INTRODUCTION TUTORIAL 3:

Difference between ρ_k AND $\hat{\rho}_k$

$$\underline{X_{t-2} \quad X_{t-1} \quad X_t} \,\Big|\quad X_{t+1}$$

$$Corr(X_t, X_{t-1}) = \rho_1 \longrightarrow \hat{\rho}_1$$

$$Corr(X_t, X_{t-2}) = \rho_2 \longrightarrow \hat{\rho}_2$$

$$\vdots$$

$$Corr(X_t, X_{t-k}) = \rho_k$$

POPULATION PARAMETER (unknown)

ESTIMATED VALUE

SAMPLE PARAMETER

Exercise 3.1

a) $H_0: \rho_k = 0$ (corr is not significant)

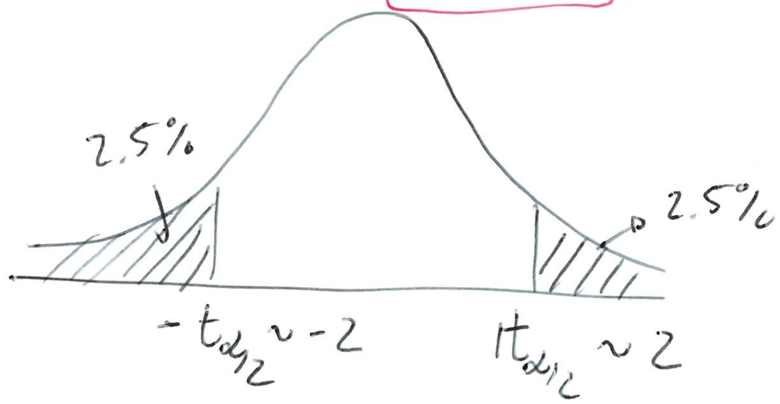
$H_1: \rho_k \neq 0$ (there is correlation)

$\downarrow$

rejection rule: $|t| > t_{\alpha/2} \sim 2$

$|t| > 2$

$\boxed{|\sqrt{n}\,\hat{\rho}_k|} > 2$



t_stat: $\dfrac{\hat{\rho}_k - \underbrace{\rho_{k,H0}}_{\to 0}}{\underbrace{\hat{\sigma}}_{1}/\sqrt{n}}$

