algebra refresher workshop

Maths DISC

Queen Mary
University of London

Drop-In Study Centre v1.1

Learning Development

Welcome to the refresher booklet for algebra skills. Maths is an integral part of all science and engineering degree programmes, so whether you like it or not, you will have to do some maths at some point in your degree, and algebra is probably the most common topic that arises. Most people have a mental block about maths along the lines of, “I can’t do it; I don’t like; and I won’t do it”, but this will not get you very far. Maths is like any other tool, a screwdriver or a spanner, to be used to get a job done and should be treated in the same way – I don’t know anyone who is frightened to use a spanner!!!

This booklet covers the basics and then uses this to guide you through the more common areas that are encountered at University. You may already think that you know some of the material and that can skip through it, but it is in your interests to read the booklet carefully and in order, attempt all the problems and check your answers.

One thing you should get used to is NO CALCULATORS. All of this work can be done in your head. It may take a little longer to start with, but with a little practise, you will find it a lot quicker.

getting started

Until now maths has generally only concerned numbers, but at University level we often want to determine unknowns, and we represent unknowns by a symbol or letter. The most common of these are x, y, and z, or a, b and c, but it is just as well to be able to recognise other symbols and the most commonly used are from the Greek alphabet:

Α	α	*alpha*	Ν	ν	*nu*
Β	β	*beta*	Ξ	ξ	*xi*
Γ	γ	*gamma*	Ο	ο	*omicron*
Δ	δ	*delta*	Π	π	*pi*
Ε	ε	*epsilon*	Ρ	ρ	*rho*
Ζ	ζ	*zeta*	Σ	σ	*sigma*
Η	η	*eta*	Τ	τ	*tau*
Θ	θ	*theta*	Υ	υ	*upsilon*
Ι	ι	*iota*	Φ	ϕ	*phi*
Κ	κ	*kappa*	Χ	χ	*chi*
Λ	λ	*lambda*	Ψ	ψ	*psi*
Μ	μ	*mu*	Ω	ω	*omega*

Now that we know what the symbols and letters just represent an unknown number, we can start working with them in the same way we would approach any maths question. *Provided the terms are the same*, the order in which we add or subtract them doesn’t matter.

e.g. $2x + 3x = 3x + 2x = 5x$

but, care has to be taken if the terms are different.

e.g. $2x^2 + 3x$ cannot be simplified further

The first term is in x^2 and the second term in x. This is like adding apples and oranges – they are different fruit and you cannot simplify further.

factorising & expanding

Factorisation is useful in tidying-up equations but is not always vital. To factorise we look for a term *common* to all parts of the equation.

e.g $y = 3x + 6z + 9$

3 *is* common to all the terms on the right (*the common factor*) and can be brought outside brackets

i.e. $y = 3(x + 2z + 3)$

The equations are *exactly the same*, but the latter can be easier to deal with.

Expansion is the opposite of factorisation. Some equations will have two or more brackets to multiply out and mistakes are easy to make if you do not approach the question with a method. Perhaps one of the easiest is to use is:

e.g $y = (x+3)\ (x+5)$

In this way ALL the terms are multiplied out:

$$y = x.x + 3.5 + 3.x + x.5$$

$$y = x^2 + 3x + 5x + 15$$

$$y = x^2 + 8x + 15$$

So, **in general**: $(x + a)\ (x + b) = x^2 + (a + b)x + ab$

Note: To avoid confusion with x, the multiplication sign is often replaced by "." (x.x) or omitted completely (ab).

Care has to be taken with algebra when dealing with signs. An unknown may be a negative number, but is represented by a positive symbol. What should be remembered is:

(1) Adding a negative number is the same as subtracting a positive number

(2) Subtracting a negative number is the same as adding a positive number

e.g. If x=8 and y=-2, then,

$x + y = 8 + (-2) = 8 - 2 = 6$

$x - y = 8 - (-2) = 8 + 2 = 10$

checkpoint 1 – factorising and expanding

Factorise the following expressions:

1.	$5a + 15ax$	2.	$4x + 2x^2$	3.	$3p - 9q$	4.	$2x - 12y$
5.	$\frac{1}{2}x + \frac{1}{4}y$	6.	$x^2 - a + \frac{1}{4}$	7.	$a^2 + 8a + 7$	8.	$16b^2 - 1$

9. $5x^2 – 4x - 1$ 10. $s^2 + 6s – 7$ 11. $3x^2 + x$ 12. $16x^4 + 4x^2$

13. $6x^2 + 13x + 6$ 14. $2a + 4b$ 15. $x^2 – 1$ 16. $-x^2 + 1$

17. $z^2 + 5z + 6$ 18. $x^2 + 2xy + y^2$ 19. ${}^{x}/_{3} + {}^{y}/_{6}$ 20. $x^2 - 4$

