ECOM073: Topics in Financial Econometrics

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Exercise 3.

Problem 3.1.

(a) The sample autocorrelation function at lags $1, 2, ..., 9, 10$ was computed from the sample with $N = 100$ observations.

The following values were obtained:

$$0.16, 0.15, 0.05, 0.12, 0.1, 0.05, 0.01, 0.011, 0.009, 0.04.$$

In addition, it is known that the Ljung-Box statistic $Q(m)$ computed for $m = 8$ lags has p-value 0.60.

Test for no correlation in this time series at 5% significance level.

(b) Assume that sample size is $N = 100$, and the sample autocorrelation function at lags $1, 2, ..., 9, 10$ is taking values

$$-0.4, 0.12, -0.05, 0.2, 0.1, 0.05, 0.01, 0.011, 0.009, 0.04.$$

In addition, it is known that the Ljung-Box statistic $Q(10)$ computed for $m = 8$ lags has p-value 0.02.

Test at 5% significance level, that this time series is a white noise.

Solution. (a) To answer this question, we need to test the null hypothesis that there is no significant correlation at any lag $k \geq 1$ at significance level 5%:

H_0 : $\rho_k = 0$ (correlation not significant at lag k)

against alternative

H_1: $\rho_k \neq 0$ (correlation significant at lag k).

Rule: Reject H_0 if

$$|\hat{\rho}_k| > 2/\sqrt{N} = 2/\sqrt{100} = 2/10 = 0.2$$

Do not reject H_0 if

$$|\hat{\rho}_k| \leq 2/\sqrt{N} = 0.2$$

We find that for all lags $k = 1, \cdots, 10$

$$|\hat{\rho}_k| < 0.2$$

which means that at lags 1 to 10 there is no correlation. Therefore the we cannot reject the hypothesis that the time series is a white noise.

How to obtain such rule?

From theory we know that if $\rho_k = 0$ then

$$t = \sqrt{N}\,\hat{\rho}_k \sim N(0,1).$$

According to statistical theory, we reject H_0 at 5% significance level, if

$$|t| \geq z_{2.5\%} \sim 2, \qquad or \quad |\hat{\rho}_k| > 2/\sqrt{N}.$$

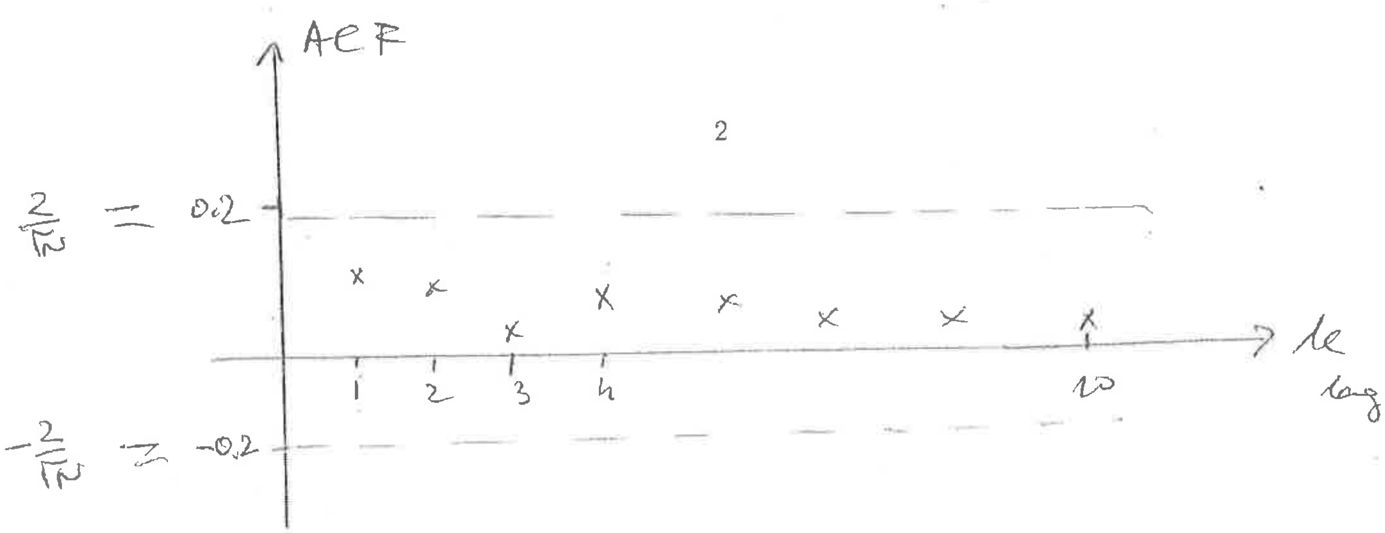
Alternative way of testing

If $n = 100$ then the 95% confidence interval for zero correlation at any lag $k = 1, 2, \cdots$ is

$$\left[\frac{-2}{\sqrt{n}}, \frac{2}{\sqrt{n}}\right] = \left[\frac{-2}{10}, \frac{2}{10}\right] = \left[-0.2, 0.2\right].$$

We draw the graph and check if any of sample correlations lies outside the band. We see that all of them are inside. So we have no evidence in the data for correlation in this times series.

p values 0.60 is grater than significance level 0.05. So Ljung-Box test shows that there is no significant correlation at lags 1, ..., 8, which also suggests that the time series is a white noise.