$$t = \frac{\hat{\rho}_k}{1/\sqrt{n}} = \sqrt{n}\,\hat{\rho}_k$$

reject H_0 IF $|\sqrt{n}\,\hat{\rho}_K| > 2$

$$|\hat{\rho}_K| > \frac{2}{\sqrt{n}}$$

we know that $n = 100$

$$\frac{2}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.2$$

IF $K = 1 \rightarrow \hat{\rho}_K = \hat{\rho}_1 = 0.16 < 0.2$

$\downarrow$

fail to reject H_0

so corr is not statistically different from zero

IF $K = 2 \rightarrow |\hat{\rho}_K| = |\hat{\rho}_2| = 0.15 < 0.2$

$\downarrow$

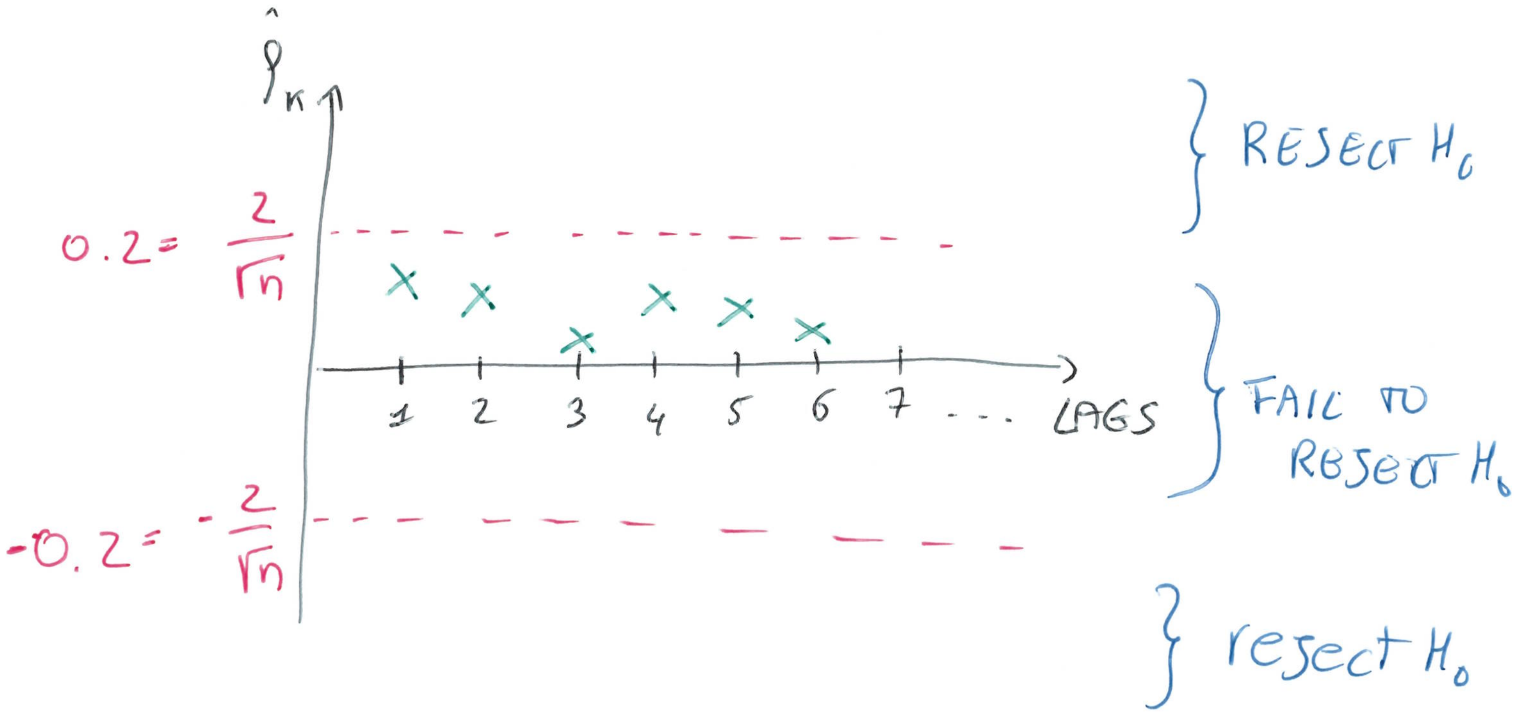
FAIL TO REJECT H_0

ALTERNATIVE WAY:

$$\left[-\frac{2}{\sqrt{n}} , \frac{2}{\sqrt{n}} \right]$$

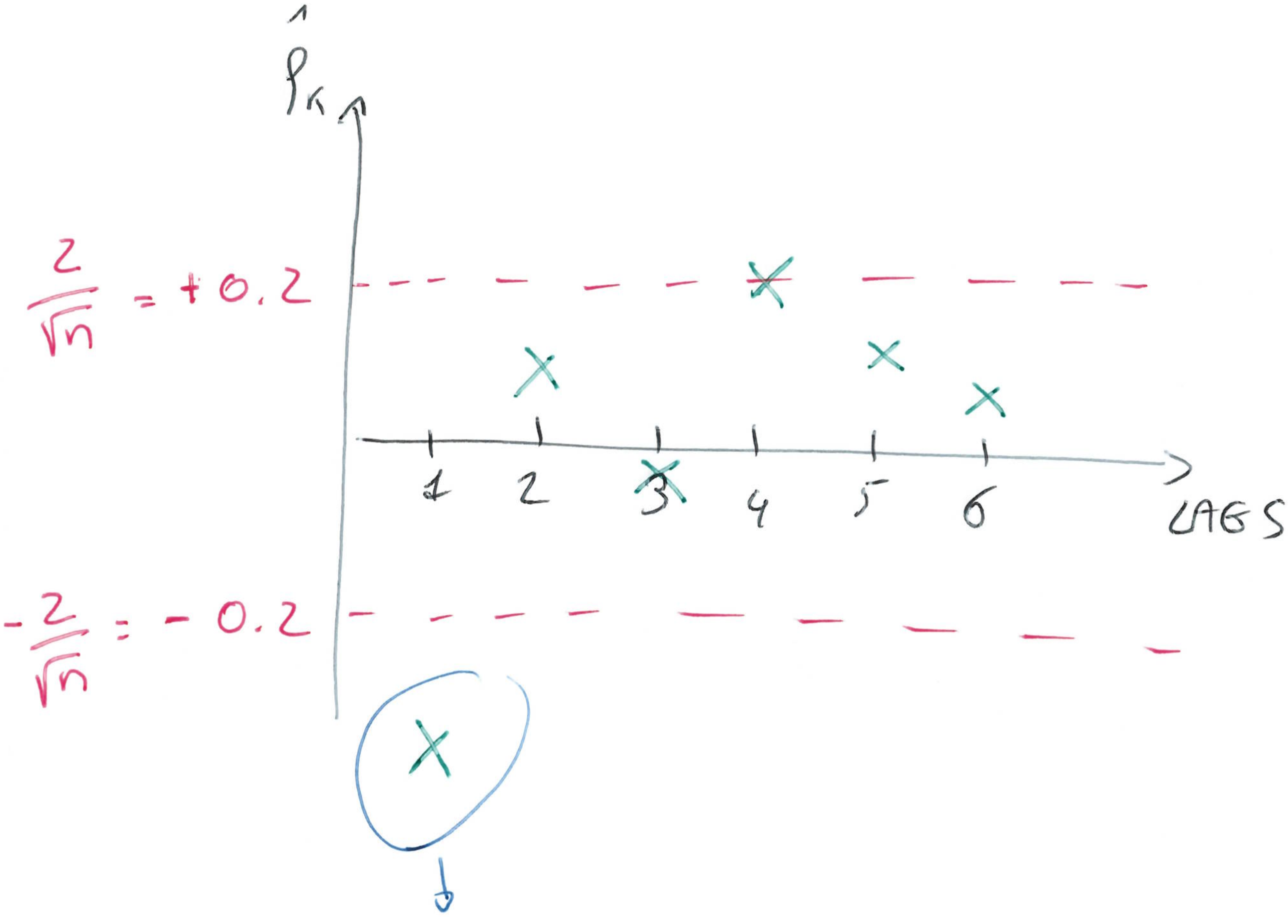
$$\downarrow$$

IF $\hat{\rho}_k$ FALLS INSIDE THIS INTERVAL WE FAIL TO REJECT H_0



THIS SERIES IS A WHITE NOISE SERIES

b)



WE CAN CONCCLUDE

rej H_0 SO ρ_k IS STATISTICALLY DIFFERENT FROM ZERO

Exercise 3.2

AC:	0.040	$= \hat{\rho}_1$
	-0.006	$= \hat{\rho}_2$
	-0.018	$= \hat{\rho}_3$
	-0.031	$= \hat{\rho}_4$
	0.021	$= \hat{\rho}_5$
	-0.041	$= \hat{\rho}_6$
	0.004	$= \hat{\rho}_7$

