Some Example Calculations of Tides

What is the height of tides in Earth raised by the Moon?

The Sun?

Assuming a fluid Earth with no internal strength, the height of the tide is

\[ H_1 = \frac{5}{2} \left( \frac{M_2}{M_1} \right) \left( \frac{R_1}{r} \right)^3 R_1 \]

For the Moon:

Raising Tides in the Earth

Body #1 -> Earth
Body #2 -> Moon

\[ r = \alpha = 60 \, \text{R}_\oplus \]

\[ \frac{M_e}{M_\oplus} = \frac{1}{81} \quad \text{R}_\oplus = 6300 \, \text{km} \]

\[ H = \frac{5}{2} \left( \frac{1}{81} \right) \left( \frac{1}{60} \right)^3 \left( 6.3 \times 10^6 \right) \]

\[ H = 0.90 \, \text{m} \]

Height raised in Earth due to Moon

Can do the same calculation for the Sun and compare
Two key points:

- The height of the tide is a strong function of the separation between bodies:
  \[ H \propto \left( \frac{1}{r} \right)^3 \]

- Tides can be reduced in height by the rigidity or internal strength of bodies.

The Love number, \( k_2 \), parametrizes the strength or rigidity of a body.

For uniform density bodies:

\[ k_2 = \frac{3/2}{1 + \frac{19\mu}{2\rho g R}} \]

\( \mu \): Internal rock strength or rigidity, has units of pressure, is measured in bars.

\( \rho \): Density at surface of body.

\( g \): Surface gravitational acceleration.

\( R \): Body radius.
THE LOVE NUMBER - CONTINUED

For fluids with no strength \( \mu = 0 \)

\[ K_2 = 3/2 \]

For bodies with rigidity much greater than gravitational compression \( \mu \gg \rho g R \)

or \( \mu \to \infty \) \( K_2 \to 0 \)

**Modified height of equilibrium tide**

\[
H_1 = \frac{5}{3} K_2 \left( \frac{M_2}{M_1} \right) \left( \frac{R_2}{r} \right)^3 R_1
\]

Love number of body \#1 (the one being referred to)

The rigidity or strength of a body reduces the height of the tide.

For fluids \( \mu \to 0 \), \( K_2 \to 3/2 \), and

\[
H_1 = \frac{5}{2} \left( \frac{M_2}{M_1} \right) \left( \frac{R_2}{r} \right)^3 R_1
\]

is recovered.