

1a)

$$\alpha + \beta = -\frac{b}{a} = \frac{8}{2} = 4$$

$$\alpha \times \beta = \frac{c}{a} = \frac{-1}{2} = -\frac{1}{2}$$

$$-\frac{a}{1} = \gamma + \delta = \alpha + \frac{1}{\beta} + \beta + \frac{1}{\alpha} = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta} = 4 + \frac{4}{-\frac{1}{2}} = -4$$

$$q = 4$$

$$r = \gamma\delta = \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \frac{1}{\alpha\beta} + \alpha\beta + 2 = \frac{1}{-\frac{1}{2}} - \frac{1}{2} + 2 = -\frac{1}{2}$$

So the new quadratic is:

$$x^2 + 4x - \frac{1}{2}$$

How to check this on your own?

1) From wolframalpha website:

just type $2x^2 - 8x - 1 = 0$

and it will find the solutions to $2x^2 - 8x - 1 = 0$,

$$\alpha = \frac{3}{2}\sqrt{2} + 2, \beta = 2 - \frac{3}{2}\sqrt{2}$$

The same for:

$$\text{Solutions to } x^2 + 4x - \frac{1}{2} = 0 \text{ are: } \frac{3}{2}\sqrt{2} - 2, -\frac{3}{2}\sqrt{2} - 2$$

Then check the relation between the roots (also in wolframalpha type just the numbers, here the sum):

$\gamma = \alpha + \frac{1}{\beta} = \frac{3}{2}\sqrt{2} + 2 + \frac{1}{2 - \frac{3}{2}\sqrt{2}} = -\frac{3}{2}\sqrt{2} - 2$ which is the same as the root of the new quadratic.

Similarly for the $\delta = \beta + \frac{1}{\alpha} = 2 - \frac{3}{2}\sqrt{2} + \frac{1}{\frac{3}{2}\sqrt{2} + 2} = \frac{3}{2}\sqrt{2} - 2$ which is the same as the other root of new quadratic.