Maths & Stats Pre-Sessional Tutorial

Mock Exam - Solutions

QUESTION 1

Which of the following central tendency statistics is affected by an outlier?

Correct Answer: Mean

Explanation: The mean is calculated by adding up all the values in a dataset and then dividing by the number of values. Outliers are extreme values that differ significantly from the majority of the data points. When you have outliers in your dataset, they can influence the mean because they pull the average towards their extreme values. Other measures of central tendency, such as the median, are less influenced by outliers. The median is the middle value when the data is arranged in numerical order. It's not affected by extreme values, making it more robust in the presence of outliers compared to the mean.

QUESTION 2

Which one of these variables is a continuous random variable?

Correct Answer: The time randomly selected students take to complete an exam.

Explanation: A continuous variable is a variable that can take on any value within a range. A continuous variable takes on an infinite number of possible values within a given range. Continuous variables are measured, while discrete variables are counted. Discrete variable is a variable that takes on distinct, countable values.

All the variables are discrete variables except “The time randomly selected students take to complete an exam” which is continuous.

QUESTION 3

The Correlation coefficient between X and Y is equal to zero, i.e., corr(X, Y) = 0. What do you conclude?

Correct Answer: There is no relationship between X and Y.

Explanation: Correlation coefficients measure the strength of the relationship between two variables. The correlation coefficient goes from -1 to +1.

The extreme values of -1 and 1 indicate a perfectly linear relationship where a change in one variable is accompanied by a perfectly consistent change in the other. For these relationships, all of the data points fall on a line.

A coefficient of zero represents no linear relationship. As one variable increases, there is no tendency in the other variable to either increase or decrease.

QUESTION 4

What is the mean of the following numbers: 23, 45, 87, 40, 50?

Correct Answer: 49.

Explanation: $\mu = \frac{1}{N}\sum x_1 = \frac{1}{5}(23 + 45 + 87 + 40 + 50 += 49$

QUESTION 5

The sample mean is…

Correct Answer: an unbiased estimator of the population mean.

Explanation: The average value of the observations in a sample is called sample mean.

The sample mean is a random variable that is an estimator for the population mean. If we take different samples, we can calculate different sample means. The expected value of all these sample means is equal to the population mean μ. Therefore, the sample mean is an unbiased estimator of the population mean.

QUESTION 6

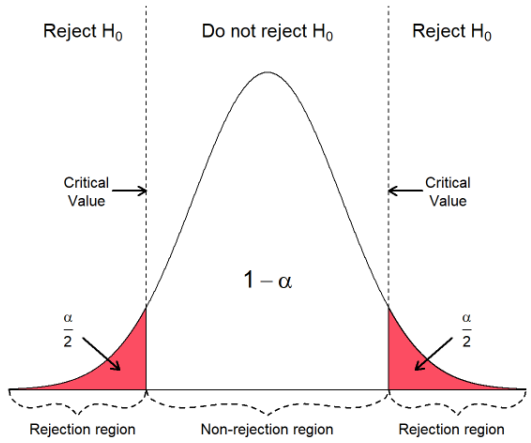
The value of the test statistic which separates the rejection region and the non-rejection region is called:

Correct Answer: critical value.

Explanation: Critical value is a cut-off value that is used to mark the start of a region where the test statistic, obtained in hypothesis testing, is unlikely to fall in. This region is called rejection area.

Critical value can be defined as a value that is compared to a test statistic in hypothesis testing to determine whether the null hypothesis is to be rejected or not. If the value of the test statistic (in absolute terms) is lower than the critical value, then the null hypothesis cannot be rejected.

For a two-sided test, we have:



QUESTION 7

A business student claims that , on average, an MBA student is required to prepare more than five business cases per week. To examine the claim, a statistics professor asks to a sample of 10 MBA students to report the number of cases they prepare weekly. The professor obtains that the selected 10 students prepared on average 5.7 cases per week.

Assuming that the number of cases is normally distributed with a standard deviation of 1.5 and the critical value is 1.64, what can you conclude?

Correct Answer: Fail to reject the null hypothesis, so the student's claim is not true.

Explanation:

$$H_0: \mu = 5$$

$$H_1: \mu > 5$$

We then calculate the z-statistic: $z - stat = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{5.7-5}{1.5/\sqrt{10}} = 1.47$

This is a one-tailed test.

We reject the null hypothesis if the test statistic is greater than the critical value (1.64).

