

numeracy refresher workshop



Drop-In Study Centre v1.1

Learning Development

Welcome to the refresher booklet for basic maths skills. Maths is an integral part of all science and engineering degree programmes, so whether you like it or not, you will have to do some maths at some point in your degree. Most people have a mental block about maths along the lines of, "I can't do it; I don't like; and I won't do it", but this will not get you very far. Maths is like any other tool, a screwdriver or a spanner, to be used to get a job done and should be treated in the same way – I don't know anyone who is frightened to use a spanner!!!

This booklet covers the basics. These are simple, and may seem obvious, but in many cases can be easily forgotten if not used regularly, or most likely misunderstood at some point, making any further work difficult. You may already think that you know this and can skip through it, but it is in your interests to read the material carefully, attempt all the problems and check your answers.

One thing you should get used to is NO CALCULATORS. All of this work can be done in your head. It may take a little longer to start with, but with a little practise, you will find it a lot quicker.

OK, we all have seen numbers before, but they can be classified differently.

numbers

Integers are whole numbers with a regular interval of one unit. i.e. 1, 2, 3, 4, 5 etc.

Real numbers are all the integers and those numbers that fall in between them. There is a set of numbers classified as **complex**, but you will not need to know about them just yet.

Rational numbers are those that can be expressed as the ratio of two integers, and when expressed as decimals either terminate or repeat i.e. $\frac{1}{4} = 0.25$ (terminates), $\frac{1}{9} = 0.11111$ (repeats).

Irrational numbers are those that are not rational, i.e. cannot be expressed as a ratio of two integers. You will have already seen π which is irrational, and roots are generally irrational, e.g. $\sqrt{2}$

checkpoint 1 - numbers

Classify the following numbers:

- | | | | | | | | | | |
|----|---------------|----|------|----|------------|----|------------|-----|---------------|
| 1. | 8 | 2. | 0.33 | 3. | $\sqrt{6}$ | 4. | 1.23 | 5. | $\frac{2}{5}$ |
| 6. | $\frac{3}{7}$ | 7. | 365 | 8. | 0.125 | 9. | $\sqrt{3}$ | 10. | 0.66 |

Now we are familiar with the types of numbers, we can start to do some simple maths with them, but there is an order in which we should perform functions, the rules of arithmetic.

1. The Commutative Law

When adding or multiplying, the **order** in which the numbers are placed **doesn't matter** to the final outcome.

$$2 + 3 = 3 + 2 = 5 \quad \text{and likewise, } 2 \times 3 = 3 \times 2 = 6$$

The numbers **commute** and the operations are **commutative**.

When subtracting or dividing, the **order does matter**.

$$2 - 3 = -1 \neq 3 - 2 = 1 \quad \text{and } 2 \div 3 \neq 3 \div 2$$

The operations are not commutative.

2. The Associative Law

When adding or multiplying together more than two numbers, brackets can be inserted to break up the stages that the maths is performed without affecting the result.

$$2 + 3 + 4 = 2 + (3 + 4) = (2 + 3) + 4 = 9$$

$$2 \times 3 \times 4 = 2 \times (3 \times 4) = (2 \times 3) \times 4 = 24$$

These results are **associative**.

When subtracting or dividing numbers, brackets cannot be inserted to break up the stages, as the results are different.

$$2 - 3 - 4 = -5 \quad \text{whilst } 2 - (3 - 4) = 3$$

$$2 \div 3 \div 4 \text{ is not the same as } 2 \div (3 \div 4)$$

3. The Distributive Law

When multiplying or dividing the contents of a bracket by a factor outside the bracket, **ALL** terms inside the bracket must be multiplied or divided.

$$2 \times (3 + 4) = (2 \times 3) + (2 \times 4) = 14$$

$$\text{Likewise, } \frac{1}{2} \times (3 + 4) = (\frac{1}{2} \times 3) + (\frac{1}{2} \times 4) = \frac{7}{2} = 3\frac{1}{2}$$

This rule does not apply to powers or roots (see indices).

