Queen Mary
University of London

May Examination Period 2024

ECN225 Econometrics 2 Duration: 2 hours

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

Answer ALL questions. The questions carry equal marks. JUSTIFY ALL YOUR ANSWERS. Cross out any answers that you do not wish to be marked. There are tables of critical values from the standard normal, $F_{m,\infty}$, ADF, χ^2_m, and QLR distribution provided on pages 6-7. Also, the cumulative standard normal distribution tables are provided on pages 8-9.

Only nonprogrammable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM

Examiner: Dr. Jinu Lee

Question 1 (25 marks)

Suppose you set out to forecast the unemployment rate in the United States (UrateUS), using quarterly data from the first quarter of 1960 (1960:Q1) to the fourth quarter of 1999 (1999:Q4).

(a) Table 1 gives changes in the United States aggregate unemployment rate for the period 1999:Q1-Q4 and levels of the current and lagged unemployment rates for 1999:Q1. Fill in the blanks for the missing unemployment rate levels. **(5 marks)**

Table 1: Changes in Unemployment Rates in the United States 1999:Q1 to 1999:Q4

Quarter(t) (t)	Unemployment Rate ($UrateUS_t$, %)	First Lag ($UrateUS_{t-1}$, %)	Change of Unemployment Rate ($\Delta UrateUS_t$, %)
1999:Q1	4.3	4.4	-0.1
1999:Q2			0.0
1999:Q3			-0.1
1999:Q4			-0.1

(b) Table 2 presents the first four autocorrelations for the United States aggregate unemployment rate and its change for the time period 1960 (first quarter) to 1999 (fourth quarter). Explain briefly what these two autocorrelations measure. **(5 marks)**

Table 2: First Four Autocorrelations of the US Unemployment Rate and Its Change, 1960:Q1-1999:Q4

Lag	Unemployment Rate ($UrateUS_t$)	Change of Unemployment Rate ($\Delta UrateUS_t$)
1	0.97	0.62
2	0.92	0.32
3	0.83	0.12
4	0.75	-0.07

(c) Suppose you decide to estimate an AR(1) and an AR(4) in the change in the United States unemployment rate to forecast the aggregate unemployment rate. The results are as follows:

$$\widehat{\Delta UrateUS_t} = \underset{(0.022)}{-0.003} + \underset{(0.106)}{0.621}\Delta UrateUS_{t-1}, \quad R^2 = 0.393, \quad SER = 0.255$$

$$\widehat{\Delta UrateUS_t} = \underset{(0.022)}{-0.005} + \underset{(0.125)}{0.663}\Delta UrateUS_{t-1} - \underset{(0.139)}{0.082}\Delta UrateUS_{t-2} + \underset{(0.117)}{0.106}\Delta UrateUS_{t-3} - \underset{(0.091)}{0.176}\Delta UrateUS_{t-4}, \quad R^2 = 0.416, \quad SER = 0.253$$

Considering the actual unemployment rate for 2000:Q1 is 4.0%, what is your forecast for the unemployment rate level in 2000:Q1? Compare the forecast error of the AR(4) model with the forecast error of the AR(1) model. You may refer to the information provided in Tables 1 and 2 if necessary. **(5 marks)**

(d) If much of the forecast error arises as a result of future error terms dominating the error resulting from estimating the unknown coefficients, then what is your best guess of the RMSFE (Root Mean Squared Forecast Error) for the both models in part (c)? Justify it. **(5 marks)**

(e) Further, suppose you consider more lags for the autoregressive model. Discuss and explain how to determine the order of an autoregression model. **(5 marks)**

Question 2 (25 marks)

(a) Consider the stationary AR(1) model $Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t$. Show that $E(Y_t) = \frac{\beta_0}{1-\beta_1}$. **(5 marks)**

(b) Show that the random walk with no drift has a unit root. Further, prove that its variance increases linearly over time. **(5 marks)**

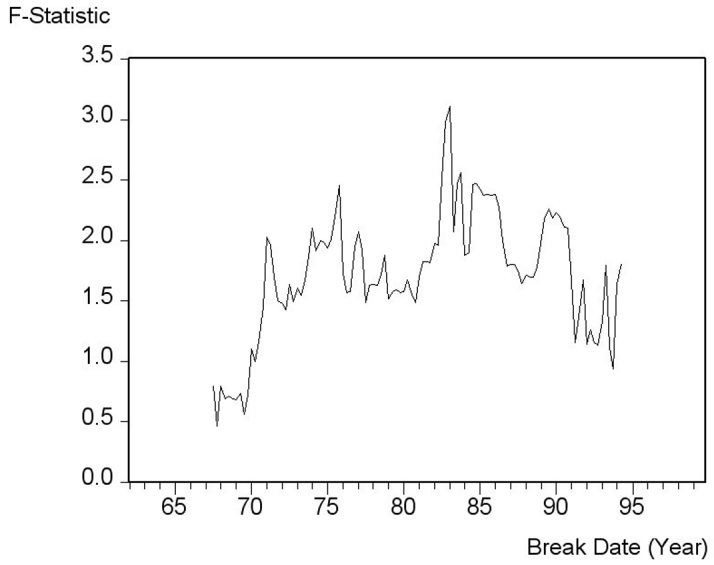
(c) Let X_t and Y_t be time series with a unit root, i.e. series that have a stochastic trend. Is it a good idea to run the OLS regression of Y_t on X_t? Why or why not? **(5 marks)**

