Queen Mary University of London

Main Examination period Sample Paper – May/June – Semester B

MTH4115 / MTH4215: Vectors and Matrices

Examiners: C. Garetto, W. Huang, M. Lewis

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You will have a period of **3 hours** to complete the exam and submit your solutions.

You should attempt ALL questions. Marks available are shown next to the questions.

The exam is closed-book, and **no outside notes are allowed**.

Only approved non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Question 1 [25 marks].

(a) Let $A = (-1, 3, -2)$, $B = (0, 1, 5)$ and $C = (-2, 1, 7)$. Compute **[10]**

$$\frac{|\overrightarrow{BA}|}{|\overrightarrow{AC}|^2} \,.$$

(b) Let $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$, and P be the point in $\mathbb{R}^3$ with position vector $\mathbf{p} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$.

(i) Write the parametric equations of the line l through P in the direction of the vector $\mathbf{u}$. **[5]**

(ii) Does the point $Q = (1, 2, 1)$ lie on the line l? Justify your answer with a short argument. **[5]**

(c) Let $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and let R be the point in $\mathbb{R}^3$ with coordinates (a, b, c). Prove that $|\mathbf{v}|$ is the length of the segment OR. **[5]**

Question 2 [25 marks].

In a three dimensional space $\mathbb{R}^3$, consider plane Π_1 given by the Cartesian equation $x + y + z = 6$, plane Π_2 given by the Cartesian equation $x + 2y + 3z = 14$, and plane Π_3 given by the Cartesian equation $x + 3y + 2z = 13$.

(a) Write down the linear system A, whose solutions are the intersection of these three planes. Write down the associated homogeneous system B to this linear system A. **[5]**

(b) Bring the augmented matrix of the homogeneous system B obtained in (a) to row echelon form. State the leading and free variables of the system in this form, and find all solutions of B. **[10]**

(c) Based on the solutions of B, state how many points are in the intersection of the three planes. Write down all solutions of the linear system A. **[10]**

Question 3 [25 marks].

(a) Let

$$C = \begin{pmatrix} 2 & 1 & -9 \\ -3 & 1 & 2 \\ 5 & -4 & 0 \end{pmatrix} .$$

Evaluate C^T, C^TC, $\frac{1}{2}(C + C^T)$. **[4]**

(b) Prove that for any square matrix A, the matrices A^TA and $\frac{1}{2}(A + A^T)$ are both symmetric. **[6]**

(c) If we take $B = \frac{1}{2}(A + A^T)$, then prove $(A - B)^T = B - A$. **[5]**

(d) Are the matrices A^TA and AA^T always equal? Either prove this result or state a counter-example. **[4]**

(e) Prove that if A is invertible, then so is A^TA. **[6]**

Question 4 [25 marks].
Consider the matrix

$$A = \begin{pmatrix} 1 & -1 & -4 \\ 0 & 7 & -5 \\ 3 & 4 & 9 \end{pmatrix} .$$

(a) Find elementary matrices E_1, E_2, E_3 such that $U = E_3E_2E_1A$, where U is an upper triangular matrix. **[8]**

(b) Evaluate the determinant of A and state whether A is invertible. **[7]**

(c) Evaluate the determinant of the following matrix: **[10]**

$$B = \begin{pmatrix} 7 & 1 & -1 & -4 \\ 8 & 1 & -1 & -4 \\ 5 & 0 & 14 & -10 \\ 9 & 3 & 4 & 9 \end{pmatrix} .$$

End of Paper.