From these rules we get an order in which to approach various operations within an expression. **B**rackets should be dealt with first, then **I**ndices, **D**ivision, **M**ultiplication, **A**ddition and finally **S**ubtraction. This gives us the acronym **BIDMAS**. Remember this and you won't go far wrong.

checkpoint 2 – rules of arithmetic

Evaluate the following:

1. $6 + (4 - 3)$
2. $\frac{1}{2}(7 - 2)$
3. $(3 + 4) \div 3$
4. $3 \times (8 - 5)$
5. $2 + (2 \times 2)$
6. $(2 + 2) \times 2$
7. $(2 + 3) \times (4 - 1)$
8. $(4 \div 2 \times 3 - 1) \div 5$
9. $(7 \times 3 + 3) \div 8$
10. $8 \div 2 + 3 \times 3 - 7$

fractions

We have already encountered a couple of fractions previously under rational numbers, $\frac{1}{4}$ and $\frac{1}{9}$. Fractions are composed of a numerator (the number on the top) and a denominator (that on the bottom) with a fraction bar between them, and represent a part of a whole, such as one slice from a pizza. They cannot be simplified any further: $\frac{2}{4}$ is the same as $\frac{1}{2}$ as both the numerator and denominators are divisible by 2, whereas, $\frac{2}{3}$ cannot be simplified further.

Proper fractions are those where the numerator is smaller than the denominator, and these fractions always have a value less than 1. e.g. $\frac{3}{4}$

Improper Fractions are those where the numerator is greater than or equal to the denominator and these always have a value greater than 1. e.g. $\frac{7}{5}$

Mixed numbers are a combination of a whole number and a fraction. e.g. $1 \frac{1}{2}$

checkpoint 3 – classification of fractions

Classify the following

1. $2 \frac{3}{4}$
2. $\frac{3}{2}$
3. $\frac{7}{8}$
4. $\frac{6}{5}$
5. $\frac{11}{15}$
6. $1 \frac{2}{3}$
7. $\frac{2}{5}$
8. $\frac{25}{13}$
9. $3 \frac{1}{2}$
10. $\frac{13}{11}$

working with fractions

You should be able to convert between improper fractions and mixed numbers without any difficulty, but there is no harm in just recapping the procedure.

Remember, with an improper fraction, the numerator is greater than the denominator. e.g. $\frac{7}{5}$ This can be re-written as:

$$\frac{5+2}{5} = 1 + \frac{2}{5}$$

Of course, this works the other way around, so you look at the denominator, convert the whole number to a fraction (in the example above this will be $\frac{5}{5}$ ths) and then add the two fractions.

This brings us neatly onto **ADDING** and **SUBTRACTING** fractions. Fractions can only be added or subtracted if the denominator is the **same**. When there are two or more different denominators, then the lowest common denominator has to be determined **before** any addition or subtraction can take place.

e.g. Add $\frac{4}{5}$ and $\frac{5}{9}$. As we can see the denominators are not the same and the lowest common denominator is $5 \times 9 = 45$

In order to put both fractions into 45ths, we need to multiply $\frac{4}{5}$ by 9 and $\frac{5}{9}$ by 5 i.e.

$$\frac{4}{5} \times 9 = \frac{36}{45} ; \quad \frac{5}{9} \times 5 = \frac{25}{45}$$

Now we have two fractions that can be added

$$\frac{36}{45} + \frac{25}{45} = \frac{36+25}{45} = \frac{61}{45} = 1 + \frac{16}{45}$$

To **MULTIPLY** fractions, the numerators are multiplied together as are the denominators.

e.g. Multiply $\frac{4}{5}$ and $\frac{5}{9}$. In this case the numerators are 4 and 5, and the denominators are 5 and 9.