(d) Suppose you have collected quarterly data on UK inflation (Inf) from 1962 to 1999 to forecast UK inflation. You run the following regression, where the numbers in parenthesis are homoskedasticity-only robust standard errors:

$$\widehat{\Delta Inf}_t = \underset{(0.28)}{0.49} - \underset{(0.05)}{0.10} Inf_{t-1} - \underset{(0.09)}{0.39} \Delta Inf_{t-1} - \underset{(0.09)}{0.33} \Delta Inf_{t-2} - \underset{(0.09)}{0.21} \Delta Inf_{t-3} + \underset{(0.08)}{0.05} \Delta Inf_{t-4}.$$

Test for the presence of a stochastic trend in the inflation time series (Inf). Also, does the fact that you use quarterly data suggest including four lags in the above regression, or how should you determine the number of lags? **(5 marks)**

(e) There is some evidence that the Phillips curve has been unstable during the 1962 to 1999 period for the United States, and in particular during the 1990s. Suppose you set out to investigate whether or not this instability also occurred in the United Kingdom. You consider testing for a break in the stability of the UK unemployment rates ($Urate_t$) based on the ADL(4,4) model for the inflation changes (ΔInf_t). The following graph shows the $F-$statistics testing for a break at different dates over the period:



Given the QLR statistic with 15% trimming turns to be 3.11, first state the null and alternative hypotheses and test for the hypothesis of a break in the ADL(4,4) regression. Also, provide comments on the figure. **(5 marks)**

Question 3 (25 marks)

A researcher investigating the determinants of crime in the United Kingdom has data for 42 police regions over 22 years. She estimates by OLS the following regression:

$$\ln cmrt_{it} = \alpha_i + \phi_t + \beta_1 unrtm_{it} + \beta_2 proyth_{it} + \beta_3 \ln pp_{it} + u_{it}, \quad i = 1, \cdots, 42, \quad t = 1, \cdots, 22,$$

where the variables are as follows:
$cmrt$ the crime rate per head of population,
$unrtm$ is the unemployment rate of males,
$proyth$ is the proportion of youths,
pp is the probability of punishment measured as (number of convictions)/(number of crimes reported).
α_i equals one for area i and is zero otherwise for all i
ϕ_t is one in year t and zero for all other years for $t = 2, \cdots, 22$.

Note that α and ϕ are area and year fixed effects and ϕ_1 is not included.

(a) What is the purpose of excluding ϕ_1? What are the terms α and ϕ likely to pick up? Discuss the advantages of using panel data for this type of investigation. **(5 marks)**

(b) In principle, the given model can be estimated by OLS including additional area and year binary variables instead of the area and year fixed effects. However, this needs to include 41 area binary indicators and 21 year binary indicators. Discuss and explain how to alternatively estimate the identical coefficients, β_1, β_2, and β_3, addressing this problem. **(5 marks)**

(c) Estimation by OLS using heteroskedasticity and autocorrelation-consistent standard errors results in the following output, where the coefficients of the fixed effects are not reported:

$$\widehat{\ln cmrt}_{it} = \underset{(0.109)}{0.063} unrtm_{it} + \underset{(0.179)}{3.739} proyth_{it} - \underset{(0.024)}{0.588} \ln pp_{it}, \quad R^2 = 0.904.$$

Comment on the results. In particular, what is the effect of a ten percent increase in the probability of punishment? **(5 marks)**

(d) To test for the relevance of the area fixed effects, you restrict the regression by dropping all entity fixed effects and including a single constant. The relevant F-statistic is 135.28. What are the degrees of freedom? What is the critical value from your F table? **(5 marks)**

(e) You want to analyze what happens to the coefficients and their standard errors when the equation is re-estimated without fixed effects. In the resulting regression, $\hat{\beta}_2$ and $\hat{\beta}_3$ do not change by much, although their standard errors are roughly double. However, $\hat{\beta}_1$ is now 1.340 with a standard error of 0.234. Why do you think that is?

