Chapter 4
GALACTIC CHEMICAL EVOLUTION

Chemical evolution:

- how the abundances of the chemical elements have built up over time
- in our Galaxy and other galaxies
- elemental abundances in stars and in the ISM
- how the elemental abundances correlate with time, location in galaxy, stellar velocities etc.

Definitions:

- ***metals*** – elements other than H and He also referred to as ***heavy elements***
- ***metallicity*** - fraction of heavy elements by mass
- $X \equiv$ mass fraction of Hydrogen
- $Y \equiv$ mass fraction of Helium
- $Z \equiv$ mass fraction of heavy elements

with $X + Y + Z = 1$

Nucleosynthesis

Nucleosynthesis – the processes by which heavy elements are created

Primordial nucleosynthesis (Big Bang):

$\rightarrow$ 93 % H, 7 % He + trace of Li^7 by number
$\rightarrow$ 77 % H, 23 % He by mass

Nucleosynthesis in stars and supernovae then creates heavy elements (and some additional helium):

- **enriched material produced in stellar interiors through nuclear reactions**

Solar abundances:

$\rightarrow$ 70 % H, 28 % He, 2 % metals by mass
i.e. X = 0.70, Y = 0.28, Z = 0.02

Various nuclear processes are involved:

PP chain – series of reactions that fuse protons into He

CN and CNO cycles – also fuse protons to produce He, with C, N, O and Fe as possible by-products

He-burning in red giants – producing Be, C, possibly continuing to burn C and O to produce Mg and Si

α-capture (capture of He nucleii by other nucleii) produces isotopes consisting of multiples of α-particles, called α-elements

Some isotopes are built up by *neutron capture* - capture of neutrons by other nucleii – particularly heavier elements, beyond Zn:

s-process - slow capture within stars
r-process - rapid capture under extreme conditions e.g. supernovae

Chemical Enrichment of Galaxies

Stars can eject heavy elements (metals) into the ISM via mass loss (stellar winds) and supernovae

New stars formed from the ISM will be enriched in heavy elements and eventually recycle this material back into the ISM, which itself becomes progressively more metal-rich

Supernovae are important to chemical enrichment:

- **eject large quantities of metals into ISM**
- **can themselves generate heavy elements via nucleosynthesis**

TYPE Ia supernovae

– binary systems containing a white dwarf
– eject enriched material into ISM $\gtrsim 10^8$ yr after star formation
– rich in Fe

TYPE II supernovae

– produced by massive stars $M \gtrsim 8M_\odot$
– eject enriched material into ISM $\sim 10^7$ yr after star formation
– rich in C, N, O

Main sequence lifetime $T_{\rm MS}$ of a star is a very strong function of the stellar mass M:

– Stars with $M \lesssim 0.9M_\odot$ have $T_{\rm MS} >$ age of the Universe

So mass that goes into low mass stars is lost from the recycling process and remains locked up in long-lived stars

Samples of low mass stars preserve abundances of the ISM from which they formed (if enriched material is not dredged up from the stellar interior)

This is true for G dwarfs for example:
– gas in the photosphere is almost unchanged in chemical composition from the gas from which they formed

So a sample of G dwarfs provides samples of chemical abundances through the history of the Galaxy

In contrast, observations of interstellar gas in a galaxy provide information on *present-day* abundances and on the current state of chemical evolution

Chemical Abundances

Chemical abundances can be measured in stellar photospheres from the *strengths* of *absorption* lines

Strengths of stellar spectral lines depend on:

– abundance of the element responsible
– effective temperature of the star
– acceleration due to gravity at surface
– small-scale turbulence in stellar atmosphere

Abundances can also be determined from strengths of *emission* lines from interstellar gas e.g. from H II regions

Abundance ratios by number are expressed relative to the Sun using a parameter [A/B], where A and B are the chemical symbols of two elements :

$$[\mathbf{A}/\mathbf{B}] = \log_{10}\left(\frac{N(\mathrm{A})}{N(\mathrm{B})}\right) - \log_{10}\left(\frac{N(\mathrm{A})}{N(\mathrm{B})}\right)_{\odot}$$

where $\odot$ represents the abundance ratio in the Sun.

