January Examination Period 2025

ECN382 Portfolio Management *Duration: 2 hours*

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Answer ALL questions

Non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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Examiner: Dr Debapriya Paul

Question 1

Annualised parameter estimates for stocks A, B, and C are as follows:

$$
\begin{aligned}
\mathbb{E}[R_A] &= 25\%, & \mathbb{E}[R_B] &= 14\% & \mathbb{E}[R_C] &= 6\% \\
\sigma_A &= 45\%, & \sigma_B &= 20\% & \sigma_C &= 10\% \\
& & \rho_{A,B} &= 0.60, & \rho_{A,C} &= 0.30 \\
& & & & \rho_{B,C} &= 0.25
\end{aligned}
$$

Consider two portfolios. Portfolio 1 is equally weighted across the three stocks. Portfolio 2 is invested in stocks A and C only, with \$3 invested in stock C for every \$1 invested in stock A.

a) Using matrix representation, show the vector of expected returns, the variance-covariance matrix, and for each portfolio, the vector of portfolio weights with respect to all 3 stocks. **[10 marks]**

b) Calculate the correlation between the returns of the two portfolios. **[10 marks]**

c) The parameter estimates for the stocks are historic averages and covariances derived from monthly returns data spanning 20 years. To obtain more reliable parameter estimates, an analyst suggests increasing the sampling frequency (e.g. using daily instead of monthly returns data). Evaluate the effectiveness of the analyst's recommended approach and propose alternative method(s) to overcome any identified shortcomings. **[5 marks]**

Question 2

You are evaluating the performance of three actively managed funds, X, Y and Z. The results of Single Index Model (SIM) regressions of historical returns for the three funds are given in Table 1 below.

Table 1

	Fund X	Fund Y	Fund Z
Alpha	1.5%	−1.2%	0%
Beta	0.75	1.50	0.95
Residual Standard Error	25%	10%	12%

Assume all regression parameter estimates are persistent and that idiosyncratic returns are uncorrelated across assets. It is also known that the market excess returns have an expected value of 6% and a standard deviation of 20%. For each question below, provide a brief explanation and show your computations.

(a) Based on the information provided in Table 1, which fund appears to offer the best risk-adjusted return profile if you were to select one as your sole risky asset? Justify your choice. **[10 marks]**

(b) You want to optimally combine all three funds with an existing passive fund that holds the market portfolio. If you use the Treynor-Black approach to carry out this optimisation, how much would your resultant optimal risky portfolio be under-/over-weight in each fund? **[10 marks]**

(c) Calculate the increase in Sharpe ratio achieved through the optimal combination in (b). **[5 marks]**

Question 3

a) Table 2 below presents the results of an analysis of the performance of portfolios sorted by: (i) past twelve-month returns (excluding the past month); and (ii) past sixty-month returns (excluding the past twelve months). This table is from the paper "Multifactor Explanations of Asset Pricing Anomalies" by Fama and French (1996) discussed in class. Portfolio 1 denotes the decile of stocks with lowest returns during the portfolio-formation period, whereas portfolio 10 contains the decile with the highest returns. The table shows the results of Fama-French 3-factor model regressions on the returns of each portfolio, where "a", "b", "s", and "h" refer to the relevant coefficients in the Fama-French 3-factor model (as shown in the regression specification below), and "$t()$" provides the associated t-statistics.

$$R_p - r_f = a_p + b_p[R_m - r_f] + s_p SMB + h_p HML + \varepsilon_P$$

Table 2

	Portfolio formation months are *t*-12 to *t*-2				Portfolio formation months are *t*-60 to *t*-13			
	1	2	9	10	1	2	9	10
a	−1.15	−0.39	0.33	0.59	−0.18	−0.16	−0.07	−0.12
b	1.14	1.06	1.10	1.13	1.13	1.09	1.06	1.15
s	1.35	0.77	0.51	0.68	1.50	0.83	0.45	0.50
h	0.54	0.35	0.23	0.04	0.87	0.54	−0.00	−0.26
$t(a)$	−5.34	−3.05	3.88	4.56	−0.80	−1.64	−0.92	−1.36
$t(b)$	21.31	33.36	51.75	35.25	20.24	44.40	60.49	53.04
$t(s)$	17.64	16.96	16.89	14.84	18.77	23.63	18.06	16.33
$t(h)$	6.21	6.72	6.68	0.70	9.59	13.67	−0.14	−7.50

