Topic: TESTING FOR CORRELATION

Exercise 3.1

a) THE VALUES OF SAMPLE AUTOCORRELATION FUNCTION AT LAGS 1, 2, ..., 10 ARE:

0.16 ; 0.15 ; 0.05 ; 0.12 ; 0.1 ; 0.05 ; 0.01 ; 0.011 ; 0.009 ; 0.04

TEST FOR NO CORRELATION IN THIS TIME SERIES AT 5% SIGNIFICANCE LEVEL

$\downarrow$

$H_0 : \rho_k = 0$ (correlation not significant at lag k)

$H_1 : \rho_k \neq 0$ (correlation significant at lag k, so correlation is statistically different from zero at lag k)

where $k = 1, 2, .., 10$

- TEST STATISTIC

IF X_t ARE iid VARIABLES, THEN THE STATISTIC

$$t = \sqrt{n}\,\hat{\rho}_K \sim N(0,1)$$

THIS TEST STATISTIC IS USED TO CONDUCT SIGNIFICANCE TESTS FOR AUTOCORRELATION COEFFICIENTS BY CONSTRUCTING A NON-REJECTION REGION (LIKE A CONFIDENCE INTERVAL) FOR AN ESTIMATED AUTOCORRELATION COEFFICIENT

- REJECTION RULE

WE REJECT H_0 AT 5% SIGNIFICANCE LEVEL

IF $|t| > z_{\alpha/2}$ IF $\alpha = 5\%$ THEN $z_{\alpha/2} = 1.96$

$$|t| > 1.96 \sim 2$$

$$|\sqrt{n}\,\hat{\rho}_K| > 2$$

$$|\hat{\rho}_K| > \frac{2}{\sqrt{n}}$$ → this is the rejection rule

SO :

WE REJECT H_0 IF $|\hat{\rho}_k| > \frac{2}{\sqrt{n}}$

WE DO NOT REJECT H_0 IF $|\hat{\rho}_k| \leq \frac{2}{\sqrt{n}}$

Alternative way :

THE DECISION RULE IS TO REJECT THE NULL HYPOTHESIS OF ZERO CORRELATION AT ANY LAG IF THE SAMPLE CORRELATION COEFFICIENT LIES OUTSIDE THE RANGE

$$\left[-\frac{2}{\sqrt{n}} ; \frac{2}{\sqrt{n}}\right]$$

IN OUR CASE

WE HAVE $n = 100$ OBSERVATIONS AND WE WANT TO TEST:

$H_0 : \rho_K = 0$

$H_1 : \rho_K \neq 0$

RESECTION RULE:

WE RESECT H_0 IF:

$$|\hat{\rho}_K| > \frac{2}{\sqrt{n}} = \frac{2}{\sqrt{100}} = \frac{2}{10} = 0.2$$