So the ratio of iron to hydrogen in a star relative to the Sun is written as

$$[\mathbf{Fe/H}] \;=\; \log_{10}\left(\frac{N(\mathrm{Fe})}{N(\mathrm{H})}\right) \;-\; \log_{10}\left(\frac{N(\mathrm{Fe})}{N(\mathrm{H})}\right)_{\odot}$$

The [Fe/H] parameter for the Sun is 0 by definition (as are all other ratios)
Examples:

Mildly metal-poor star in the Galaxy:

$$[\mathbf{Fe/H}] \simeq -0.3$$

Very metal-poor star in Galactic halo:

$$[\mathbf{Fe/H}] \simeq -1.5 \textbf{ to } -2$$

Metal-rich star in the Galaxy:

$$[\mathbf{Fe/H}] \simeq +0.3$$

Interstellar gas in the Galaxy has near-solar metallicity:

$$[\mathbf{Fe/H}] \simeq 0$$

The Simple Model of Galactic Chemical Evolution

The *Simple Model* of chemical evolution simulates the build up of the metallicity Z in a volume of space

The Simple Model makes the following assumptions:

- the volume initially contains only unenriched gas – initially there are no stars and no heavy elements
- the volume of space where the evolution takes place is a 'closed box' – no gas enters or leaves the volume
- the gas in the volume is well mixed – the same chemical composition throughout
- instantaneous recycling occurs – following star formation, all newly created heavy elements that enter the ISM from stars do so immediately
- the fraction of newly-synthesised heavy elements ejected into the ISM after material forms stars is constant over time

These assumptions, although slightly naive, allow some important predictions about the variation of the metallicity in the interstellar gas in terms of the amount of star formation that has taken place

The Simple Model predicts that the metallicity distribution for a sample of long-lived stars is given by:

$$\frac{N(Z)}{N_1} = \frac{1 - e^{-Z(t)/p}}{1 - e^{-Z_1/p}}$$

where :

$N(Z)$ **is the number of these stars having metallicity** Z **and less**

N_1 **is the value of** $N(Z)$ **today**

Z_1 **is the value of** Z **today**

p **is the 'yield'**

Consider a volume within a galaxy, small enough to be homogeneous, but large enough to contain a good sample of stars

Initially, at time $t = 0$ (before star formation starts):

$M_{\rm stars}(0) = 0$

$Z = 0$ **(metallicity of gas)**

$M_{\rm gas}(0) = M_{\rm total}$ **(volume contains pure gas initially)**

Then at some later time, t :

$M_{\rm stars}(t)$ **is the mass in stars**

$M_{\rm gas}(t)$ **is the mass in gas**

$Z(t)$ **is the metallicity of the gas**

$M_{\rm total}$ **is the total (baryonic) mass**

where the total mass at any time, $M_{\rm total}$ is:

$$M_{\rm total} = M_{\rm stars}(t) + M_{\rm gas}(t)$$

Note that :

$M_{\rm stars}(t)$ **and** $M_{\rm gas}(t)$ **are both functions of time**

But because of the closed-box assumption:

$M_{\rm total}$ **is constant**

Let $M_{\rm metals}$ be the mass of heavy elements in the gas within the volume at time t

The heavy element mass fraction of the gas is therefore:

$$Z(t) \equiv \frac{M_{\rm metals}(t)}{M_{\rm gas}(t)}$$

Now consider a time interval from t to $t + \delta t$

Star formation will occur during this time interval, with stars forming from gas

Let the change in $M_{\rm stars}$ be $\delta M_{\rm stars}$
Let the change in $M_{\rm gas}$ be $\delta M_{\rm gas}$

Some stars will eject enriched gas back into the interstellar medium (through supernovae and mass loss)

