Queen Mary
University of London

January Examination Period 2025

ECN322 Topics in Econometrics **Duration: 2 hours**

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Answer **all** questions. **JUSTIFY ALL YOUR ANSWERS.** Cross out any answer that you do not wish to be marked.

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Examiner: Liudas Giraitis

Question 1 (35 marks)

Matrix A is a 2×2 symmetric matrix:

$$A = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$$

a) Find the spectral decomposition of A .

[8 marks]

b) Find the trace and determinant of A.

[6 marks]

c) Find the rank of A.

[3 marks]

d) Is matrix A positive definite, positive semidefinite or neither?

[5 marks]

e) Is matrix A invertible? If yes, find the inverse matrix, otherwise provide your explanation why it is not invertible.

[4 marks]

f) Could matrix A be a variance-covariance matrix of a 2×1 random vector?

[4 marks]

g) Find the expression of the spectral decomposition of A^{100}.

[5 marks]

Question 2 (35 marks)

Consider the linear regression model in matrix form

$$Y = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

where $E(\boldsymbol{\varepsilon}|X) = \mathbf{0}$, and $\boldsymbol{\varepsilon}|X \sim N(0, I)$. Y and $\boldsymbol{\varepsilon}$ are $n \times 1$ vectors, $\boldsymbol{\beta}$ is a $(k+1) \times 1$ vector, and X is a $n \times (k+1)$ matrix.

a) Write down the expression of the sum of squared errors $S(\boldsymbol{\beta})$.

[4 marks]

b) Show that the OLS estimator $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ is $\hat{\boldsymbol{\beta}} = (X'X)^{-1}X'Y$.

[6 marks]

c) Show that $E(\hat{\boldsymbol{\beta}}|X) = \boldsymbol{\beta}$.

[6 marks]

d) True (T) or False (F) (explanation needed)

i) The OLS estimator is unbiased because it gives the closest estimator of the true parameter comparing to the other methods.

ii) The OLS estimator exists when X has full rank.

[6 marks]

Furthermore, $\boldsymbol{\varepsilon}|X \sim \mathcal{N}(0, I)$ indicates to us that the asymptotic distribution of OLS estimator $\hat{\boldsymbol{\beta}}$ is $\hat{\boldsymbol{\beta}} \sim \mathcal{N}(0, (X'X)^{-1})$. Now assume we have a linear regression model for time series $\{y_t\}$ and $\{x_t\}$ as below:

$$y_t = \beta x_t + \varepsilon_t, \quad t = 1, \cdots, 500,$$

where $\beta = 0.5$. We would like to do 1000 **Monte Carlo experiments** to check whether the mean of the distribution of $\hat{\beta}$ is zero or not in EViews. (Let x_t be drawn from i.i.d. $\mathcal{N}(0, 1)$)

e) Describe how you design your Monte Carlo experiments. Carefully describe the steps involved in the investigation.

[10 marks]

f) We can also compute the 90% confidence interval for the true parameter β. Briefly explain how will you do this.

[3 marks]

Question 3 (30 marks)

Birthweight of infants is associated with demographic characteristics and maternal behaviour. We are interested in factors causing **low birthweight** (less than 2500 grams). We will do some regression analysis using June 1997 Detailed Natality Data published by the National Center for Health Statistics in the US. The dependent variable in these regressions is birthweight, and we consider regressors including baby's gender, monther's age, mother's marital status and prenatal medical care etc.

a) In this empirical example, we have some dummy variables (e.g. baby's gender,) and some non-dummy variables (e.g. monther's age). Briefly explain what is dummy variable.

[4 marks]

b) Denote birthweight for baby i as y_i, and collect the regressors in vector x_i. We have following the mean regression model,

$$y_i = \beta' x_i + \varepsilon_i, \quad E(\varepsilon_i | x_i) = 0.$$

Now if we would also like to do linear quantile regression, write down the expression of the model in the similar way as in mean regression model. Make sure you define every symbol used in your answer.

[4 marks]

We apply linear quantile regression regression and mean regression to our data, and get the regression results for regressor "Boy" as in Figure 1, including the estimator of β and its 90% confidence intervals. The horizontal axis represents the quantile level of birthweight from 0 to 1 (excluding 0 to 1). The vertical axis represents weight (unit in gram).

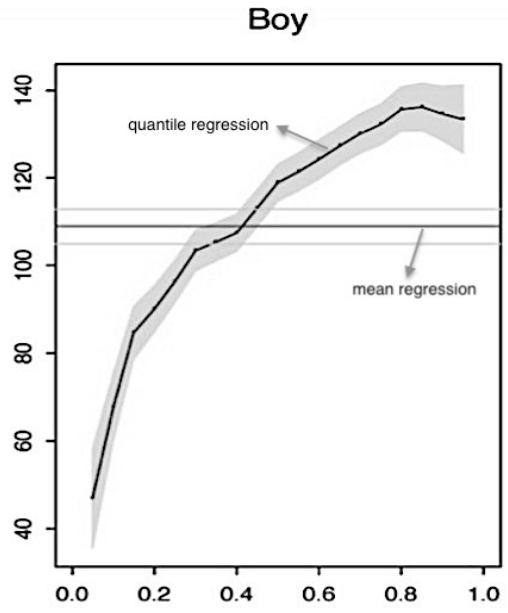

Figure 1: Quantile regression results – Boy

c) What conclusions can you draw from the mean regression results in Figure 1?

[4 marks]

d) What conclusions can you draw from the quantile regression results in Figure 1?

[8 marks]

e) In this example, which regression model is better? Explain your answer.

[6 marks]

f) Omitted variable bias may occur in this empirical study. Describe the conditions that cause this bias.

[4 marks]

End of paper