Queen Mary
University of London

January Examination Period 2024-25

ECN241 Asset Pricing *Duration: 2 hours*

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

Answer ALL questions

Cross out any answers that you do not wish to be marked. You will find an appendix of 3 pages with a list of useful formulas at the end of the exam sheet.

Non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM

Examiner: Jan Toczynski

Question 1

a) Explain, in no more than 100 words, the Efficient Market Hypothesis. How does strong-form market efficiency differ from weak-form market efficiency? **[10 marks]**

b) Consider the following scenario. The central bank is holding a press conference in which it will announce that it decided to increase the interest rate. How is the reaction of the market in the minutes following the announcement likely to differ depending on whether this decision is expected or unexpected by the market? **[5 marks]**

[Total 15 marks]

Question 2

Suppose a retail investor has a utility function $U = E(r) - A\sigma^2$, where E(r) is the expected return for a single risky asset, σ^2 is the variance of returns, and A is the investor's risk aversion coefficient assumed to be 1.

a) Draw the indifference curve for $u = 0.05$ in the return - risk (standard deviation) plane (hint: calculate the utility for three different points on the x-axis). In the plot, draw an arrow indicating the direction in which utility values grow. **[8 marks]**

b) Calculate, by solving the utility maximization problem, the optimal allocation to the risky asset if the risk-free rate is 3%, the expected return of the risky asset is 10%, the standard deviation of the risky asset is 20%. **[9 marks]**

c) Assume now that the retail investor from point b) can borrow at the interest rate r_m. What would r_m need to be for the investor to leverage his portfolio? Would this be a realistic scenario? **[8 marks]**

[Total 25 marks]

Question 3

Consider the following two risky assets: A (expected return of 9% and standard deviation of 18%) and B (expected return of 11% and standard deviation of 25%).

a) Calculate the weights of the minimium variance portfolio assuming that the correlation between returns of assets A and B is $\rho = -1$. What is the variance of returns for this portfolio? **[5 marks]**

b) Considering the portfolio from point a) and the fact that the risk-free rate is 7% (you can both borrow and lend at the risk-free rate), is there an arbitrage opportunity in this market? Explain. **[7 marks]**

c) Explain, in no more than 100 words, why uncorrelated firm-specific risks can be diversified away by adding more stocks to the portfolio while systemic risk cannot be diversified away. **[8 marks]**

[Total 20 marks]

Question 4

Recall the single index model formula:

$$r_i = a_i + \beta_i r_m + e_i,$$

where r_m is the market factor and r_i is return of stock i. Error terms e_i are assumed to be uncorrelated between firms. Your portfolio has been constructed by assuming that the single index model is valid. The portfolio holds two assets: asset X has $\beta_X = 1.2$ and an idiosyncratic variance $\sigma^2(e_X) = 0.08$ and asset Y has $\beta_Y = 0.15$ and an idiosyncratic variance $\sigma^2(e_Y) = 0.12$.

a) Explain, in no more than 100 words, how the Single Index Model simplifies the calculation of portfolio variance compared to the Markowitz Model when managing a portfolio of multiple risky assets. **[10 marks]**

b) Calculate the covariance between returns of assets X and Y assuming that the market variance σ^2_m is 0.04. **[5 marks]**

c) Suppose that in reality $\rho_{e_Y e_X}$, the correlation between the error terms of asset X and asset Y, is 0.4 instead of zero. Recalculate the covariance between r_X and r_Y (hint: use the properties of covariance and remember that $Cov(r_m, e_i) = 0$). Discuss the implications of this non-zero correlation on the risk and diversification benefits of the portfolio. **[10 marks]**

[Total 25 marks]

Question 5

Are the following statements about the CAPM model implications true or false? Explain your answers:

a) "If the risk premium increases and the risk-free rates stays the same then high beta stocks will become relatively more attractive". **[5 marks]**

b) "According to the CAPM, adding more stocks to a portfolio will help reduce its market risk". **[5 marks]**

c) "If portfolios A and B have the same beta then they must have the same level of risk". **[5 marks]**

[Total 15 marks]

End of Paper - An appendix of 3 pages follows

Appendix

Key Formulas and Equations

These key formulas are provided for convenience and they might need to be adjusted or transformed depending on the context - you should at all times rely on your understanding of what they represent.

Annuity formula

$$PV_0 = A \times \left(1 - \frac{1}{(1+r)^n}\right) \times \frac{1}{r}$$

Equity Valuation

The constant-growth version of the dividend discount model (DDM) asserts that if dividends are expected to grow at a constant rate g forever, the intrinsic value of the stock is determined by the formula:

$$V_0 = \frac{D_1}{k-g}$$

The dividend growth rate g is given by the following relationship:

$$g = \text{ROE} \times b$$

where ROE stands for the return on equity and b is the plowback ratio.
The present value of growth opportunities (PVGO) is given by the following relationship:

$$\text{PVGO} = P_0 - \frac{E_1}{k}$$

where E_1 stands for the earnings at the end of period 1, and P_0 is the current stock price.
The Price-Earnings ratio is:

$$\frac{P_0}{E_1} = \frac{1-b}{k - \text{ROE} \times b}$$

The Free Cash Flow to the Firm (FCFF) is given by the following formula:

$$\text{FCFF} = \text{EBIT}(1-t_c) + \text{Depreciation} - \text{Capital Expenditures} - \text{Increases in NWC}$$

where EBIT stands for Earnings before Interest and Taxes, t_c is the corporate tax rate, and NWC is the net working capital.