We firstly need to express the change δZ in the metallicity of the interstellar gas in terms of $\delta M_{\rm stars}$ and $\delta M_{\rm gas}$

We have $Z = \mathbf{fn}(M_{\rm metals}, M_{\rm gas})$

so the derivative of Z with respect to time is

$$\frac{{\rm d}Z}{{\rm d}t} = \frac{\partial Z}{\partial M_{\rm metals}} \frac{{\rm d}M_{\rm metals}}{{\rm d}t} + \frac{\partial Z}{\partial M_{\rm gas}} \frac{{\rm d}M_{\rm gas}}{{\rm d}t} .$$

Differentiating $Z(t) \equiv \frac{M_{\rm metals}(t)}{M_{\rm gas}(t)}$ we get:

$$\frac{\partial Z}{\partial M_{\rm metals}} = \frac{1}{M_{\rm gas}} \quad \textbf{and} \quad \frac{\partial Z}{\partial M_{\rm gas}} = -\frac{M_{\rm metals}}{M_{\rm gas}^2}$$

which gives:

$$\frac{{\rm d}Z}{{\rm d}t} = \frac{1}{M_{\rm gas}} \frac{{\rm d}M_{\rm metals}}{{\rm d}t} - \frac{M_{\rm metals}}{M_{\rm gas}^2} \frac{{\rm d}M_{\rm gas}}{{\rm d}t}$$

For a small time interval δt we have:

$$\delta Z = \frac{\delta M_{\rm metals}}{M_{\rm gas}} - \frac{M_{\rm metals}}{M_{\rm gas}^2} \delta M_{\rm gas}$$

and since $Z = (\frac{M_{\rm metals}}{M_{\rm gas}})$ we get:

$$\delta Z = \frac{\delta M_{\rm metals}}{M_{\rm gas}} - Z \frac{\delta M_{\rm gas}}{M_{\rm gas}}$$

which is the change in Z in time δt

We need to distinguish between the total mass in stars $M_{\rm stars}$ at time t

and

the total mass that has taken part in star formation $M_{\rm SF}$ up to time t

When a mass $\delta M_{\rm SF}$ goes into stars during star formation, the total mass in stars will change by an amount less than this, because material from the new stars is ejected back into the interstellar gas

So $\delta M_{\rm SF} > \delta M_{\rm stars}$ and $M_{\rm SF} > M_{\rm stars}$

Let α be the fraction of mass participating in star formation that remains locked up in long-lived stars (and stellar remnants). So:

$$\delta M_{\rm stars} \;=\; \alpha\, \delta M_{\rm SF}$$

(with $0 < \alpha < 1$, constant)

The mass of newly-synthesised heavy elements ejected back into the ISM is proportional to the mass that goes into stars (from the Simple Model assumptions)

(Note: it naturally follows that

$$\delta M_{\rm SF} \;= (1-\alpha)\, \delta M_{\rm SF} \;+\; \alpha\, \delta M_{\rm SF}$$

from the definition of α above)

Let the mass of newly-synthesised heavy elements ejected into ISM be $p\,\delta M_{\rm stars}$, where p is the yield (a constant)

The change $\delta M_{\rm metals}$ in mass of heavy elements in the gas in a time interval δt will be due to:

- **loss of heavy elements in the gas that goes into star formation:**

$$-\,\delta M_{\rm SF}\,M_{\rm metals}/M_{\rm gas} \;=\; -\,\delta M_{\rm SF}\,Z$$

- **gain of old heavy elements used in star formation and then ejected back into the gas unchanged:**

$$Z\,(1-\alpha)\,\delta M_{\rm SF}$$

  where $(1-\alpha)$ is fraction of mass going into stars that is ejected back

- **gain of newly synthesised heavy elements from stars that are ejected into the gas:**

$$p\,\delta M_{\rm stars}$$

  (from the definition of the yield p above)