$$\frac{4}{5} \times \frac{5}{9} = \frac{4 \times 5}{5 \times 9} = \frac{20}{45} = \frac{4}{9}$$

To **DIVIDE** fractions, the divisor is inverted, and then the two fractions are multiplied together, but remember to cancel where appropriate.

e.g. Divide $\frac{4}{5}$ by $\frac{5}{9}$. In this case the divisor is $\frac{5}{9}$, which, when inverted becomes $\frac{9}{5}$, so...

$$\frac{4}{5} \div \frac{5}{9} = \frac{4}{5} \times \frac{9}{5} = \frac{36}{25} = 1 + \frac{11}{25}$$

checkpoint 4 – fractions

- | | | | |
|--------------------------------------|---------------------------------|----------------------------|--------------------|
| 1. $11/7 + 1/3$ | 2. $1/3 + 1/2$ | 3. $3/4 \times 7/8$ | 4. $2/5 \div 1/10$ |
| 5. $1/9 - 2/3$ | 6. $18/7 \div 3/2$ | 7. $1/2 \times 1/2$ | 8. $4/5 - 1/3$ |
| 9. $(3/4 - 1/2) \times 1/3$ | 10. $(2/5 + 1/6) \div 1/2$ | 11. $(2/5 - 1/6) \div 1/2$ | |
| 12. $(1/3 + 1/2) \times (1/4 + 2/3)$ | 13. $(2/3 + 1/4) : (1/2 + 1/3)$ | | |

fractions to decimals

Fractions and decimals are closely related, and any fraction can be written as a decimal. Some will be simple with exact answers and others will have an infinite number of digits. You will already be aware of the most common, $\frac{1}{2} = 0.5$, $\frac{3}{4} = 0.75$, and don't have to think about them, all that is happening is the numerator is being divided by the denominator. If the fraction is proper, the equivalent decimal will be less than one, and if it is, the decimal will be greater than one.

e.g. Convert $2/7$ to a decimal.

First of all, the fraction is proper, so we expect a decimal that is smaller than 1. Now we have to divide 2 by 7.

$$\begin{array}{r} 0.285714 \\ 7 \overline{) 2.000000} \end{array}$$

Notice that I have added a string of zeroes after the decimal point. This doesn't change the number 2, just makes it possible to divide in the normal way. The only difference is that the decimal point is included both in the number being divided and the quotient, or answer, above. I could also continue with the division if I wished, but in this case, 6 decimal places are enough to give the idea. It is often the case with fractions that are converted to decimals that they do not complete. There are three ways in which this can happen. They can either recur. $1/9$ is a good example of this as the answer is $0.11111\dot{1}$. The $\dot{}$ is used to denote recurrence. They can repeat, and $2/7$ is an example of this. If we had continued the division above we would have got $0.285714285714\dots$. Notice how the next 6 decimal places are exactly the same. To denote this we place a bar over the top of those digits that repeat i.e.

$$2/7 = 0.\overline{285714}$$

Finally, the decimals can go on forever without repeating. In this case, the decimal can be quoted to a certain number of decimal places and rounded. Rounding, decimal places, and significant figures are covered in a later section of this booklet.

decimals

If a number includes a decimal point with digits to the right of the decimal point, it is said to be written in **decimal notation**, or is a **decimal**, for short. This can become confusing as the decimal point is also often referred to as the decimal. Throughout this section, the word decimal will be used to refer to the number, and decimal point will refer to the point.

If we consider whole numbers, the position of a digit indicates the number of 1s, 10s, 100s etc., that there are in that number, and similarly, the position of the digits to the right of the decimal point indicate the 1/10ths, 1/100ths, 1/1000ths, etc., there are in the number. e.g. The number 123.456 is made up of the following:

100s	10s	1s	1/10ths	1/100ths	1/1000ths
1	2	3.	4	5	6

We can now look at the **ADDITION** and **SUBTRACTION** of decimals. When adding two decimals, the numbers being added are called addends and the answer the sum. The most important point when either adding or subtracting decimals is to line up the decimal point, i.e. keep the decimal point from each addend, directly above one another, and then just add up the columns, remembering, where necessary, to carry over a ten.

e.g. Add 0.125 and 3.256

$$\begin{array}{r} 0.125 \\ + 3.256 \\ \hline = 3.381 \end{array}$$

By keeping the decimal point in place throughout, there will be no mistakes in the answer.