Expand the following expressions:

21. 3(ab) 22. 3(x + y) 23. x(a – b) 24. 5(p + q)

25. $6(2ab^2)$ 26. 7(xy) 27. $3(6d^3c)$ 28. 2(a – 4b)

29. (am)n 30. (2 + a)(2 – a) 31. $(x + y)^2$

32. (x + 2)(x – 3) 33. a(a +2x) 34. (3 – y)(y + 5)

35. (x + 3)x 36. (s + 1)(s + 2)(s + 3) 37. (5 – 3a)(a + 1)

38. $(x + 1)^3$ 39. (½x + 1)(½x + 4) 40. (9 + y)(2 – y)

fractions

Of course, we can also have algebraic fractions, and this is where a lot of mistakes arise. If we have two fractions 1/x and 1/y, then

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{x+y} \qquad \text{WRONG!}$$

Fractions can only be added if the DENOMINATOR is the same (the bit underneath the line), we must find the *lowest common denominator*.

When we multiply something by 1, we do not alter it, so if we multiply by a/a (which is 1) then we do not change the fraction.

$$\frac{1}{x} = \frac{a}{ax} \qquad \text{for any value of a}$$

So for our example above, if we multiply 1/x by y/y and 1/y by x/x we do not change either term, but…

$$\frac{y}{xy} + \frac{x}{yx} = \frac{y + x}{yx} \qquad \text{RIGHT!}$$

Of course, the same applies when you are subtracting fractions, the lowest common denominator must be determined first.

checkpoint 2 – algebraic fractions

Express the following as a single fraction:

1. ${}^{1}/_{2x} + {}^{1}/_{3y}$ 2. ¼ . 2(a + b) 3. $\frac{2}{3}$ p – q)

4. $\frac{3}{x}$. (x + 4)/7 5. $1/(\frac{x}{y})$ 6. $\frac{x}{y} \cdot \frac{(x+1)}{(y+1)}$

7.	$\frac{5/6}{(t+4)}$	8.	$\frac{3/4}{(a-1)}$	9.	$\frac{(a-1)}{(3/4)}$	10.	$\frac{1}{z} \cdot \frac{x+3}{z+1}$	11.	$\frac{3}{a+3} \div \frac{x}{2a+6}$
12.	$\frac{6}{2x+1} \div \frac{3x}{4}$	13.	$\frac{3}{a+3} + \frac{x}{2a+6}$			14.	$\frac{1}{z} + \frac{x+3}{z+1}$	15.	$\frac{6}{2x+1} - \frac{3x}{4}$
16.	$\frac{1}{x+3} + \frac{2}{x+2}$	17.	$\frac{x+1}{x+3} + \frac{x+2}{x+4}$			18.	$\frac{2}{3x+2} - \frac{3}{x+3}$	19.	$\frac{a}{7} \cdot \frac{11}{21}$

general rules

General Rules of Algebra

(1) An equation is unaffected if:

The same amount is added to or subtracted from each side

e.g. $y = x + 2$ add 4 throughout, $y + 4 = x + 6$

Each side is multiplied or divided by the same amount.

e.g. $y = 2x$ multiply throughout by 4, $4y = 8x$

(2) Any term that appears on *both* the top and bottom of the *same* side of an equation can be cancelled.

$$\frac{8x}{8} = \frac{y}{4}$$

The 8s on the left hand side of the equation cancel to give…

$$x = \frac{y}{4}$$

(3) Both sides of an equation can be inverted and the resulting equation will hold

If $x = a + b$ then, $\frac{1}{x} = \frac{1}{a+b}$

Note that a + b is treated as one term!

checkpoint 3 – general rules

Simplify the following:

1.	$\frac{9x}{3}$	2.	$\frac{2}{8a}$	3.	$\frac{2x^4}{6x}$	4.	$\frac{10}{5y}$	5.	$\frac{3}{4a} \cdot \frac{2b}{6}$
6.	$\frac{12x}{6} \cdot \frac{3}{6x}$	7.	$\frac{7x}{4y} \cdot \frac{2}{14x}$			8.	$\frac{16x^2}{8}$	9.	$\frac{2}{14x} \div \frac{4}{21x^3}$

Often we have to deal with two equations that contain the same terms, and it is possible to add or subtract the equations so that on of the unknown terms is eliminated. When we use two or more equations to solve for an unknown, we are solving using **simultaneous equations**.

e.g If we consider the equations:

$$y = 3x + 2 \quad (1)$$
and
$$y = 10 - x \quad (2)$$

To find x and y we have to use both equation (1) and (2) at the same time – *simultaneously*.

If we subtract equation (2) from equation (1) we can eliminate y and find x

(1) - (2)
$$\begin{array}{rl} & y = 3x + 2 \\ - & y = -x + 10 \\ \hline & 0 = 4x - 8 \end{array}$$

as $y - y = 0$, $3x - (-x) = 4x$ and $2 - 10 = -8$

We can now re-arrange this to get $4x = 8$, and this gives us $x = 2$.