Therefore, in time interval δt:

$$\delta M_{\rm metals} \;=\; -\,Z\,\delta M_{\rm SF} \;+\; Z\,(1-\alpha)\,\delta M_{\rm SF} \;+\; p\,\delta M_{\rm stars}$$

which gives on expanding and cancelling:

$$\begin{aligned}\delta M_{\rm metals} \;&=\; -\,Z\,\alpha\,\delta M_{\rm SF} \;+\; p\,\delta M_{\rm stars}\\ &=\; -\,Z\,\delta M_{\rm stars} \;+\; p\,\delta M_{\rm stars}\end{aligned}$$

(substituting for $\alpha\,\delta M_{\rm SF} = \delta M_{\rm stars}$)

Now dividing by $M_{\rm gas}$:

$$\frac{\delta M_{\rm metals}}{M_{\rm gas}} \;=\; -\,Z\,\frac{\delta M_{\rm stars}}{M_{\rm gas}} \;+\; p\,\frac{\delta M_{\rm stars}}{M_{\rm gas}}$$

But the changes in masses are related by $\delta M_{\rm total} = \delta M_{\rm stars} + \delta M_{\rm gas} = 0$ **for a closed box**

So:

$$\delta M_{\rm stars} \;=\; -\,\delta M_{\rm gas}$$

$$\therefore \quad \frac{\delta M_{\rm metals}}{M_{\rm gas}} \;=\; Z\,\frac{\delta M_{\rm gas}}{M_{\rm gas}} \;-\; p\,\frac{\delta M_{\rm gas}}{M_{\rm gas}}$$

Substituting this into $\delta Z \;=\; \frac{\delta M_{\rm metals}}{M_{\rm gas}} \;-\; Z\,\frac{\delta M_{\rm gas}}{M_{\rm gas}}$

$$\delta Z \;=\; Z\,\frac{\delta M_{\rm gas}}{M_{\rm gas}} \;-\; p\,\frac{\delta M_{\rm gas}}{M_{\rm gas}} \;-\; Z\,\frac{\delta M_{\rm gas}}{M_{\rm gas}}$$

$$\therefore \quad \delta Z \;=\; -\,p\,\frac{\delta M_{\rm gas}}{M_{\rm gas}}$$

Converting this to a differential and integrating from time 0 to t:

$$\int_0^{Z(t)} {\rm d}Z' \;=\; -\int_{M_{\rm gas}(0)}^{M_{\rm gas}(t)} p\,\frac{{\rm d}M'_{\rm gas}}{M'_{\rm gas}}$$

$$\therefore \quad Z(t) \;-\; 0 \;=\; -\,p\,\Big[\ln M'_{\rm gas}\Big]_{M_{\rm gas}(0)}^{M_{\rm gas}(t)}$$

This gives:

$$\boxed{\;Z(t) \;=\; -\,p\,\ln\left(\frac{M_{\rm gas}(t)}{M_{\rm gas}(0)}\right)\;}$$

Since $M_{\rm gas}(0) = M_{\rm total}$ (a constant) for all t (because we have a closed box that initially contained only gas), we can rewrite this equation using the gas fraction $\mu \equiv M_{\rm gas}(t)/M_{\rm total}$, where $M_{\rm total}$ is a constant :

$$\boxed{Z(t) \;\; = \;\; -\, p \;\ln \mu}$$

This is a useful and straightforward expression:

– both Z and μ can be measured
– and the equation does not depend on time or star formation rate explicitly

We now need to consider how the mass in stars, $M_{\rm stars}$, depends on metallicity, Z

Since we already know that:

$$Z(t) \;\; = \;\; -\, p \;\ln \left(\frac{M_{\rm gas}(t)}{M_{\rm gas}(0)} \right)$$