WE FIND THAT FOR ALL LAGS $K = 1, 2, \dots, 10$

$$|\hat{\rho}_K| < 0.2$$

WHICH MEANS THERE IS NO CORRELATION AT LAGS 1 TO 10

if $K = 1 \rightarrow \hat{\rho}_1 = 0.16 < 0.2$

if $K = 2 \rightarrow \hat{\rho}_2 = 0.15 < 0.2$

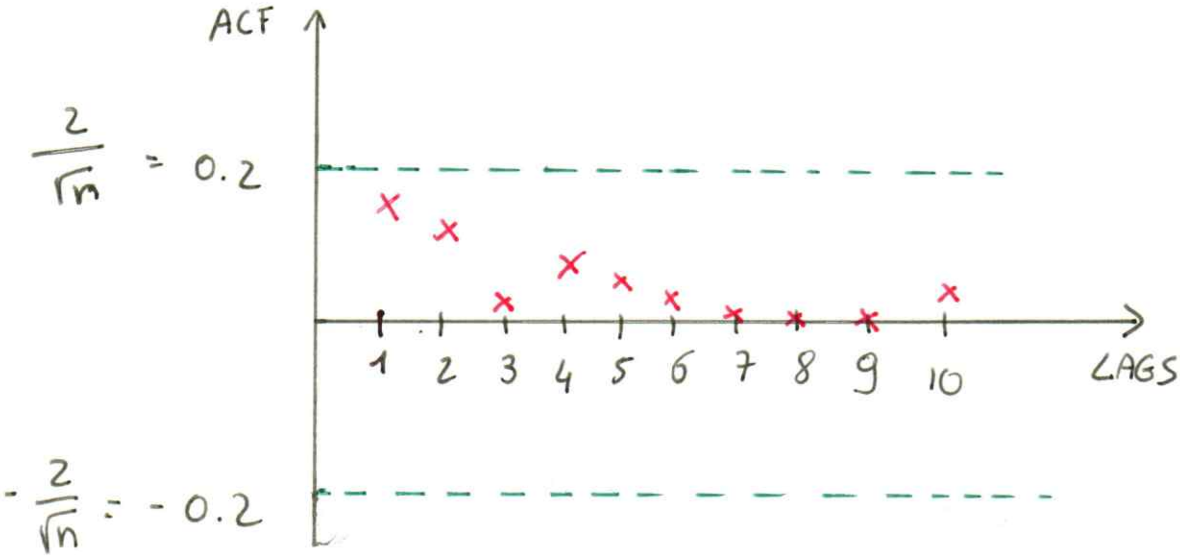
if $K = 3 \rightarrow \hat{\rho}_3 = 0.05 < 0.2$

$\vdots \qquad \vdots$

THEREFORE WE CANNOT RESECT THE HYPOTHESIS THAT THE TIME SERIES IS A WHITE NOISE

Alternative way of testing

$$\left[-\frac{2}{\sqrt{n}}, \frac{2}{\sqrt{n}}\right] = \left[-\frac{2}{\sqrt{100}}, \frac{2}{\sqrt{100}}\right] = [-0.2, 0.2]$$



WE DO NOT REJECT H_0

→ WE ALSO KNOW THAT Q-STATISTIC $Q(8)$ HAS A P-VALUE OF 0.60

THE Q STATISTIC IS ALSO CALLED LJUNG-BOX STATISTIC CAN BE USED TO TEST FOR CORRELATION NOT AT ONE LAG, BUT AT FEW LAGS SIMULTANEOUSLY.

AS FOR ANY JOINT HYPOTHESIS TEST, ONLY ONE AUTOCORRELATION COEFFICIENT NEEDS TO BE STATISTICALLY SIGNIFICANT FOR THE TEST TO RESULT IN A REJECTION.

$Q(m)$

$$H_0: \rho_1 = \rho_2 = \rho_3 = \dots \rho_m = 0$$

$$H_1: \rho_1 \neq 0 \text{ OR } \dots \text{ OR } \rho_m \neq 0$$

IN OUR CASE

$m = 8$

$$H_0: \rho_1 = \rho_2 = \dots = \rho_8 = 0$$

$$H_1: \rho_1 \neq 0 \text{ OR } \rho_2 \neq 0 \text{ OR } \dots \text{ OR } \rho_8 \neq 0$$

REJECTION RULE:

IF P_VALUE $< \alpha$ → REJECT H_0

IF P_VALUE $\geq \alpha$ → DO NOT REJECT H_0

SINCE P-VALUE $= 0.60 > \alpha = 0.05$

THEN WE DO NOT REJECT H_0 (no autocorrelation)