~~th~~ AC has been calculated in Eviews using Correllogram

$\boxed{\text{LAGS TO INCLUDE: } \ln(996) \sim 7}$

$\boxed{0.0634}$

this is $\frac{2}{\sqrt{n}}$ when $n = 996$

a) $H_0: \rho_k = 0$

$H_1: \rho_k \neq 0$

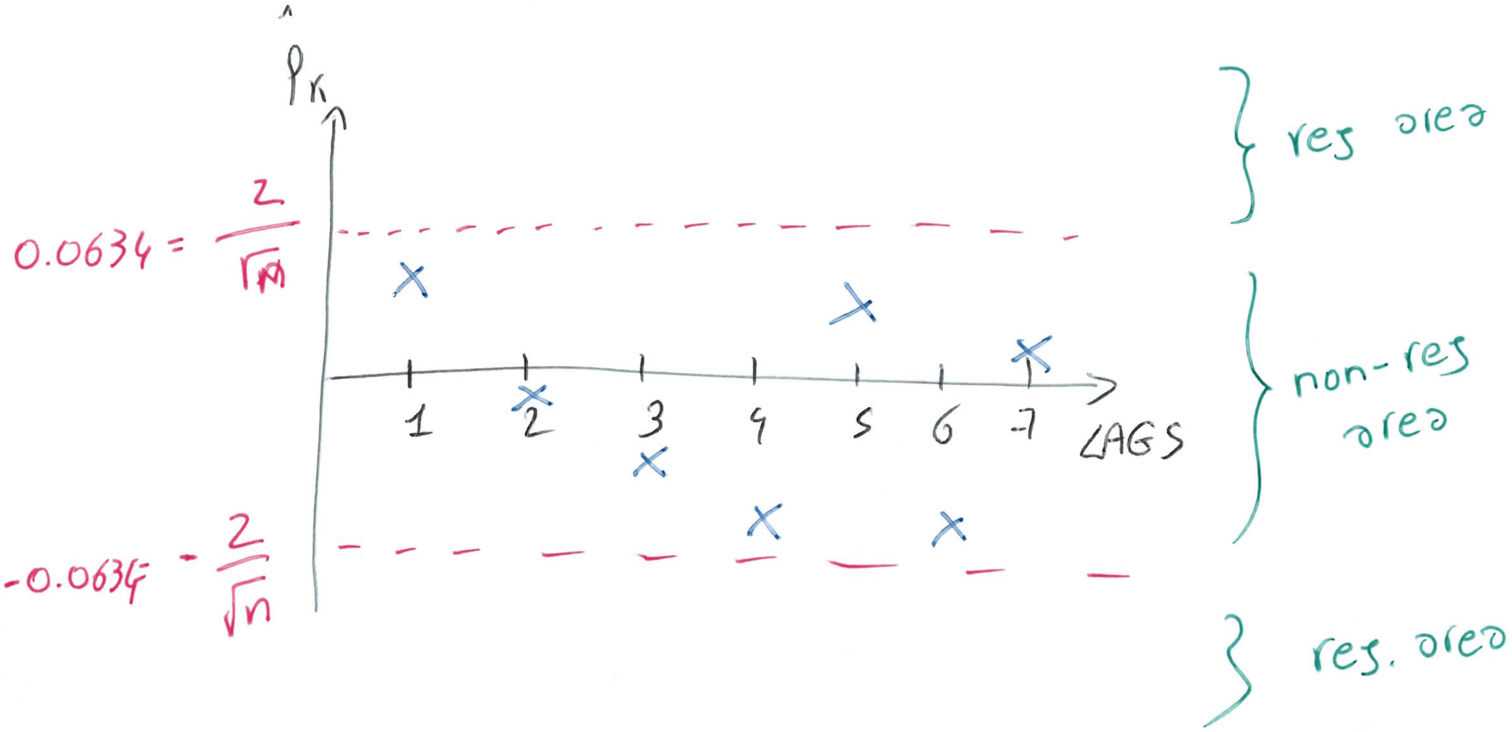
$\boxed{\text{SINGLE HYPOTHESIS}}$

rejection rule: reject H_0 if $|\hat{\rho}_k| > \frac{2}{\sqrt{n}}$

obs = 996 (= n)

rej H_0: IF $|\hat{\rho}_k| > \frac{2}{\sqrt{996}} =$

$= \underline{\underline{0.0634}}$



$\hat{\rho}_k$
$0.0634 = \frac{2}{\sqrt{n}}$
$-0.0634 = \frac{2}{\sqrt{n}}$
1
2
3
4
5
6
7
LAGS
rej area
non-rej area
rej. area