Substituting $M_{\rm gas(t)} \;\; = \;\; M_{\rm gas}(0) \;\; - \;\; M_{\rm stars}(t)$

we obtain:

$$Z(t) \;\; = \;\; -\, p \;\ln \left[\frac{M_{\rm gas}(0) - M_{\rm stars}(t)}{M_{\rm gas}(0)} \right]$$

which rearranges to:

$$\boxed{\frac{M_{\rm stars}(t)}{M_{\rm gas}(0)} \;\; = \;\; 1 \;\; - \;\; {\rm e}^{-Z(t)/p}}$$

This gives a very useful prediction of how the fraction of the mass of the volume that is in stars builds up with metallicity

Does not involve star formation rate as a function of time

$M_{\rm stars}(t)/M_{\rm gas}(0)$ **increases from zero at time** $t = 0$**, and can become very large if most of the gas is used up in star formation**

Today, at time t_1**, we have a metallicity** Z_1 **and a mass in stars** $M_{\rm stars1}$**. Therefore today, we have:**

$$\frac{M_{\rm stars1}}{M_{\rm gas}(0)} \;=\; 1 \;-\; {\rm e}^{-Z_1/p}$$

Dividing this equation with the previous one gives:

$$\boxed{\frac{M_{\rm stars}(t)}{M_{\rm stars1}} \;=\; \frac{1 \;-\; {\rm e}^{-Z(t)/p}}{1 \;-\; {\rm e}^{-Z_1/p}}}$$

This is a prediction of how the mass in stars at any time varies with the metallicity

Consider observing some subsample of long-lived stars of similar mass

The number $N(Z)$ having a metallicity $\leq Z$ will be proportional to the mass in these stars

$$N(Z) \propto M_{\rm stars}(t)$$

So:

$$\frac{N(Z)}{N_1} = \frac{M_{\rm stars}(t)}{M_{\rm stars1}}$$

where N_1 is the value of $N(Z)$ today, i.e. $N(Z_1)$

So we get finally:

$$\boxed{\frac{N(Z)}{N_1} = \frac{1 - {\rm e}^{-Z(t)/p}}{1 - {\rm e}^{-Z_1/p}}} \qquad (1)$$

(note that $N(Z)$ is a *cumulative* number)

This gives a specific prediction of the cumulative number of stars as a function of metallicity

In practice, it's often easier to work with the differential distribution dN/dZ, which expresses the number of stars with metallicity Z as a function of Z

Comparing the Simple Model with Observations: Galactic Stellar Abundances and the G-Dwarf Problem

The previous equation predicts the number of stars $N(Z)$ having a metallicity $\leq Z$ as a function of Z for a sample of long-lived stars based on the Simple Model

How does the prediction compare with observation?

Need metallicity data for relatively large numbers of stars

So generally use photometry rather than spectroscopy
– though photometric estimates tend to give abundances of elements with strong absorption rather than the overall metallicity

So iron abundances [Fe/H] are generally quoted for studies of the metallicity distribution

Use G- or K-type main sequence (dwarfs) stars because of long lifetimes

G dwarfs are generally preferred because:
– greater luminosity and
– metallicity estimates better calibrated

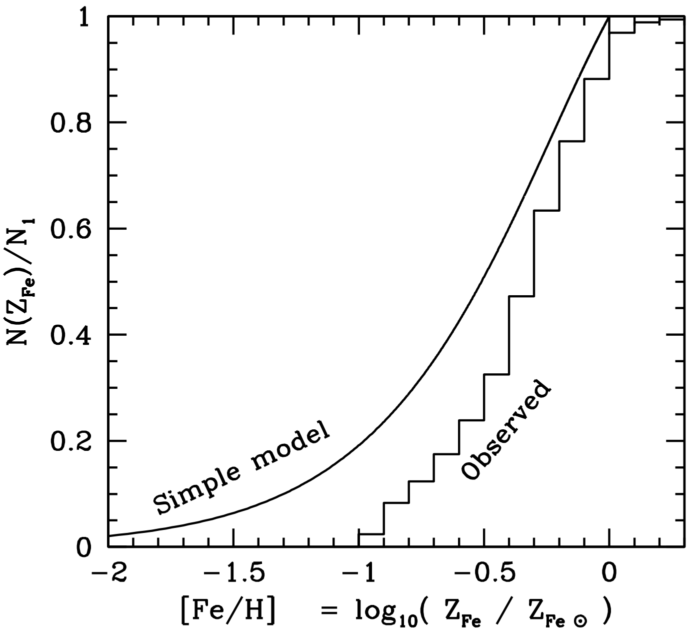
Results show that the Simple Model works reasonably well for metal-poor globular clusters

