.3	<i>Log-Linearized System</i>	367
14.2.4	<i>Business Cycle Statistics and Model Results</i>	372
14.2.5	<i>Impact of a Permanent TFP Shock</i>	375
14.2.6	<i>Impact of a Temporary TFP Shock</i>	376
14.2.7	<i>Effects of Persistence and Critiques of the RBC Model</i>	377
14.3	Extensions of the Basic RBC Model	378
14.3.1	<i>Various Utility Functions</i>	378
14.3.2	<i>Capacity Utilization</i>	383
14.3.3	<i>Capital or Investment Adjustment Costs</i>	384
14.3.4	<i>Stochastic Trends</i>	389
14.3.5	<i>Other Sources of Shocks</i>	391
14.4	Exercises	394
15	Bayesian Estimation of DSGE Models Using Dynare	397
15.1	Principles of Bayesian Estimation	398
15.2	Bayesian Estimation of DSGE Models	399
15.2.1	<i>Numerical Solution and State-Space Representation</i>	400
15.2.2	<i>Evaluating the Likelihood Function</i>	401
15.2.3	<i>Computing the Posterior</i>	403
15.3	An Example	405
15.3.1	<i>Dynare Codes</i>	405
15.3.2	<i>Dynare Output</i>	409
15.3.3	<i>Stochastic Trends</i>	410
15.4	Exercises	411
16	Overlapping Generations Models	413
16.1	Exchange Economies	413
16.1.1	<i>A Special Case and Multiple Equilibria</i>	415
16.1.2	<i>Existence and Efficiency</i>	420
16.2	Production Economies	425
16.2.1	<i>Multiple Equilibria</i>	427
16.2.2	<i>Dynamic Efficiency</i>	431
16.3	Asset Price Bubbles	432
16.4	Exercises	436
17	Incomplete Markets Models	439
17.1	Production Economies	439
17.1.1	<i>Income Fluctuation Problem</i>	440
17.1.2	<i>Production</i>	441

17.1.3	<i>Stationary Recursive Equilibrium</i>	442
17.1.4	<i>Computation and Implications</i>	443
17.2	Endowment Economies	447
17.2.1	<i>Risk-Free Rate</i>	447
17.2.2	<i>Fiat Money</i>	449
17.2.3	<i>Interest on Currency</i>	449
17.2.4	<i>Seigniorage</i>	452
17.3	Aggregate Shocks	454
17.3.1	<i>Recursive Equilibrium</i>	454
17.3.2	<i>Krusell–Smith Method</i>	455
17.4	Exercises	457
18	Search and Matching Models of Unemployment	459
18.1	A Basic DMP Model	460
18.1.1	<i>Steady State</i>	462
18.1.2	<i>Transitional Dynamics</i>	464
18.1.3	<i>Large Firms</i>	466
18.1.4	<i>Efficiency</i>	468
18.2	Endogenous Job Destruction	469
18.2.1	<i>Steady State</i>	472
18.2.2	<i>Transitional Dynamics</i>	475
18.3	Unemployment and Business Cycles	475
18.3.1	<i>Households</i>	475
18.3.2	<i>Firms</i>	477
18.3.3	<i>Nash Bargained Wages</i>	479
18.3.4	<i>Equilibrium</i>	480
18.4	Exercises	480
19	Dynamic New Keynesian Models	481
19.1	A Basic DNK Model	482
19.1.1	<i>Households</i>	482
19.1.2	<i>Final Goods Firms</i>	483
19.1.3	<i>Intermediate Goods Firms</i>	484
19.1.4	<i>Central Bank</i>	486
19.1.5	<i>Sticky-Price Equilibrium</i>	486
19.1.6	<i>Flexible-Price Equilibrium</i>	487
19.1.7	<i>Log-Linearized System</i>	488
19.2	Monetary Policy Design	493
19.2.1	<i>Efficient Allocation</i>	493
19.2.2	<i>Quadratic Approximation to Utility</i>	494
19.2.3	<i>Commitment versus Discretion</i>	498

19.3	Fiscal Stimulus	501
19.3.1	<i>A Neoclassical Model</i>	502
19.3.2	<i>Monopolistic Competition</i>	503
19.3.3	<i>A DNK Model</i>	504
19.3.4	<i>Zero-Interest-Rate Lower Bound</i>	507
19.3.5	<i>Duration of Fiscal Stimulus</i>	511
19.3.6	<i>Government Purchases and Welfare</i>	514
19.4	A Medium-Scale DSGE Model	517
19.4.1	<i>Households</i>	518
19.4.2	<i>Firms</i>	521
19.4.3	<i>Monetary and Fiscal Policies</i>	523
19.4.3	<i>Aggregation and Equilibrium</i>	523
19.5	Exercises	524
IV	Further Topics	525
20	Recursive Utility	527
20.1	Deterministic Case	528
20.1.1	<i>Koopmans's Utility</i>	528
20.1.2	<i>Construction</i>	530
20.2	Stochastic Case	533
20.2.1	<i>Epstein–Zin Preferences</i>	533
20.2.2	<i>Ambiguity Aversion</i>	539
20.2.3	<i>Temporal Resolution of Uncertainty</i>	546
20.3	Properties of Recursive Utility	549
20.3.1	<i>Concavity</i>	549
20.3.2	<i>Risk Aversion</i>	550
20.3.3	<i>Utility Gradients and Pricing Kernels</i>	550
20.4	Portfolio Choice and Asset Pricing	554
20.4.1	<i>Optimality and Equilibrium</i>	554
20.4.2	<i>Log-Linear Approximation</i>	557
20.4.3	<i>Long-Run Risk</i>	563
20.5	Pareto Optimality	569
20.5.1	<i>Lucas–Stokey Approach</i>	570
20.5.2	<i>Dumas–Wang–Uppal Approach</i>	572
20.6	Exercises	573
21	Dynamic Games	575
21.1	Repeated Games	576
21.1.1	<i>Perfect Monitoring</i>	576

21.1.2	<i>Equilibrium Payoff Set</i>	578
21.1.3	<i>Computation</i>	581
21.1.4	<i>Simple Strategies</i>	583
21.1.5	<i>Imperfect Public Monitoring</i>	583
21.2	Dynamic Stochastic Games	587
21.3	Application: The Great Fish War	588
21.4	Credible Government Policies	591
21.4.1	<i>One-Period Economy</i>	591
21.4.2	<i>Infinitely Repeated Economy</i>	593
21.4.3	<i>Equilibrium Value Set</i>	595
21.4.4	<i>Best and Worst SPE Values</i>	597
21.4.5	<i>Recursive Strategies</i>	599
21.5	Exercises	601
22	Recursive Contracts	603
22.1	Limited Commitment	604
22.1.1	<i>A Dynamic Programming Method</i>	605
22.1.2	<i>A Lagrangian Method</i>	607
22.1.3	<i>An Alternative Characterization</i>	608
22.2	Hidden Action	609
22.3	Hidden Information	615
22.3.1	<i>Characterizations</i>	617
22.3.2	<i>Long-Run Poverty</i>	621
22.4	Exercises	622
	Mathematical Appendixes	625
A	Linear Algebra	627
B	Real and Functional Analysis	633
C	Convex Analysis	643
D	Measure and Probability Theory	651
	References	661
	Matlab Index	683
	Name Index	685
	Subject Index	691