January Examination Period 2024-25

ECN115—Mathematical Methods in Economics and Finance ***Duration: 2 hours***

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Answer ALL questions

Calculators are not permitted in this examination. Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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Examiner: Evgenii Safonov

Question 1 [19 marks]

a) For a sequence a_n, define what means that a_n has a finite limit a.

[7 marks]

b) Suppose that a sequence a_n has a finite limit a. Thus, $a_n \longrightarrow a$. Suppose also that $a_n \geq 0$ for all n. Show that $a \geq 0$.

Hint: you can start by assuming, towards a contradiction, that $a < 0$. What would this imply for the values of a_n for large enough n?

[7 marks]

c) Suppose two sequences c_n and b_n have finite limits c, b correspondingly. Thus, $c_n \longrightarrow c$ and $b_n \longrightarrow b$. Suppose also that $c_n \geq b_n$ for all n. Use the result from problem (b) to show that $c \geq b$.

Hint: consider a sequence $a_n = c_n - b_n$.

[5 marks]

Question 2 [25 marks]

a) Suppose (one-variable) function f is defined on some open interval that includes point x. Define what means that f has a derivative at point x.

[7 marks]

Use the rules of differentiation and known derivatives (no need to prove anything) to solve the following problems.

b) Calculate the derivative of the function $g : (0, \infty) \to \mathbb{R}$ at point $x = 1$, where

$$g(x) \ = \ \frac{3}{\sqrt{x}} + 4\sqrt{x}$$

[5 marks]

c) Calculate the derivative of the function $h : \mathbb{R} \to \mathbb{R}$ at point $x = 1$, where

$$h(x) \ = \ 5^{2x}$$

[6 marks]

d) Calculate the derivative of the function $w : \mathbb{R}\backslash\{0\} \to \mathbb{R}$ at point $x = -1$, where

$$w(x) \ = \ \ln\left(1 - \frac{1}{x^2+1}\right)$$

Hint: You can simplify the function w using the known properties of the logarithm before taking the derivative of w.

[7 marks]

Question 3 [34 marks]

Let $a \in \mathbb{R}$ be a parameter. Consider function $h : (6-\sqrt{5}, 6+\sqrt{5}) \to \mathbb{R}$ given by

$$h(x) \;=\; a \cdot \ln\left(\frac{-(x-6)^2+5}{2}\right)$$

a) Find all values of the parameter a for which the function h is continuous on its domain.

[3 marks]

b) Maximize the function h over the interval $[5, 8]$. That is, find

$$\max_{x\in[5,8]} h(x), \qquad \arg\max_{x\in[5,8]} h(x).$$

Your answer should depend on the value of the parameter a.

Hint: consider 3 cases: $a > 0$, $a = 0$, $a < 0$.

[9 marks]

c) Minimize the function h over the interval $[5, 8]$. That is, find

$$\min_{x\in[5,8]} h(x), \qquad \arg\min_{x\in[5,8]} h(x).$$

Your answer should depend on the value of the parameter a.

Hint: consider 3 cases: $a > 0$, $a = 0$, $a < 0$.

[9 marks]

Consider the function $f : (-\infty, 6+\sqrt{5}) \to \mathbb{R}$ given by the following formula:

$$f(x) \;=\; \begin{cases} -|x-2|+1 & if \quad x < 5 \\ a \cdot \ln\left(\dfrac{-(x-6)^2+5}{2}\right) & if \quad x \geq 5 \end{cases}$$

where the notation $|x-2|$ denotes the absolute value of $x-2$.

d) Find all values of the parameter a for which the function f is continuous on its domain.

[3 marks]

e) Use your results from problem (b) to maximize the function f over the interval $[0, 8]$. That is, find

$$\max_{x\in[0,8]} f(x), \qquad \arg\max_{x\in[0,8]} f(x).$$

Your answer should depend on the value of the parameter a.

[10 marks]

Question 4 [22 marks]

Let parameters a, b be such that $a > 0, b \geq 1$.

a) Find all $x \geq 0$ such that $e^{ax^2} \leq e^{ax}$.

[3 marks]

b) Find all $x \geq 0$ such that $e^{ax^2} \leq xe^{ax^2}$.

[3 marks]

c) Using your results from problems (a), (b), show that

$$\int_0^b e^{ax^2} dx \;\leq\; \frac{1}{2a}e^{ab^2} + \frac{1}{2a}e^a - \frac{1}{a}$$

[11 marks]

d) Suppose now that the parameter c is such that $c \geq 1$. Use the expression from problem (c) to provide an upper bound on the value of the definite integral

$$\int_{-c}^{b} e^{ax^2} dx$$

[5 marks]

End of Paper