b) ASSUME $n = 100$ AND THE SAMPLE AUTOCORRELATION FUNCTION IS

$-0.4,\ 0.12,\ -0.05,\ 0.2,\ 0.1,\ 0.05,\ 0.01,\ 0.011,\ 0.009,\ 0.04$

AND LJUNG-BOX STATISTIC $Q(10)$ HAS P-VALUE $= 0.02$

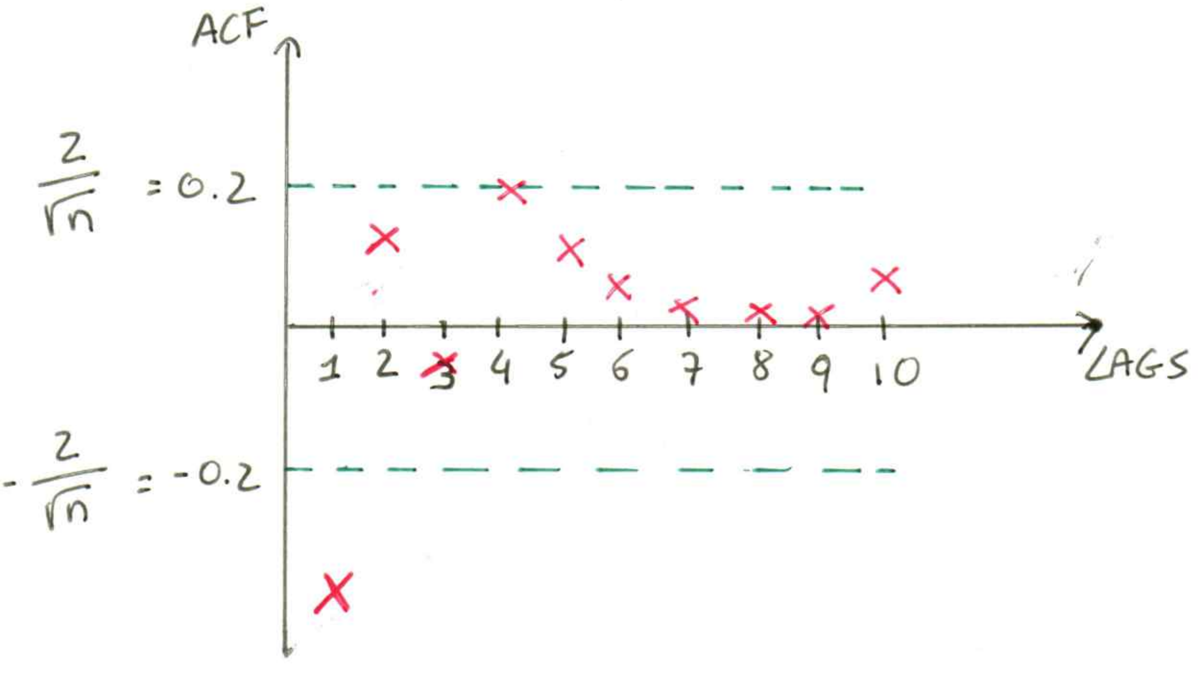
TEST AT 5% SIGNIFICANCE LEVEL, THAT THE TIME SERIES IS A WHITE NOISE.

$\downarrow$

$H_0 : \rho_K = 0$

$H_1 : \rho_K \neq 0$

$$\left[-\frac{2}{\sqrt{n}}, \frac{2}{\sqrt{n}}\right] = \left[-\frac{2}{\sqrt{100}}, \frac{2}{\sqrt{100}}\right] = [-0.2, 0.2]$$



CORRELATION AT LAG 1 IS SIGNIFICANT

IN FACT

$$|\hat{\rho}_1| = 0.4 > \frac{2}{\sqrt{n}} = 0.2$$

WE CAN CONCLUDE THAT TIME SERIES IS <u>NOT</u> A WHITE NOISE

→ Q - statistic

$H_0: \rho_1 = \rho_2 = \ldots\ldots = \rho_{10} = 0$

$H_1: \rho_1 \neq 0$ OR $\rho_2 \neq 0$ OR .... OR $\rho_{10} \neq 0$

P-VALUE $= 0.02$

$\alpha = 0.05$

SINCE P-VALUE $< \alpha$ THEN WE REJECT H_0

THE TEST SHOWS THAT THERE IS CORRELATION, SO THE TIME SERIES IS NOT A WHITE NOISE

Exercise 3.2

a) test for autocorrelation in r_t

TO PLOT CORRELOGRAM IN EVIEWS

open file in Eviews
↓
double click on variable ibmrtn
↓
VIEW
↓
CORRELOGRAM
lags to include : 7

A GOOD RULE OF THUMB TO SELECT THE NUMBER OF LAGS IS TAKING $\ln(n)$.
IT IS IMPORTANT TO SELECT THE CORRECT NUMBER OF LAGS BECAUSE OTHERWISE OUR RESULTS COULD BE MISLEADING.