Discuss the results in Table 2 and explain what they imply about the return premia associated with the momentum and long-run return reversal anomalies. **[15 marks]**

b) Explain how the BAB ("betting-against-beta") factor is constructed and describe the empirical anomaly that motivates the BAB factor. Give one explanation for this anomaly. **[10 marks]**

Question 4

a) Table 3 below shows the annualised volatilities (σ), market capitalisation weights (w_{eq}), and market capitalisation-implied expected excess returns (Π) of the equity indices of seven major industrial countries. Values are shown in percent.

Table 3

	σ	w_{eq}	Π
Australia	16.0	2.0	3.4
Canada	20.3	3.1	2.1
France	24.8	6.4	3.6
Germany	27.1	6.9	1.3
Japan	21.0	10.1	5.5
UK	20.0	11.4	2.5
USA	18.7	60.1	5.7

You have developed 3 views about the future expected excess returns of these indices over the next investment period, and you want to use the Black-Litterman framework to implement your views and decide your investment allocation across the 7 countries' equity markets. Your views are:

- Canadian equities will deliver expected excess returns of 4%.
- Australian equities will underperfom Japanese equities by 2%.
- US equities will outperform a value-weighted average of European stocks (France, Germany, and the UK) by 5.7%.

i) Specify your views in terms of the P matrix and Q vector discussed in lectures.

ii) Qualitatively describe how your optimal portfolio weights would differ from your prior weights. Assume the prior uncertainty parameter $\tau = 0.05$.

iii) Suppose that your confidence in both the market prior and your own views doubles. How would this affect your optimal portfolio weights? Explain.

[15 marks]

b) At the start of 2020, International Business Machine's (IBM's) pension funds were under-funded by \$9.2 billion (i.e., the value of the funds' liabilities exceeded the value of their assets by \$9.2 billion). Despite IBM injecting \$10.5 billion during the year, its pension funds remained underfunded at year-end. Knowing that interest rates fell during 2020, explain what must have happened. Describe a strategy discussed in lectures that IBM could have used to hedge its interest rate risk, and discuss the limitations of this strategy. **[10 marks]**

End of Paper, an Appendix of 2 pages follows

Appendix - Key Formulas

A selection of formulas used in this module are presented below. This list is not exhaustive – the exam may require the use of formulas and approaches that are not shown below. At the same time, the mere presence of an equation in the list below does not necessarily imply that it will be relevant to any of the questions in the exam.

Statistical properties

Given random variables X, Y, and Z, and constants a, b, c, and d, the following properties hold:

$$\mathbb{E}\left[aX + bY\right] = a\mathbb{E}\left[X\right] + b\mathbb{E}\left[Y\right]$$

$$\operatorname{Cov}\left(aX + bY + c\,, dZ\right) = \operatorname{Cov}\left(aX, dZ\right) + \operatorname{Cov}\left(bY, dZ\right) + \operatorname{Cov}\left(c, dZ\right) = ad\operatorname{Cov}\left(X, Z\right) + bd\operatorname{Cov}\left(Y, Z\right)$$

$$\operatorname{Var}(aX + bY + c) = a^2\operatorname{Var}(X) + b^2\operatorname{Var}(Y) + 2ab\operatorname{Cov}(X, Y)$$