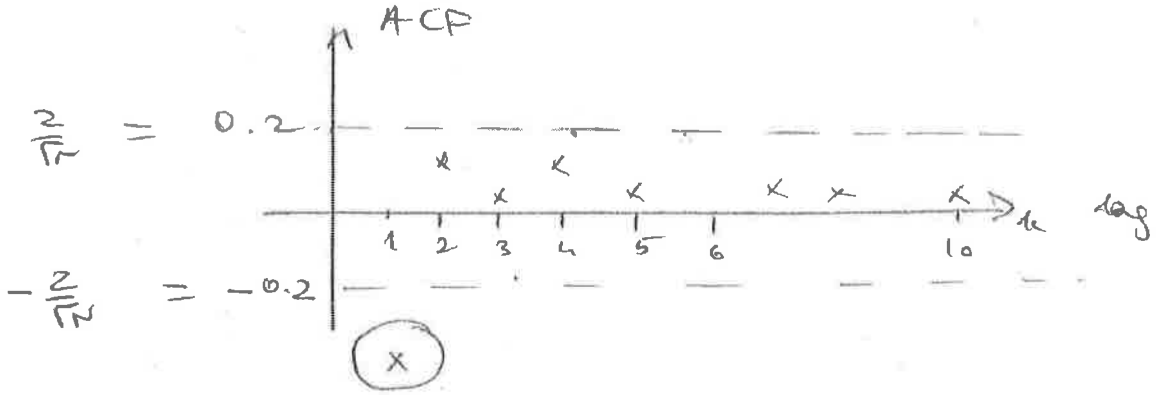


(b) Since $N = 100$ is same as above, we can use the same rule as in case (a).

We find that: $|\hat{\rho}_1| = 0.4 > 2/\sqrt{N} = 2/\sqrt{100} = 2/10 = 0.2$

So correlation at lag 1 is significant, and therefore time series is not a white noise.

p values 0.02 is smaller than significance level 0.05. So Ljung-Box test shows that there is significant correlation at some lag 1, ..., 8, which also suggests that the time series is not a white noise.



Problem 3.2. In **d-ibm3dx7008.txt** you will find the daily simple stock returns r_t of IBM for the period 1926-2008

(a) use e-views, to test for serial-correlation in r_t.
(b) use e-views, to test for serial-correlation in r_t^2.

Comment how you reached your decision, and what you are finding.

Solution. (a) Data set has $N = 996$ observations. Below you we have the e-views output of ACF function for 12 lags.

Notice that $2/\sqrt{N} = 2/\sqrt{996} = 0.0634$. Notice that all sample autocorrelations in table satisfy

$$|\hat{\rho}_k| < 0.0634.$$

Hence, sample ACF are not significant at lags 1-12. They show no correlation.

The output includes ACF and Ljung-Box-test results, denoted by Q. We can use it for testing for correlation. Its output we should read as follows:

Line 1: $m = 1$, $p = 0.207$ which shows no correlation in lag 1 at significance level 5%, since $p > 0.05$

Line 2: $m = 2$, $p = 0.442$ which shows no correlation in lag 1 and 2, since $p > 0.05$

Line 3: $m = 3$, $p = 0.581$ which shows no correlation in lag 1, 2 and 3, since $p > 0.05$.

....

Line 10: $m = 12$, $p = 0.173$ which shows no correlation in lag 1 to 10, since $p < 0.05$.

We stopped at 10 since $\ln(n) = \ln(996) \sim 7$. We could go for larger m, but then Q test results will be not reliable.

Why? Notice that p value decreases when m increases. For m very large, we may found $p < 0.05$, which would lead to wrong conclusion the series is correlated. Such decision would be wrong, because we used m which is too large, i.e. $m >> \log N$

Answer: We found that times series r_t is a white noise.

Date: 01/31/12 Time: 17:49
Sample: 1926M01 2008M12
Included observations: 996

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.040	0.040	1.5919	0.207
		2	-0.006	-0.008	1.6329	0.442
		3	-0.018	-0.017	1.9568	0.581
		4	-0.031	-0.030	2.9353	0.569
		5	0.021	0.023	3.3682	0.643
		6	-0.041	-0.043	5.0229	0.541
		7	0.004	0.007	5.0407	0.655
		8	0.067	0.067	9.6226	0.293
		9	0.054	0.049	12.538	0.185
		10	0.038	0.032	13.990	0.173

(b). Now we test for correlation in r_t^2. From finance we know that r_t^2 may be correlated (this is called ARCH effect). We discuss it later.

We found that

- $$|\hat{\rho}_k| > 0.0634$$

  for all lags 1 to 9 except lag 7.

- Lung-Box test has p-values 0 for $m = 1, ..., 10$.

That shows significant correlation in r_t^2. This times series is not a white noise.