IN OUR CASE:
THE DATASET CONTAINS 996 OBSERVATIONS

so $\ln(996) \sim 7$

WE WANT TO TEST:

$H_0 : \rho_K = 0$

$H_1 : \rho_K \neq 0$

WE REJECT H_0 IF $|\hat{\rho}_K| > \frac{2}{\sqrt{n}}$

$|\hat{\rho}| > \frac{2}{\sqrt{996}} = 0.0634$

- IF $K = 1 \to |\hat{\rho}_1| = 0.04 < 0.0634$ SO NO CORRELATION

$K = 2 \to |\hat{\rho}_2| = 0.006 < 0.0634$ SO NO CORRELATION

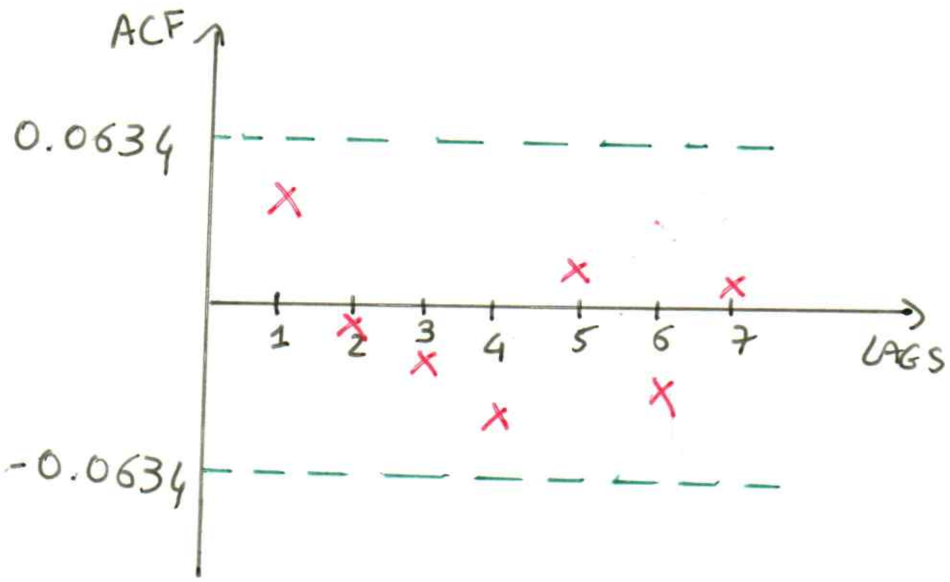
$\vdots$

$K = 7 \to |\hat{\rho}_7| = 0.004 < 0.0634$ SO NO CORRELATION

$\downarrow$

FOR ALL LAGS WE FAIL TO REJECT H_0

SO IBM RETURNS IS A WHITE NOISE



→ Q- STATISTIC

$m = 1$ $\quad H_0 : \rho_1 = 0$

$H_1 : \rho_1 \neq 0$

$Q(1) \rightarrow$ P-VALUE $= \underline{\underline{0.207 > 0.05}}$ [do not reject]

$m = 2$ $\quad H_0 : \rho_1 = \rho_2 = 0$

$H_1 : \rho_1 \neq 0$ OR $\rho_2 \neq 0$

$Q(2) \rightarrow$ P. VALUE $= \underline{\underline{0.442 > 0.05}}$ [do not reject]

⋮

$m = 7$ $\quad H_0 : \rho_1 = \rho_2 = \dots = \rho_7 = 0$

$H_1 : \rho_1 \neq 0$ OR .... OR $\rho_7 \neq 0$

$Q(7) \rightarrow$ P. VALUE $= \underline{\underline{0.655 > 0.05}}$ [do not reject]

THEREFORE THE SERIES IS A WHITE NOISE

Topic: MOVING AVERAGE MODELS

A MOVING AVERAGE MODEL IS SIMPLY A LINEAR COMBINATION OF WHITE NOISE PROCESSES, SO THAT X_t DEPENDS ON THE CURRENT AND PREVIOUS VALUES OF A WHITE NOISE DISTURBANCE TERM.