Subtraction is done in the same way, aligning the decimal point, and then subtracting as you would normally, however, in subtraction, the terms used for the numbers are minuend and subtrahend, where the minuend is the number that is having the other subtracted from it, and the subtrahend is the number being subtracted. A handy hint for addition and subtraction is to always make the number of decimal places the same.

e.g Subtract 2.568 from 3.14

In this case, 3.14, the minuend, has 2 decimal places (dp) and 2.568 is the subtrahend and has 3dp. In order to give then the same number of dp we can say:

$$\begin{array}{r} 3.140 \\ + 2.568 \\ \hline = 0.572 \end{array}$$

Note that 3.14 was written as 3.140 to clarify the position of the decimal point.

decimals

MULTIPLYING decimals can be done as if the numbers are whole, with the decimal point inserted into the product as the final step. It's position is determined by adding how many places the right-most non-zero digit lies to the right of the decimal point in each factor.

e.g. Multiply 2.202×1.7

First of all we look at the numbers without the decimal points i.e. 2202 and 17 and multiply them as we would normally:

$$\begin{array}{r} 2202 \\ \times \quad 17 \\ \hline 15414 \\ + \quad 22020 \\ \hline = \quad 37434 \end{array}$$

Now we count up the number of decimal places in the original numbers. 2.202 has 3 dp and 1.7 1dp, totalling 4dp. We then count across from the right hand side of the above answer 4 places and insert the decimal point. This gives us the answer 3.7434. so $2.202 \times 1.7 = 3.7434$

Note that if we have 3.156×2.300 we can either work through as above using the figures 3156 and 2300, and then adjust the decimal point by 6 places in the answer, OR we can say that 2.300 is the same as 2.3, because the last two digits are zero. We then would multiply 3156 and 23, and adjust the decimal point by 4 places. Either way will give the same answer.

DIVISION of decimals is accomplished by treating the divisor as if it is a whole number. The easiest way to do this is by multiplying both the numbers by a factor of ten until they are whole numbers.

e.g. Divide 9.75 by 0.3

This is the same as writing the fraction $9.75/0.3$. If we multiply anything by 1, we get what we started, so if we multiply the above fraction by $10/10$ (or 1) we will have the division $97.5 \div 3$.

$$3 \overline{) 97.5}$$

So 9.75 divided by 0.3 is 32.5. One point to remember is to include zeroes where necessary in the quotient. In this way you can be certain of placing the decimal point correctly.

e.g. Divide 1.2 by 4

$$4 \overline{) 1.2}$$

The zero in the quotient places the decimal point correctly.

checkpoint 5 – decimals

1. $0.25 + 0.34$ 2. $0.125 + 0.543$ 3. $1.273 - 0.112$
4. 0.34×0.71 5. $0.14 - 1.246$ 6. 1.53×0.691
7. $0.6632 - 1.1001$ 8. $6.271 + 3.142$ 9. 0.932×10
10. $1.68 \div 0.21$ 11. $1.715 \div 0.245$ 12. $1.316 \div 100$
Convert to decimals:
13. $1/8$ 14. $1\ 3/5$ 15. $(1/2 + 1/3)$ 16. $11/4$ 17. $8/7$

rounding decimals

When we '**round**' a number, we replace an exact value with an approximation. Rounding a number is always done to a specified place value and in a specific way. If the digit immediately to the right of the specified place value is less than 5, the digit that is actually in the place value is not changed during rounding. If the digit to the right of the place value is 5 or more, then the digit in the specified place value is increased by 1. All of the digits to the right of the specified place value are then eliminated. Rounding decimals is basically the same as rounding whole numbers, except that if the decimal is being rounded to a place to the right of the decimal point, with any digits to the right of that point not replaced with zeros but simply eliminated.

e.g. Round the numbers 314159 to the nearest thousand, and 3.14159 to the nearest hundredth.

