WEEK 12 Tutorial

TAKE-HOME MESSAGE from W 11

Shortest-path DISTANCES

$$d = \{ d_{ij} \}$$

$$\downarrow$$

DIAMETER

$$D = \max_{i,j} \{ d_{ij} \}$$

CHARACTERISTIC PATH LENGTH

$$\ell = \langle d_{ij} \rangle$$

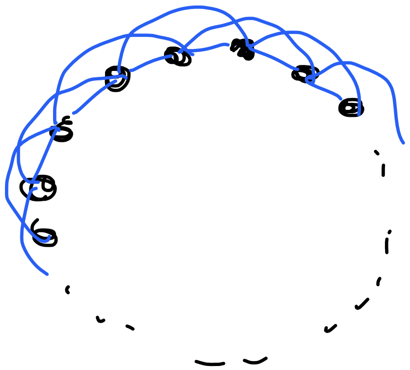
TRIANGLES

$$C_{WS} = \frac{1}{N} \sum_i C_i$$

CLUSTERING COEFFICIENT

$$C_i = \begin{cases} \dfrac{T_i}{\dfrac{k_i(k_i-1)}{2}} & \text{if } k_i > 1 \\ 0 & \text{otherwise} \end{cases}$$

REGULAR LATTICES



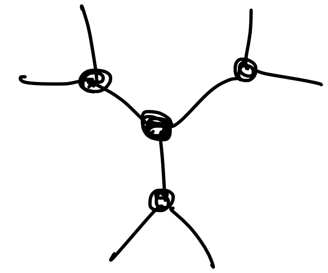
of degree K

$$C = \frac{3}{4}\frac{K-2}{K-1} \qquad D \simeq \frac{N}{K}$$

"large"

✓ $\boxed{B}$

CAYLEY TREES



of degree K

$$C = 0 \qquad D \simeq 2\frac{\ln N}{\ln(K-1)} \quad N \gg 1$$

POISSON NETWORKS

$$C \simeq \frac{\langle k \rangle}{N} = p \qquad \ell \simeq \frac{\ln N}{\ln \langle k \rangle} \quad N \gg 1$$

"Small" $\rightarrow$ SWDP

✓ $\boxed{A}$

Real networks here $\begin{cases} \boxed{A}\ \text{SWDP} \equiv \ell, D \text{ small} \\ + \\ \boxed{B}\ C \text{ large} \end{cases} \longrightarrow$ W.S. model

FEEDBACK ON ASSESSED COURSEWORK 5

At time $t = 1$ we start with a network of $\begin{cases} m_0 = 2 & \text{nodes} \\ m_0 = 1 & \text{link} \end{cases}$

At each time $t > 1$:

① A new node with $m = 1$ links is added to the network

② The new link is attached to an existing node i with

$$\Pi_i = \frac{a_i}{Z}$$

$a_i \in \mathbb{N}^+$ drawn from $\Pi(a)$

$$Z = \sum_{J = 1, 2, \dots, N(t-1)} a_J$$

$\boxed{QA}$ M-F solution $\to$ $k_i(t)$ assuming $Z \simeq \bar{a}\, t$

with $\bar{a}$ = average over $\Pi(a)$

In M-F $\quad \dfrac{dk_i}{dt} = \tilde{\Pi}_i$

the expected increase in the # of links of node i at time t

$$\tilde{\Pi}_i = m \cdot \Pi_i = 1 \cdot \frac{a_i}{Z}$$

m: # of new links

Π_i: probability that a new link connects to node i

Hence

$Z = \bar{a}\, t$

$$\begin{cases} \dfrac{dk_i}{dt} = \dfrac{a_i}{Z} = \dfrac{a_i}{\bar{a}\, t} & \text{for } t > t_i \\ k_i(t_i) = m = 1 \end{cases}$$

t_i: time of arrival of node i

Integrating from time t_i to time t

$$\int_{k_i(t_i)}^{k_i(t)} dk_i' = \frac{a_i}{\bar{a}} \int_{t_i}^{t} \frac{dt'}{t'}$$

$k_i(t_i) = m = 1$

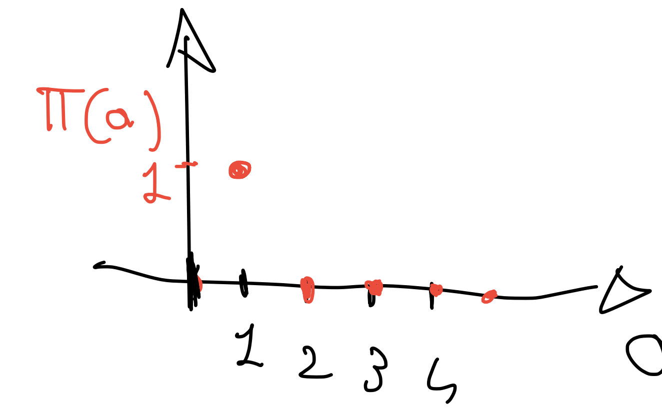
$$(*) \quad k_i(t) = 1 + \frac{a_i}{\bar{a}} \ln\left(\frac{t}{t_i}\right)$$

$t \geq t_i$

QB Derive $P(k)$ for $t \gg 1$ in M-F

assuming $\Pi(a) = \begin{cases} 1 & \text{for } a = 1 \\ 0 & \text{for } a \neq 1 \end{cases}$

$z \simeq \bar{a}\, t$



$a_i = 1 \quad \forall i \qquad$ Hence $\bar{a} = 1 \qquad z \simeq \bar{a}\, t = t$

$$\mathrm{Prob}\big(\underbrace{k_i(t)}_{\text{use } \circledast} > k\big) = \mathrm{Prob}\Big(1 + \underbrace{\frac{a_i}{\bar{a}}}_{1} \ln\Big(\frac{t}{t_i}\Big) > k\Big) =$$

$\ln\left(\frac{t}{t_i}\right) > k - 1$

$\frac{t}{t_i} > e^{k-1}$

$$= \mathrm{Prob}\left(t_i < t\, e^{-k+1}\right)$$

$t_i < t\, e^{-k+1}$

Using the approximation $\mathrm{Prob}(t_i < \tau) \simeq \dfrac{\tau}{t}$ for $t \gg 1$

we have

$$\mathrm{Prob}\left(k_i(t) > k\right) \simeq \frac{\cancel{t}\, e^{-k+1}}{\cancel{t}} = e^{-k+1}$$

Finally the degree distribution can be obtained as

$$P(k) = \frac{d}{dk} \underbrace{\mathrm{Prob}\left(k_i(t) \le k\right)}_{= 1 - \mathrm{Prob}\left(k_i(t) > k\right)} = \frac{d}{dk}\left[1 - e^{-k+1}\right] = e^{-k+1}$$

$$P(k) \simeq e^{-k+1} \qquad \text{for } t \gg 1$$