If we now substitute this value into both equations (1) and (2) we can get the value of y.

In (1) $\quad y = 3(2) + 2 = 8$
and in (2) $\quad y = 10 - 2 = 8$

Both ways we get $y = 8$ and we know that $x = 2$ and $y = 8$ are the correct answers. *Always* substitute into both equations to check your answer is correct. Often, it is necessary to multiply equations by a number throughout so that the unknown term disappears, and in these cases it is wise to label the 'new' equations to avoid confusion. In general we need ***n*** equations to be able to determine ***n*** unknown terms.

checkpoint 4 – simultaneous equations

Solve the following equations simultaneously for all the unknowns:

1. $4a + 3b = 24$; $2a + 3b = 18$
2. $3x - y = 11$; $2x + y = 9$
3. $3p + 4q = 18$; $4p - 3q = -1$
4. $7s + t = 9$; $3s + t = -3$
5. $3x + 2y = 18$; $4x - y = 2$
6. $3x - 2y = 9$; $x + 2y = 15$
7. $a + 3b = 12$; $a + b = 10$
8. $4\alpha + 2\beta = 10$; $3\alpha + 5\beta = 11$
9. $2x - y - z = 5$; $4y + 3z = 5$; $x + 2y = 7$
10. $x + y = 12$; $x - y = 2$
11. $x + 2y = 4$; $x + 3z = 5$; $2y - z = 1$
12. $p + 3q = 6$; $2p + q = 7$
13. $2a + 4b - c = 1$; $a + 2b + 3c = -3$; $b - a - 4c = -10$

The general form of a quadratic equation is:

$$ax^2 + bx + c = 0$$

where a, b, and c are known numbers or *coefficients*. Sometimes it possible to factorise the equation into a product of the two roots simply by observation.

$$(x + d)\,(x + e) = 0$$

In this case the sum of the numeric terms will be b/a and the product will be c/a i.e.

$$d + e = b/a \qquad\qquad de = c/a$$

e.g. If we consider the quadratic: $x^2 + x - 6 = 0$

The sum of the two numeric terms would be 1 (the coefficient of x) and the product would be –6. For this to be the case, the two terms would be:

$$(x + 3)\,(x - 2) = 0$$

Things are not normally this simple, and to be able to solve a quadratic equation easily, we have to use other methods.

Another way to solve quadratic equations is by using the formula (derived by completing the square:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The two roots of the quadratic equation are given by the ± sign in front of the square root term (called the *discriminant*). If...

$b^2 - 4ac > 0$ The roots are real and different
$b^2 - 4ac = 0$ The roots are real and the same
$b^2 - 4ac < 0$ The roots are imaginary

e.g Find the roots of the quadratic equation:

$$3x^2 + 4x - 5 = 0$$

a = 3, b = 4, and c = -5 *Remember the minus sign!*

In the equation we have:

$$x = \frac{-4 \pm \sqrt{4^2 - 4.3.(-5)}}{2.3}$$

$$x = \frac{-4 \pm \sqrt{16 + 60}}{6} \qquad = \frac{-4 \pm \sqrt{76}}{6}$$

Therefore $x = \dfrac{-4 + \sqrt{76}}{6}$ or $\dfrac{-4 - \sqrt{76}}{6}$

$x = 0.786$ or -2.120

checkpoint 5 – quadratic equations

Use any method to evaluate the roots of the following quadratic equations:

1.	$x^2 + 2x - 8 = 0$	2.	$x^2 - 5x - 5 = 1$
3.	$a^2 + 12a + 36 = 0$	4.	$s^2 - 8x + 16 = 0$
5.	$p^2 + 11p - 12 = 0$	6.	$a^2 - 6a - 11 = 5$
7.	$2x^2 + 1 = 3x$	8.	$4x^2 - 9x = -2$
9.	$x^2 - 3x = 4$	10.	$2x - 1 + 3x^2 = 0$

Solve the following for the unknown:

11.	$x(x - 2) =0$	12.	$x(4x + 1) = 3x$
13.	$2x - 8 = -x^2$	14.	$10 + x^2 = -7x$
15.	$x(x + 3) = 4$	16.	$2x - 8 = -x^2$
17.	$4x^2 - 3 = 11x$	18.	$x(2 - x) = 1$
19.	$2x^2 + 6x = 0$	20.	$2(x^2 + 2) = x(x - 4)$

You should now check all of your answers.

If you are still encountering problems with the topics covered in this booklet, then visit the Drop-In Study Centre, situated on the first floor of the Main Library - West reading rooms - LD1, at the times given on the website. http://www.library.qmul.ac.uk/learning_support/disc