**SEJ014 Principles of Mathematics**

# Powers and Logarithms: Key Facts

| NAME | MEANING | REMEMBER |
|---|---|---|
| **Exponentiation** | **base** $a$ is raised to **power** $x$ | $a^x$ means "$a$ to the power $x$" |
| **Logarithm** | the power of **base** $a$ that gives **argument** $x$ | $\log_a x$ means "the power of $a$ that is $x$" |

Exponentiation is defined for **any power**. Logarithms are **not defined for non-positive arguments** (unless we go beyond the real numbers, and except for 'artificial' examples — see (2) below).

Here are the key facts about exponentiation and logarithms:

| | **Power** | **Log** |
|---|---|---|
| 1 | $a^0 = 1$<br>$1^0 = (-1)^0 = 0^0 = 10000^0 = 1$ | $\log_a 1 = 0$<br>$\log_1 1 = \log_{-1} 1 = \log_{-100} 1 = 0$ |
| 2 | $a^1 = a$<br>$1^1 = 1,\ 0^1 = 0,\ (-1)^1 = -1$ | $\log_a a = 1$<br>$\log_1 1 = 1,\ \log_{-1}(-1) = 1 \text{ (or } -1),\ \log_{1/2} \frac{1}{2} = 1$ |
| 3 | $a^{-1} = \frac{1}{a}$<br>$2^{-1} = \frac{1}{2},\ \left(\frac{1}{3}\right)^{-1} = 3,\ (-1)^{-1} = -1$ | $\log_a \left(\frac{1}{a}\right) = -1$<br>$\log_2 \left(\frac{1}{2}\right) = \log_{1/2} 2 = -1$ |
| 4 | $a^{1/2} = \sqrt{a}$<br>$2^{1/2} = \sqrt{2},\ 4^{1/2} = 2,\ 8^{1/2} = \sqrt{8} = 2\sqrt{2}$ | $\log_a \left(\sqrt{a}\right) = \frac{1}{2}$<br>$\log_{10} \left(\sqrt{10}\right) = \log_4 2 = \log_9 3 = \frac{1}{2}$ |
| 5 | $a^{p/q} = \sqrt[q]{a^p}$<br>$2^{4/3} = \sqrt[3]{2^4} = \sqrt[3]{16} = 2\sqrt[3]{2},\ (-1)^{5/3} = \sqrt[3]{(-1)^5} = \sqrt[3]{-1} = -1$ | $\log_a \left(\sqrt[q]{a^p}\right) = \frac{p}{q}$<br>$\log_2 \left(\sqrt[3]{16}\right) = \frac{4}{3},\ \log_{10} \sqrt[4]{\frac{1}{10}} = -\frac{1}{4}$ |
| 6 | $a^m \times a^n = a^{m+n}$<br>$2^4 \times 2^5 = 2^{4+5} = 2^9,\ 2^4 \times 2^{-5} = 2^{4-5} = 2^{-1} = \frac{1}{2}$ | $\log_a(xy) = \log_a x + \log_a y$<br>$\log_{10} 40 = \log_{10} 10 + \log_{10} 4 = 1 + \log_{10} 4$ |
| 7 | $a^m / a^n = a^{m-n}$<br>$2^4/2^5 = 2^{4-5} = 2^{-1} = \frac{1}{2},\ 2^1/2^{-5} = 2^{1-(-5)} = 2^6$ | $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$<br>$\log_{10} \left(\frac{x}{100}\right) = \log_{10} x - \log_{10} 100 = -2 + \log_{10} x$ |
| 8 | $(a^m)^n = a^{mn}$<br>$\left(2^3\right)^4 = 2^{3\times4} = 2^{12},\ \left(2^3\right)^{-2} = 2^{3\times-2} = 2^{-6} = \frac{1}{2^6}$ | $\log_a (x^y) = y \log_a x$<br>$\log_5 \left(25^k\right) = k \times \log_5 25 = k \times 2 = 2k$ |
| 9 | $a^{\log_a x} = x$<br>$2^{\log_2 4} = 2^2 = 4,\ 2^{\log_2(1/2)} = 2^{-1} = \frac{1}{2}$ | $\log_a (a^x) = x$<br>$\log_5 \left(5^k\right) = k \times \log_5 5 = k \times 1 = k$ |

**Note:** $\log_{10}$ is abbreviated to 'log' on most calculators.

The abbreviation 'ln' stands for $\log_e$, the **natural logarithm** where $e = 2.71828$ to 5 decimal places. This is also available on calculators. Taking logs to other bases is covered in Maths 1 (some calculators will handle any base).