THEN

$$\boxed{X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q}}$$

IS A qth ORDER MOVING AVERAGE MODEL, DENOTED AS MA (q)

THIS MODEL CAN BE ALSO EXPRESSED AS:

$$X_t = \mu + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t$$

→ PLEASE NOTE THAT ε_t IS A WHITE NOISE PROCESS WITH $E(\varepsilon_t) = 0$ AND $Var(\varepsilon_t) = \sigma^2$

Exercise 3.3

CONSIDER THE MA(1) TIME SERIES:

$$X_t = \varepsilon_t + \theta \varepsilon_{t-1} \qquad \text{where } \varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$$

a) FIND $E(X_t)$

WE KNOW THAT IF $E(\varepsilon_t) = 0$, THEN $E(\varepsilon_{t-i}) = 0$ FOR ALL i.

THEREFORE THE EXPECTED VALUE OF THE ERROR TERM IS ZERO FOR ALL TIME PERIODS.

$$E(X_t) = E(\varepsilon_t + \theta \varepsilon_{t-1})$$

$$= E(\varepsilon_t) + E(\theta \varepsilon_{t-1}) =$$

$$= \underbrace{E(\varepsilon_t)}_{=0} + \theta \underbrace{E(\varepsilon_{t-1})}_{=0} = 0$$

BECAUSE ε_t IS A WHITE NOISE PROCESS

SO

$$E(X_t) = 0$$

b) FIND $Var(X_t)$

$$Var(X_t) = E\left[(X_t - E(X_t))^2\right]$$

WE KNOW THAT $E(X_t) = 0$ THEREFORE

$$Var(X_t) = E\left[(X_t - E(X_t))^2\right] = E\left[(X_t)^2\right]$$

$$= E\left[(\underbrace{\varepsilon_t + \theta\varepsilon_{t-1}}_{X_t})^2\right] =$$

$$= E\left[\varepsilon_t^2 + \theta^2\varepsilon_{t-1}^2 + 2\theta\varepsilon_t\varepsilon_{t-1}\right] =$$

$$= \underbrace{E\left[\varepsilon_t^2\right]}_{=\sigma_\varepsilon^2} + \theta^2\underbrace{E\left[\varepsilon_{t-1}^2\right]}_{=\sigma_\varepsilon^2} + 2\theta\underbrace{E\left[\varepsilon_t\varepsilon_{t-1}\right]}_{=0}$$

THIS IS THE VARIANCE IN FACT

$$Var(\varepsilon_t) = E\left[(\varepsilon_t - E(\varepsilon_t))^2\right] =$$

$$= E(\varepsilon_t^2)$$

IF $Var(\varepsilon_t) = \sigma_\varepsilon^2$ THEN

$$Var(\varepsilon_{t-i}) = \sigma_\varepsilon^2 \quad \forall i$$

THIS IS THE AUTOCOVARIANCE OF ε_t.

SINCE ε_t IS A WHITE NOISE PROCESS THEN

$$Cov(\varepsilon_t, \varepsilon_{t-i}) = E(\varepsilon_t \cdot \varepsilon_{t-i}) = 0$$

SO

$$Var(X_t) = \sigma_\varepsilon^2 + \theta^2 \sigma_\varepsilon^2 + 0$$

$$= (1 + \theta^2)\, \sigma_\varepsilon^2$$

c) FIND THE AUTOCOVARIANCE FUNCTION γ_K

$$Cov(X_t, X_{t-K}) = E\left[(X_t - \underbrace{E(X_t)}_{=0})(X_{t-K} - \underbrace{E(X_{t-K})}_{=0})\right] =$$

$$= E\left[X_t \cdot X_{t-K}\right] =$$

$$= E\left[(\varepsilon_t + \theta \varepsilon_{t-1})(\varepsilon_{t-K} + \theta\, \varepsilon_{t-K-1})\right]$$

$$= E\left[\varepsilon_t \varepsilon_{t-K} + \theta \varepsilon_{t-1} \varepsilon_{t-K} + \theta \varepsilon_t \varepsilon_{t-K-1} + \theta^2 \varepsilon_{t-1} \varepsilon_{t-K-1}\right] =$$

$$= E[\varepsilon_t \varepsilon_{t-K}] + \theta E[\varepsilon_{t-1} \varepsilon_{t-K}] + \theta E[\varepsilon_t \varepsilon_{t-K-1}] + \theta^2 E[\varepsilon_{t-1} \varepsilon_{t-K-1}]$$