The Free Cash Flow to Equity (FCFE) is given by the following formula:

$$\text{FCFE} = \text{FCFF} - \text{Interest Expense}(1-t_c) + \text{Increases in net debt}$$

Historical Returns

The Reward-to-Volatility (Sharpe) ratio of a portfolio P is given by the following equation:

$$\text{Sharpe ratio} = \frac{\text{Portfolio risk premium}}{\text{Standard deviation of excess return}} = \frac{E(r_P) - r_f}{\sigma_P}$$

where $E(r_P)$ stands for the expected returns of portfolio P and σ_P for the standard deviation of portfolio P's excess returns, while r_f is the risk-free rate.

The real rate of return r_{real} is:

$$r_{\text{real}} = \frac{1 + \text{nominal return}}{1 + \text{inflation rate}} - 1$$

Mathematical Formulas

Variance properties:

$$Var(a \times X + b \times Y) = a^2 Var(X) + b^2 Var(Y)$$

Covariance properties:

$$Cov(aX + bY, cW + dV) = acCov(X, W) + adCov(X, V) + bcCov(Y, W) + bdCov(Y, V)$$

$$Cov(aX, bY) = abCov(X, Y)$$

Portfolio Allocation

The standard deviation of the portfolio P with weights w_A and w_B would be:

$$\sigma_P = \sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B}$$

where σ_P, σ_A, and σ_B are, respectively, the standard deviations of the expected returns of portfolio P, security A, and security B, and $\rho_{A,B}$ is the coefficient of correlation between the expected returns of the two securities A and B.

The formula to calculate the covariance of returns on funds A and B is:

$$\text{Cov}(r_A, r_B) = \rho_{A,B} \sigma_A \sigma_B$$

For asset allocation with two risky assets (say S and B), the investment proportions w_S and w_B in the optimal risky portfolio P, of the two risky funds S and B, are given by:

$$w_S = \frac{[E(r_S) - r_f]\sigma_B^2 - [E(r_B) - r_f]\text{Cov}(r_S, r_B)}{[E(r_S) - r_f]\sigma_B^2 + [E(r_B) - r_f]\sigma_S^2 - [E(r_S) - r_f + E(r_B) - r_f]\text{Cov}(r_S, r_B)}$$

and

$$w_B = 1 - w_S$$

The investment proportions $w_{\text{min}(S)}$ and $w_{\text{min}(B)}$ in the minimum variance portfolio are computed as follows:

$$w_{\text{min}(S)} = \frac{\sigma_B^2 - \text{Cov}(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2\text{Cov}(r_S, r_B)}$$

and

$$w_{\text{min}(B)} = 1 - w_{\text{min}(S)}$$

Single index model

In the single index model, the excess returns of stock i over the risk-free rate r_f are given by: $R_i = r_i - r_f$. The excess returns on the market index (say M) are given by $R_M = r_m - r_f$. $R_i(t)$ and $R_m(t)$ stand for the excess returns of stock i and of the market index in month t. The index model can be written as the following regression equation:

$$R_i(t) = \alpha_i + \beta_i R_m(t) + e_i(t)$$

Under the assumption of no correlation between the error terms the total risk of asset i (σ_i^2) is:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma^2(e_i)$$

The adjusted R^2 from the single-index model regression R_i^2 is the ratio of systematic variance to total variance:

$$R_i^2 = \frac{\beta_i^2 \sigma_m^2}{\beta_i^2 \sigma_m^2 + \sigma^2(e_i)}$$

When using the index model, we can calculate the covariance of any pair of stocks (say i and j) using the following relationship:

$$\text{Cov}(r_i, r_j) = \beta_i \beta_j \sigma_m^2$$

When using the index model, we can calculate the correlation of any pair of stocks (say i and j) using the following relationship:

$$\text{Corr}(r_i, r_j) = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j} = \text{Corr}(r_i, r_m) \times \text{Corr}(r_j, r_m)$$

CAPM

The Capital Asset Pricing Model (CAPM) implies that the risk premium on any individual asset or portfolio, say i, is the product of the risk premium on the market portfolio $[E(r_M) - r_f]$ and the beta coefficient (the CAPM beta):

$$E(r_i) - r_f = \beta_i [E(r_M) - r_f]$$

where the beta coefficient β_i is the covariance of the asset return with that of the market portfolio as a fraction of the variance of the return on the market portfolio:

$$\beta_i = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2}$$