Looking at the whole number first, the hundreds part of the number is a 1. This is less than 5, so the thousands place will not be affected, making the number 314000.

Considering the decimal, we can see that the thousandth place is a 1, which is less than 5, so the hundredth place is unaffected making the answer 3.14.

Notice how the rounding in each case is the same, it is only the position of the decimal point that is different. Like most mathematics, the idea is simple, but subtle differences make it look difficult!

Inclusion of zeros. The number of decimal places quoted gives some idea to the accuracy of the number. In this sense, 23 is not as accurate as 23.000 as the latter has been quoted to 3 decimal places. Rounding always occurs in the last decimal place, making it the least accurate. When you are asked to give an answer to a problem to 2 decimal places, you have to work to 3 decimal places to ensure that the final answer is quoted accurately. Even if the answer is a whole number, it should still be quoted with the appropriate number of zeroes after the decimal place to denote the accuracy.

checkpoint 6 – rounding decimals

Evaluate to 2 decimal places:

1. 0.137 2. 1.245 3. 6.8739 4. 0.1455

Evaluate to 3 decimal places:

5. 1.11177 6. 0.3478 7. 8.0001 8. 1.7431

estimation

Estimation is a hugely powerful tool that is overlooked. Most people rely on calculators far too much to understand whether an answer is correct or not. It only takes a slip of the finger, or a slight lapse in concentration to input the wrong figure and out comes the wrong answer. **Calculators never make mistakes, but the operators often do!**

The quickest way to estimate any maths problem is to round the figures involved to numbers that can be worked with easily. The answer will not be accurate, but will be in the right 'ball park'.

e.g. Multiply 9.98 by 2.47.

This can easily be done the conventional way and will give the result 24.65 (2dp), but if I consider 9.98 to be almost 10, and 2.47 to be almost 2.5, I can instantly say the answer will be around 25. Of course, 24.65 is around 25, so I know that I have done the working correctly.

The more that you practise working without a calculator by using simple numbers to estimate an answer, the easier it will become, and the better your overall work will become. In most cases in science and engineering, it is better to have a feel for the problem, and be able to understand what is going on, than to have an accurate answer. An accurate answer is the icing on the cake, though!

checkpoint 7 – estimation

Estimate the following:

1. 4.79×3.41 2. 12.132×6.01 3. $371.047 - 65.999$
4. $16.85 - 2.67$ 5. $85.94 + 27.45$ 6. $2.545 + 13.666$
7. $17.989 \div 6.012$ 8. $14.23 \div 6.69$ 9. 6.254×3.142
10. $(2.35 + 3.65) \times 4.88$ 11. $(8.654 - 4.444) \div 2.156$

decimals to fractions

Converting decimals to fractions is very simple. The decimal portion of a decimal number, that is, the digits to the right of the decimal point, are read as some number of portions, with the name of the portions dependent on the place value of the right-most digit.

e.g. Write 1.25 as a fraction.

This is one whole unit, and 25 hundredths. Writing this as a fraction is $\frac{25}{100}$ and we can see that both the numerator and denominator are divisible by 25. This would give $\frac{1}{4}$, so the final fraction would be $1\frac{1}{4}$.

The more and more decimal places that there are, the larger the denominator has to become to accommodate the numerator. This doesn't make the conversion any more difficult though, but does take a little bit more experience to spot any factors that will simplify the fraction.

e.g. Convert the decimal 0.123456 to a fraction.