Date: 01/31/12 Time: 17:51
Sample: 1926M01 2008M12
Included observations: 996

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.114	0.114	12.882	0.000
		2	0.095	0.083	21.951	0.000
		3	0.065	0.046	26.139	0.000
		4	0.089	0.071	34.048	0.000
		5	0.072	0.048	39.219	0.000
		6	0.086	0.061	46.710	0.000
		7	0.030	-0.001	47.603	0.000
		8	0.204	0.186	89.674	0.000
		9	0.080	0.031	96.183	0.000
		10	0.037	-0.013	97.573	0.000

Problem 3.3. Consider the MA(1) time series

$$X_t = \varepsilon_t + \theta\varepsilon_{t-1},$$

where ε_t is white noise sequence with mean 0 and variance σ_ε^2.
1. Show that following.

- (a) Find the man EX_t.
- (b) Find the variance $Var(X_t)$
- (c) Find the auto-covariance function γ_k and autocorrelation function ρ_k. Start with lag $k = 1$, then for lags $k = 2, 3, \cdots$.

2. Is (X_t) a covariance stationarity time series?

Solution 2. Solving this problem we shall use the following properties of the white noise ε_t: $E\varepsilon_t = 0$, $Var(\varepsilon_t) = \sigma_u^2$, and $E[\varepsilon_t\varepsilon_s] = 0$ if $t \neq s$.
(a) First we compute the mean

$$\begin{aligned}
E[X_t] &= E[\varepsilon_t + \theta\varepsilon_{t-1}] = E[\varepsilon_t] + \theta E[\varepsilon_{t-1}] = 0 + \theta(0) = 0. \\
Var(X_t) &= E[(X_t - E[X_t])^2] = E[(\varepsilon_t + \theta\varepsilon_{t-1})^2] \\
&= E[\varepsilon_t^2 + 2\theta\varepsilon_t\varepsilon_{t-1} + \theta^2\varepsilon_{t-2}^2] \\
&= E[\varepsilon_t^2] + 2\theta E[\varepsilon_t\varepsilon_{t-1}] + \theta^2 E[\varepsilon_{t-2}^2] \\
&= \sigma_\varepsilon^2 + 2\theta(0) + \theta^2\sigma_\varepsilon^2 \\
&= \sigma_\varepsilon^2(1 + \theta^2).
\end{aligned}$$

(b) To find the autocovariance at lag-1 note, that by definition, for $k \geq 1$,

$$\begin{aligned}
Cov(X_t, X_{t-k}) &= E[(X_t - E[X_t])(X_{t-1} - E[X_{t-1}])] = E[X_t X_{t-1}] \\
&= E[(\varepsilon_t + \theta\varepsilon_{t-1})(\varepsilon_{t-k} + \theta\varepsilon_{t-k-1})] \\
&= E[\varepsilon_t\varepsilon_{t-k} + \theta\varepsilon_{t-1}\varepsilon_{t-k} + \theta\varepsilon_t\varepsilon_{t-k-1} + \theta^2\varepsilon_{t-1}\varepsilon_{t-k-1}] \\
&= E[\varepsilon_t\varepsilon_{t-k}] + \theta E[\varepsilon_{t-1}\varepsilon_{t-k}] + \theta E[\varepsilon_t\varepsilon_{t-k-1}] + \theta^2 E[\varepsilon_{t-1}\varepsilon_{t-k-1}].
\end{aligned}$$

Therefore the lag-1 auto-covariance is

$$\begin{aligned}
\gamma_1 &= E[\varepsilon_t\varepsilon_{t-1}] + \theta E[\varepsilon_{t-1}\varepsilon_{t-1}] + \theta E[\varepsilon_t\varepsilon_{t-2}] + \theta^2 E[\varepsilon_{t-1}\varepsilon_{t-2}] \\
&= 0 + \theta\sigma_\varepsilon^2 + 0 + 0 = \theta\sigma_\varepsilon^2.
\end{aligned}$$

The autocorrelation at lag 1 is

$$\rho_1 = Corr(X_t, X_{t-1}) = \frac{\gamma_1}{\gamma_0} = \frac{\gamma_1}{Var(X_t)} = \frac{\theta\sigma_\varepsilon^2}{\sigma_\varepsilon^2(1+\theta^2)} = \frac{\theta}{(1+\theta^2)}.$$

(c) If $k \geq 2$, then

$$\gamma_k = E[\varepsilon_t\varepsilon_{t-k}] + \theta E[\varepsilon_{t-1}\varepsilon_{t-k}] + \theta E[\varepsilon_t\varepsilon_{t-k-1}] + \theta^2 E[\varepsilon_{t-1}\varepsilon_{t-k-1}] = 0$$

because ε_t is a white noise, and therefore $E[\varepsilon_t\varepsilon_s] = 0$ if $t \neq s$. Then the autocorrelation

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{0}{\gamma_0} = 0, \qquad k \geq 2.$$

Hence, autocorrelation cuts off to 0 after lag 1.

2. (X_t) is covariance stationary time series, because

- it has constant mean $EX_t = 0$,
- it has constant variance $Var(X_t) = (1+\theta^2)\sigma_u^2$.
- $Cov(X_t, X_{t-k}) = \gamma_k$ depend only on the lag k.