Portfolio equations

Consider a portfolio P comprising N risky assets and a risk-free asset. The expected return and variance of P are given by:

$$\mathbb{E}\left[R_P\right] = \underbrace{\left(1 - \sum_{i=1}^{N} w_i\right)}_{w_{r_f}} r_f + \sum_{i=1}^{N} w_i \mathbb{E}\left[R_i\right] = r_f + \sum_{i=1}^{N} w_i\left(\mathbb{E}\left[R_i\right] - r_f\right) = r_f + \mathbf{w}^{\mathsf{T}}\boldsymbol{\mu}_{\boldsymbol{ER}}$$

$$\operatorname{Var}(R_P) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i \sigma_{i,j} w_j = \mathbf{w}^{\mathsf{T}}\boldsymbol{\Sigma}\mathbf{w}$$

where $\mathbf{w}$ denotes the $N \times 1$ vector of portfolio weights, $\boldsymbol{\mu}_{\boldsymbol{ER}}$ denotes the $N \times 1$ vector of expected excess returns, and $\boldsymbol{\Sigma}$ denotes the $N \times N$ covariance matrix of the N risky assets.

Assuming quadratic utility with risk-aversion parameter λ, an investor's utility-maximising weights in the N risky assets are given by:

$$\mathbf{w}^* = \frac{1}{\lambda}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}_{\boldsymbol{ER}}$$

Factor models

From the specification of factor models with N assets ($i = 1, 2, \ldots, N$), K uncorrelated factors ($k = 1, 2, \ldots, K$), and N uncorrelated residual returns:

$$\sigma_i^2 = \sum_{k=1}^{K} \beta_{i,k}^2 \sigma_k^2 + \sigma_{\varepsilon_i}^2$$

$$\sigma_{i,j} = \sum_{k=1}^{K} \beta_{i,k}\beta_{j,k}\sigma_k^2 \quad \text{for } i \neq j$$

Black-Litterman model

In the version of the Black-Litterman model that we studied in this module, the posterior distributions of the expected excess return, $\boldsymbol{\mu_{ER}}$, and excess return, $\mathbf{ER}$, are given by $\boldsymbol{\mu_{ER}} \sim \mathcal{N}\left(\boldsymbol{\mu_{Post}}, \mathbf{M}^{-1}\right)$ and $\mathbf{ER} \sim \mathcal{N}\left(\boldsymbol{\mu_{Post}}, \boldsymbol{\Sigma_{Post}}\right)$, where:

$$\begin{aligned}
\boldsymbol{\mu_{Post}} &= \left[(\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}^{\mathsf{T}}\boldsymbol{\Omega}^{-1}\mathbf{P}\right]^{-1}\left[(\tau\boldsymbol{\Sigma})^{-1}\boldsymbol{\Pi} + \mathbf{P}^{\mathsf{T}}\boldsymbol{\Omega}^{-1}\mathbf{Q}\right] \\
\mathbf{M}^{-1} &= \left[(\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}^{\mathsf{T}}\boldsymbol{\Omega}^{-1}\mathbf{P}\right]^{-1} \\
\boldsymbol{\Sigma_{Post}} &= \boldsymbol{\Sigma} + \mathbf{M}^{-1}
\end{aligned}$$

Treynor-Black model

When combining an active portfolio A with a passive portfolio M, the optimal weight placed on A is given by:

$$w^* = \frac{w_0}{1 + (1-\beta_A)w_0}, \text{ where } w_0 = \frac{\frac{\alpha_A}{\sigma^2_{\varepsilon_A}}}{\frac{\mathbb{E}[r_M]-r_f}{\sigma^2_M}}$$

Duration

The Macaulay Duration, D, of a generic fixed-income security that pays cash flows of C_t at time t ($t = 1, \ldots, T$) is defined:

$$D = \sum_{t=1}^{T} w_t t \qquad \text{where} \qquad w_t = \frac{\mathrm{PV}(C_t)}{\sum_{t=1}^{T}\mathrm{PV}(C_t)}$$

where "PV" denotes present value (using a discretely compounded rate of interest).

The following relationship holds between the fixed-income security's Macaulay Duration (D) and its Modified Duration (D^*):

$$D = D^*(1+r)$$

where r is the effective discount rate per compounding period.

End of Appendix