With 6 decimal places the denominator has to be 100,000 so the fraction will be:

$$\frac{123456}{100000} = \frac{61728}{50000} = \frac{30864}{25000} = \frac{15432}{12500} = \frac{7716}{6250} = \frac{3858}{3125}$$

The most obvious factor when the numerator is even is 2, and each time that this has been the case, I have divided both top and bottom by two. I cannot find a factor for the last fraction so this must be the answer.

So, in summary, for one decimal place, the fraction will be tenths, for 2 decimal places, the fraction will be hundredths, 3 decimal places will be thousandths and so on. It is as simple as that!

checkpoint 8 – decimals to fractions

- | | | | | | | | |
|-----|-------|-----|--------|-----|--------|-----|--------|
| 1. | 0.125 | 2. | 0.3332 | 3. | 0.8655 | 4. | 1.248 |
| 5. | 0.111 | 6. | 0.875 | 7. | 0.0833 | 8. | 2.8571 |
| 9. | 2.555 | 10. | 6.800 | 11. | 3.0025 | 12. | 8.564 |
| 13. | 0.123 | 14. | 7.6666 | 15. | 0.750 | 16. | 5.789 |

indices

When we have an expression x^y , x is the **base**, and y is the **index** or **exponent** (**indices** is the plural). So when we say something is squared, it has an index of 2. There are some general rules of indices, but these only apply if you are working in the same base.

For **MULTIPLICATION** of terms with the same base, we **add** the indices.

e.g. $x^2 \cdot x^3 = x^{2+3} = x^5$ longhand this would be

$$x \cdot x \cdot x \cdot x \cdot x = x \cdot x \cdot x \cdot x \cdot x = x^5$$

With **DIVISION** of terms with the same base, we **subtract** the indices. e.g.

$$\frac{x^2}{x^3} = x^{2-3} = x^{-1}$$

In longhand,

$$\frac{x \cdot x}{x \cdot x \cdot x} = \frac{1}{x} = x^{-1}$$

Note that x^{-1} means the reciprocal of x , or $1/x$. The minus sign represents the reciprocal function, and the 1 is the power, so if we have x^{-2} , we would have the reciprocal of x^2 which is $1/x^2$.

For **POWERS** of indices, we multiply the powers together.

e.g. $(x^2)^3 = x^{(2 \times 3)} = x^6$

FRACTIONAL INDICES denote roots, and $x^{1/2}$ is the same as saying the square-root of x (\sqrt{x}). The index $1/3$ is the same as the cube-root and so forth. Fractional indices can be treated the same way as any other, so if the base is the same, the terms will multiply and divide as per the above rules.

POWER OF ZERO. This is a simple proof:

$$x^a/x^a = 1, \text{ however, } x^a/x^a = x^{a-a} = x^0 \therefore \text{it follows that } x^0 = 1$$

This is a general rule and any term to the power zero is unity.

checkpoint 9 – indices

- | | | | |
|--------------------------|------------------------------|-----------------------------------|---------------------|
| 1. $x^2 \cdot x^3$ | 2. $a^{1/2} \cdot a^2$ | 3. $y^{-1} \div y^3$ | 4. x^4/x^8 |
| 5. a^5/a^{-2} | 6. $x^2 y^4/x^3 y^3$ | 7. a^0 | 8. $(y^3)^3$ |
| 9. $(2x)^2$ | 10. $(ax)^3/a^4$ | 11. $(x^{1/3} \cdot x^{1/2})/x^4$ | 12. $1/x^6 + 1/x^5$ |
| 13. $(a^2 + 1)(a^2 - 1)$ | 14. $\sqrt{b} \cdot b^{1/2}$ | 15. $(3y)^3/9y$ | |

standard form

Standard form, or scientific notation, is used to express very large or very small numbers. A number expressed in standard form is written as a number of value 1 or greater, but less than 10, multiplied by a power of ten.

e.g. 1.23×10^2

Here we can see we have the number 1.23 multiplied by 10^2 . If we evaluate this, we get $1.23 \times 100 = 123$. Of course, for 123, this is a rather long way to express the number, but if we consider the speed of light, 299792500ms^{-1} . This is a rather large number, but can be expressed in standard form as $2.997925 \times 10^8\text{ms}^{-1}$. This number is still a little cumbersome, but if we approximate the number by rounding it slightly, we get $3 \times 10^8\text{ms}^{-1}$. This does make it much easier to handle.