(5 marks)

Question 4 (25 marks)

Suppose the probit model of *deny* against *P/I ratio* and *black*, using the 2380 observations in the Boston HMDA (Home Mortgage Disclosure Act) data set, yields the estimated regression function:

$$\widehat{Pr(deny = 1|P/I ratio, black)} = \Phi(\underset{(0.16)}{-2.26} + \underset{(0.44)}{2.74} P/I\ ratio + \underset{(0.083)}{0.71}\ black)$$

where the variables are as follows:
P/I ratio $\cdots$ ratio of the applicant's anticipated total monthly loan payments to his or her monthly income
Black $\cdots$ a binary that equals 1 if an applicant is black and 0 if white
deny $\cdots$ 1 a binary that equals 1 if a mortgage application is denied and 0 if accepted.

(a) A black mortgage applicant has a P/I ratio of 0.35. What is the probability that his application will be denied? **(5 marks)**

(b) Suppose the applicant reduced this ratio to 0.30. What effect would this have on his probability of being denied a mortgage? **(5 marks)**

(c) Does the marginal effect of the P/I ratio on the probability of mortgage denial depend on race? Explain. **(5 marks)**

Suppose a random variable Y has the following probability distribution:

$$\begin{aligned} Pr(Y = 1) &= p, \\ Pr(Y = 2) &= q, \\ Pr(Y = 3) &= 1 - p - q. \end{aligned}$$

A random sample of size n is drawn from this distribution, and the random variables are denoted $Y_1, Y_2, \cdots, Y_n$.

(d) Let n_1 and n_2 be the number of observations on the random variable Y which equals 1 and 2, respectively. Derive the likelihood function for the parameters p and q. **(5 marks)**

(e) Derive the maximum likelihood estimators of p and q. **(5 marks)**

End of Paper - An Appendix of 4 pages follows

Large Sample Critical Values for the t-statistic from the Standard Normal Distribution			
	Significance level		
	10%	5%	1%
2-Sided Test ($\neq$) Reject if $\lvert t\rvert$ is greater than	1.64	1.96	2.58
1-Sided Test ($>$) Reject if t is greater than	1.28	1.64	2.33
1-Sided Test ($<$) Reject if t is less than	-1.28	-1.64	-2.33

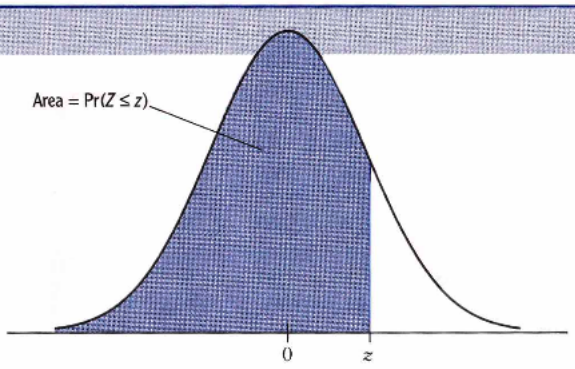
Critical Values for the $F_{m,\infty}$ Distribution			
	Significance level		
Degrees of freedom	10%	5%	1%
1	2.71	3.84	6.63
2	2.30	3.00	4.61
3	2.08	2.60	3.78
4	1.94	2.37	3.32
5	1.85	2.21	3.02
6	1.77	2.10	2.80
7	1.72	2.01	2.64
8	1.67	1.94	2.51
9	1.63	1.88	2.41
10	1.60	1.83	2.32

Large Sample Critical Values of the Augmented Dickey-Fuller Statistic			
	Significance level		
Deterministic regressors	10%	5%	1%
Intercept only	-2.57	-2.86	-3.43
Intercept and time trend	-3.12	-3.41	-3.96

Critical Values for the χ^2 Distribution			
	Significance level		
Degrees of freedom	10%	5%	1%
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.34
4	7.78	9.49	13.28
5	9.24	11.07	15.09
6	10.64	12.59	16.81
7	12.02	14.07	18.48
8	13.36	15.51	20.09
9	14.68	16.92	21.67
10	15.99	18.31	23.21

Critical Values of the QLR Statistic with 15% Trimming			
Number of Restrictions (q)	10%	5%	1%
1	7.12	8.68	12.16
2	5.00	5.86	7.78
3	4.09	4.71	6.02
4	3.59	4.09	5.12
5	3.02	3.66	4.53
6	2.84	3.37	4.12
7	2.69	3.15	3.82
8	2.58	2.98	3.57
9	2.48	2.84	3.38
10	2.40	2.71	3.23

TABLE 1 The Cumulative Standard Normal Distribution Function, $\Phi(z) = \Pr(Z \leq z)$



	Second Decimal Value of z									
z	0	1	2	3	4	5	6	7	8	9
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611

continued on next page

TABLE 1 *(continued)*

	Second Decimal Value of z									
z	0	1	2	3	4	5	6	7	8	9
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
–0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
–0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
–0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
–0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
–0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

This table can be used to calculate $\Pr(Z \leq z)$ where Z is a standard normal variable. For example, when $z = 1.17$, this probability is 0.8790, which is the table entry for the row labeled 1.1 and the column labeled 7.

End of Appendix