JOINT MULTIPLE HYPOTHESIS

WHAT IF

$H_0 : \rho_1 = \rho_2 = 0$ $\longrightarrow$ WE NEED TO USE Q-stat

χ^2_m

LAGS	Q-stat	P-value	
1	1.5919	0.207	$\longrightarrow H_0 : \rho_1 = 0$
2	1.6329	0.442	$\longrightarrow H_0 : \rho_1 = \rho_2 = 0$
3	1.9568	0.581	$\longrightarrow H_0 : \rho_1 = \rho_2 = \rho_3 = 0$

IF P-VALUE $< \alpha$ $\longrightarrow$ reject H_0

IF P-VALUE $> \alpha$ $\longrightarrow$ FAIL TO REJ H_0

Problem 3.3

$MA(1): \quad X_t = \varepsilon_t + \theta \varepsilon_{t-1} \qquad \varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$

a) $E(X_t) = ?$

$$E(\varepsilon_t + \theta \varepsilon_{t-1}) =$$

$$= \underbrace{E(\varepsilon_t)}_{0} + \theta \underbrace{E(\varepsilon_{t-1})}_{0}$$

$$= 0$$

$$\boxed{\begin{aligned} &E(aX + bY) = \\ &aE(X) + bE(Y) \end{aligned}}$$

b) $Var(X_t) = ?$

$$Var(X_t) = E\left[(X_t - \underbrace{E(X_t)}_{=0})^2\right]$$

$$= E[X_t^2]$$

$$= E[(\varepsilon_t + \theta \varepsilon_{t-1})^2] \qquad (a+b)^2 = a^2 + b^2 + 2ab$$

$$E[\varepsilon_t^2 + \theta^2 \varepsilon_{t-1}^2 + 2\theta \varepsilon_t \varepsilon_{t-1}]$$

$$= E[\varepsilon_t^2] + \theta^2 E[\varepsilon_{t-1}^2] + 2\theta E[\varepsilon_t \varepsilon_{t-1}]$$

- $E[\varepsilon_t^2] = Var(\varepsilon_t)$

$\downarrow$

$$Var(\varepsilon_t) = E\left[(\varepsilon_t - \underbrace{E(\varepsilon_t)}_{=0})^2\right] = E(\varepsilon_t^2)$$

WE KNOW $E(\varepsilon_t^2) = \sigma_\varepsilon^2$

- $E(\varepsilon_{t-1}^2) = Var(\varepsilon_{t-1})$

$\downarrow$

SINCE ε_t IS WN THEN $Var(\varepsilon_t) = Var(\varepsilon_{t-1}) = \therefore$

SO $E(\varepsilon_{t-1}^2) = \sigma_\varepsilon^2$

- $E[\varepsilon_t \, \varepsilon_{t-1}] = Cov(\varepsilon_t, \varepsilon_{t-1})$

$$Cov(\varepsilon_t, \varepsilon_{t-1}) = E\left[(\varepsilon_t - \underbrace{E(\varepsilon_t)}_{=0})(\varepsilon_{t-1} - \underbrace{E(\varepsilon_{t-1})}_{=0})\right]$$

$$= E(\varepsilon_t \, \varepsilon_{t-1}) = 0$$

X_t IS WHITE NOISE IF:

- $E(X_t) = 0$

$\gamma_0 = Var(X_t) = \sigma^2$

$\gamma_k = Cov(X_t, X_{t-k}) = 0$

$corr(X_t, X_{t-k}) = \rho_k = \dfrac{\gamma_k}{\gamma_0} = 0$

$$= \underbrace{E(\varepsilon_t^2)}_{=\sigma_\varepsilon^2} + \theta^2 \underbrace{E(\varepsilon_{t-1}^2)}_{=\sigma_\varepsilon^2} + 2\theta \underbrace{E(\varepsilon_t \varepsilon_{t-1})}_{=0}$$

$$= \sigma_\varepsilon^2 + \theta^2 \sigma_\varepsilon^2$$

$$= (1+\theta^2)\sigma_\varepsilon^2$$

σ^2 $\quad$ $\left.\begin{matrix}\sigma_\varepsilon^2 \\ \sigma_x^2\end{matrix}\right) \neq$ $\quad$ μ_x