Of course, what works with very large numbers also works with very small numbers, we will just have a negative index. If we consider the mass of the electron, we know this will be incredibly small because an electron is, so to express it in kilograms, we would have to have lots of zeroes after the decimal point before even reaching a number. In standard form it would be written:

$$9.1 \times 10^{-31}\text{kg}$$

Combined with the previous section on indices, and multiplication or division of decimals, we have a very quick way of evaluating very large or very small numbers. The powers of ten can be evaluated by adding or subtracting the indices, and the decimals, all being between 1 and 10, are simple to calculate. They are also very close to whole numbers each time, so we can even estimate them simply.

checkpoint 10 – standard form

Put the following into standard form, giving your answer to 2 decimal places:

- | | | |
|-------------------|---------------|----------------|
| 1. 234987 | 2. 0.00000254 | 3. 65784233322 |
| 4. 0.000000001445 | 5. 1236668455 | 6. 0.00012475 |
| 7. 3365 | 8. 45686332 | 9. 0.00111245 |

Expand the following from standard form:

- | | | |
|-----------------------------|-----------------------------|-----------------------------|
| 10. 1.22×10^4 | 11. 3.6665×10^{-3} | 12. 8.95×10^6 |
| 13. 9.944×10^{-3} | 14. 5.68745×10^3 | 15. 1.0×10^6 |
| 16. 8.4962175×10^2 | 17. 0.100001×10^7 | 18. 2.3354×10^{-6} |

significant figures

Finally we can deal with **SIGNIFICANT FIGURES**. Significant figures (sig. fig.) relate to the certainty or accuracy of the measurement of a number, and as the number of significant figures increases, the more certain or accurate the measurement becomes.

Scientific notation is the most reliable way of expressing a number to a given number of significant figures. In standard form, the power of ten becomes insignificant as it is a string of zeros (10^3 is 1000).

If we express the number 4000 in standard form to varying degrees of accuracy we can say:

4000 = 4×10^3	Here we have used one sig. fig.
= 4.0×10^3	Here we have used two sig. figs.
= 4.00×10^3	Here we have used three sig. figs.
= 4.000×10^3	And here we have used four sig. figs.

If we take each one in turn we can see the accuracy that applies. With the first example, 4000 to one sig. fig. lies between $3 \times 10^3 = 3000$ and $5 \times 10^3 = 5000$. Obviously the range 3000 – 5000 is very large – 2000 units -, so the accuracy of 4000 in this case is not very good. With the second example, the range starts at $3.9 \times 10^3 = 3900$ and finishes at $4.1 \times 10^3 = 4100$ making it just 200 units i.e. a tenth of the previous example, so the accuracy is much improved. The range of the next example narrows to just 20 units, and for the final example just two units, and the accuracy becomes greater and greater.

Two rules apply when significant figures are being used in calculations: For **MULTIPLICATION** and **DIVISION**, the final result is rounded to the least number of significant figures of any one term, e.g.

$$\frac{(16.04)(2.56)}{(2.789)} = 14.7$$

The answer 14.7 has 3 sig. figs. as 2.56 has 3 and both 16.04 and 2.789 have 4.