Since 1.47 < 1.64 we can conclude that we cannot reject the null hypothesis. This means the student's claim is not true.

QUESTION 8

Research for a campaign to increase mental health awareness is being carried out.

In the Liverpool area, data from 35 GP practices was collected, and the number of patients diagnosed with depression as a percentage of all patients over the past fifteen years is 26.2.

You want to test whether the percentage of people suffering from depression is different in the Liverpool area than the national average which is 21.9.

You know that the population standard deviation is 7.5.

Using a 5% significance level, what do you conclude?

Correct Answer: Reject the null hypothesis.

Explanation:

$$H_0: \mu = 21.9$$

$$H_1: \mu \neq 21.9$$

We then calculate the z-statistic: $z - stat = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{26.2-21.9}{7.5/\sqrt{35}} = 3.39$

This is a two-tailed test.

We reject the null hypothesis if the $|$test statistic$|$ is greater than the critical value.

We can find the critical values by looking at the statistical table. Any of these probabilities are fine: P(Z<z)=0.025 or P(Z>z)=0.025 or P(Z<z)=0.975. The critical values are 1.96 and -1.96

Since 3.39 > 1.96 we can conclude that we reject the null hypothesis.

QUESTION 9

The mean of two independent random variables X and Y are 2 and 4 respectively.

Let Z= 5X-2Y. The expected value of Z is?

Correct Answer: 2.

Explanation: E(Z) = E(5X - 2Y) = 5E(X) – 2E(Y) = 5*2 – 2*4 = 2

QUESTION 10

The area under a standard normal curve is

Correct Answer: 1.

Explanation: The standard normal curve is bell-shaped and is symmetric about the mean (zero). The total area under the normal curve is equal to 1. The area under the curve represents the probability of all possible values occurring, and the total probability is 1.

QUESTION 11

The length of time it takes to find a parking space at 9am follows a normal distribution with a mean of five minutes and a standard deviation of two minutes.

Find the probability that it takes more than eight minutes to find a parking space (i.e., P(X >8))

Correct Answer: 0.0668

Explanation: The Z-score is $\frac{X-\mu}{\sigma} = \frac{8-5}{2} = 1.5.$

Therefore we want to find P(Z > 1.5) = 1 - P(Z < 1.5) = 1 – 0.93319 = 0.0668

QUESTION 12

The following table lists the joint probability distribution between recipients of the Nobel prize in economics and the other five prizes, and the citizenship of the recipients, based on the 1969-2001 period.

	U.S. citizen ($Y = 0$)	Non-U.S. citizen ($Y = 1$)	Total
Economic Nobel prize ($X = 0$)	0.118	0.049	0.167
Physics, Chemistry, Medicine, Literature, and Peace Nobel prize ($X = 1$)	0.345	0.488	0.833
Total	0.463	0.537	1.00

What is the probability that a Nobel prize in Economics is a US citizen?

Correct Answer: 0.118

Explanation: You are interested in a joint probability, the probability that a Nobel prize in Economics is a US citizen. This is P(X=0 and Y = 0). From the table we can see that this probability is equal to 0.118.

QUESTION 13

Consider a line passing through the points (0.5, 0), (0, 1) and (1, -1) and having a slope -2.

Which of the following functions is representing the line?

Correct Answer: y = 1 - 2x

Explanation: The linear function can be written as y = a + bx, where a is the intercept and b is the slope. From the question, we know that the slope is -2; so b = -2. The intercept in the value of y when x=0. From the question, we know the line passes through the point (0, 1). So the intercept is equal to 1. Therefore, we can conclude that the function y = a + bx can be written as y = 1 - 2x.

QUESTION 14

You decide to calculate the log returns using the following equation: $r_t = \ln(P_t) - \ln(P_{t-1})$.

Which of the following equations is the equivalent of the above one?

Correct Answer: $r_t = ln\left(\frac{P_t}{P_{t-1}}\right)$.

Explanation: According to the rules of logarithms, we know that ln(x) – ln(y) is equivalent to ln(x/y).

For this reason, $r_t = \ln(P_t) - \ln(P_{t-1}) = ln\left(\frac{P_t}{P_{t-1}}\right)$.

QUESTION 15

What is the present value of £10,000 expected to be received in two years' time, if the business concerned requires a return of 5% on sums invested?

Correct Answer: £9,070.30

Explanation: $PV = \frac{FV}{(1+r)^t} = \frac{10{,}000}{(1+0.5)^2} = £\ 9{,}070.30$