- IF $K = 1$ ($\rightarrow$ LAG 1 AUTOCOVARIANCE)

$$\gamma_1 = Cov(X_t X_{t-1}) =$$

$$= \underbrace{E(\varepsilon_t \varepsilon_{t-1})}_{=0} + \theta \underbrace{E(\varepsilon_{t-1} \varepsilon_{t-1})}_{= E(\varepsilon_{t-1}^2) = Var(\varepsilon_{t-1}) = \sigma_\varepsilon^2} + \theta \underbrace{E[\varepsilon_t \varepsilon_{t-2}]}_{=0} + \theta^2 \underbrace{E[\varepsilon_{t-1} \varepsilon_{t-2}]}_{=0}$$

ε_t IS A WHITE NOISE SO NO AUTOCOVARIANCE

$$\gamma_1 = 0 + \theta \sigma_\varepsilon^2 + 0 + 0$$

$$= \theta \sigma_\varepsilon^2$$

- IF $K = 2$

$$\gamma_2 = Cov(X_t, X_{t-2}) =$$

$$= \underbrace{E(\varepsilon_t \varepsilon_{t-2})}_{=0} + \theta \underbrace{E(\varepsilon_{t-1}, \varepsilon_{t-2})}_{=0} + \theta \underbrace{E(\varepsilon_t \varepsilon_{t-3})}_{=0} + \theta \underbrace{E(\varepsilon_{t-1}, \varepsilon_{t-3})}_{=0}$$

$$= 0$$

- IF $K > 2$

THEN $\gamma_K = 0$

WE CAN CONCLUDE THAT

$$\gamma_1 = \theta \sigma_\varepsilon^2$$

$$\left.\begin{array}{l} \gamma_2 = 0 \\ \gamma_3 = 0 \\ \vdots \end{array}\right\} \text{ALL COVARIANCES FOR THE MA(1) PROCESS WILL BE ZERO FOR ANY LAG LENGTH, K, GREATER THAN 1}$$

$\downarrow$

in general

ALL COVARIANCES FOR THE MA(q) PROCESS WILL BE ZERO FOR ANY LAG LENGTH, K, GREATER THAN q

→ FIND THE AUTOCORRELATION FUNCTION ρ_K

THE AUTOCORRELATION AT LAG 1 IS

$$\rho_1 = Corr(X_t, X_{t-1}) =$$

$$= \frac{\gamma_1}{\gamma_0} \qquad \text{where } \gamma_0 = Var(X_t)$$

$$= \frac{\theta \sigma_\varepsilon^2}{(1+\theta^2)\sigma_\varepsilon^2} =$$

$$= \frac{\theta}{1+\theta^2}$$

- IF $K \geq 2$ THEN

$$\rho_K = corr(X_t, X_{t-K})$$

$$= \frac{\gamma_K}{\gamma_0}$$

$$= \frac{0}{\gamma_0}$$

$$= 0$$

HENCE AUTOCORRELATION CUTS OFF TO ZERO AFTER LAG 1

2) IS X_t A COVARIANCE STATIONARY TIME SERIES?

X_t IS A COVARIANCE STATIONARY TIME SERIES,

BECAUSE:

- IT HAS CONSTANT MEAN $E(X_t) = 0$
- IT HAS CONSTANT VARIANCE $Var(X_t) = (1+\theta^2)\sigma_\varepsilon^2$
- $Cov(X_t, X_{t-k}) = \gamma_k$ DEPEND ONLY ON THE LAG K