For **ADDITION** and **SUBTRACTION** the final result is rounded to the least number of decimal places, regardless of the significant figures of any one term, e.g.

$$\begin{array}{r} 2.005 \\ + 14.35 \\ \hline 16.4026 \\ \hline \end{array}$$

which rounds to 16.40 as the second number 14.35 only has 2 decimal places.

checkpoint 11 – significant figures

Give the following to 2 significant figures:

1. 2.3697 2. 587463 3. 1.23 4. 6.001

Give the following to 3 significant figures:

5. 6.35888 6. 45.476 7. 0.00225 8. 1.587

Evaluate the following giving your answer to the appropriate number of significant figures:

9. 2.35×5.112 10. $3.012 + 16.52$ 11. $27.9 \div 3.0$

12. $(1.113 \times 5.22) \div 3.221$ 13. $(8.2 \times 4.2654) \div 6.15$

15. $2.3387 - 5.360 + 2.5511$ 16. $5.20 + 7.011 - 8.555$

Well that is nearly all for this booklet. If you have worked through everything up until now, you should now be able to tackle the first section of the diagnostic test with ease, and you will also have an easier time in all of your course modules. But, of course, maths doesn't always follow a well planned out route such as that designed for this booklet, so the next section is a random selection of questions for you to do.

checkpoint 12 – pick 'n' mix

Classify the following numbers and fractions:

1. 0.25 2. 81 3. $\sqrt{7}$ 4. 0.11 5. $5/8$

6. $1 \frac{1}{2}$ 7. $13/5$ 8. $3/4$ 9. $2/7$ 10. $6 \frac{2}{3}$

Evaluate the following:

11. $6(4 + 2)$ 12. $1.65 + 3.47$ 13. $1/2 \times (1/2 + 1/3)$

14. $(5 + 7) \div 3$ 15. $2 \frac{5}{8} + 1/4$ 16. $(3 - 1) \times 5$

17. $0.35 + 1.12$ 18. $6 \div 2 + 4$ 19. $(2 - 6) \times (3 + 4)$

20. $3 \times 3 + 8$ 21. $7/8 + 1/12$ 22. $3 \frac{7}{8} - 2 \frac{1}{12}$

23. 6.33×8.45 24. $9/6 \div 1/3$ 25. $8 \div 4 + 3 - 7$

26. 0.352×1.2 27. $5/6 + 2/3$ 28. $(5 + 4) - (8 - 3)$

Convert to decimals:

29. $2/9$ 30. $1/13$ 31. $1 \frac{2}{11}$ 32. $9/7$

33. $1/6 + 1/4$ 34. $10/7 - 1/3$ 35. $1/2 \times 1/3$

36. $5/8$ 37. $4/15$ 38. $1/6$ 39. $8/3$ 40. $1/12$

Evaluate the following to 2 decimal places:

41. 1.568 42. 0.447 43. 3.664 44. 5.006

Evaluate the following to 3 decimal places:

45. 2.3564 46. 1.44458 47. 0.99865 48. 1.3654

Estimate the following:

49. 5.68×9.12 50. 0.55×20.21 51. $5.687 + 9.154$

52. $89.654 - 64.157$ 53. $654 + 4.887$ 54. $23.875 \div 8.024$

Convert the following:

55. 0.1246 56. 5.248 57. 2.665

58. 3.001 59. 8.215 60. 4.46

Evaluate the following indices:

61. $z^4 \cdot z^2$ 62. $d^{-3} \div d^3$ 63. $(a^2)^8$ 64. x^0

65. $1/k^2$ 66. $x^6 \cdot x^8 / x^5$ 67. $a^{1/4} \cdot a^{2/3}$ 68. $\sqrt[3]{(x^3)}$

Express the following in standard form with answers to 2 decimal places:

69. 3354874 70. 0.0125998 71. 123655447555

72. 0.00020014 73. 526 74. 0.000000000354

Express the following to 2 significant figures:

75. 8000 76. 2645 77. 0.0002554

78. 5.66477 79. 0.2144 80. 356874

You should now check all of your answers.

If you are still encountering problems with the topics covered in this booklet, then visit the Drop-In Study Centre, situated on the first floor of the Main Library - West reading rooms - LD1, at the times given on the website. http://www.library.qmul.ac.uk/learning_support/disc