But not for stars in the solar neighbourhood within the Galactic disk

The figure below compares predicted *cumulative* metallicity distribution of Equation (1) with observations of long-lived stars in the solar neighbourhood

The Simple Model predicts a far larger proportion of metal-poor stars than are observed

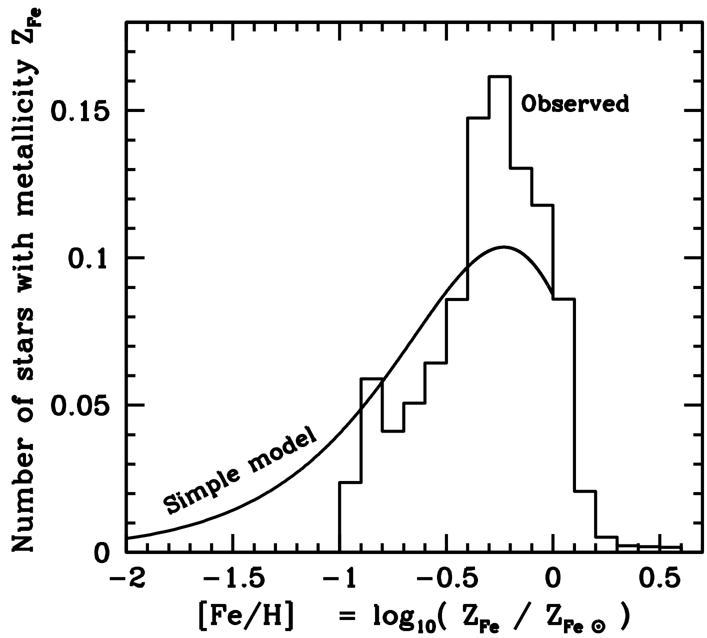
This is the *G-Dwarf Problem*



Observed *cumulative* metallicity distribution for stars in the solar neighbourhood, compared with the Simple Model for $p = 0.010$ and $Z_1 = Z_\odot = 0.017$. [The observed distribution uses data from Kotoneva et al., M.N.R.A.S., 336, 879, 2002, for stars in the Hipparcos Catalogue.]

The next figure shows predicted *differential* metallicity distribution (i.e. a histogram of star numbers as a function of [Fe/H])

– this also shows the failure of the Simple Model prediction



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Solutions to the G-Dwarf Problem

The G-dwarf problem indicates that Simple Model is an oversimplification in the solar neighbourhood

– so one or more of its assumptions must be wrong

– but which one?

A better fit to the observed data is possible by relaxing any of a number of the Simple Model assumptions

These include:

- **gas was not initially of zero metallicity**
- **there has been an inflow of very metal-poor gas (this can help, but yield p must be adjusted)**
- **there has been a variable initial mass function – could result in change in fraction α of mass that stays locked up in long-lived stars, or in change in the yield p (but no evidence for this)**
- **the samples of stars are biased against low-metallicity stars (but observers take care to correct this)**

Other possible changes that could be made to the model :

- **Allow for a loss of gas from the volume**

– plausible because supernovae following star formation could drive gas out of the volume

– could set the outflow rate to be proportional to star formation rate

– so loss of gas from the volume in a time δt is $c\delta M_{\rm stars}$ where c is a constant

– in this case the total mass $M_{\rm total}$ in the volume varies with time, unlike in the Simple Model

– however in practice, loss of enriched gas makes the G dwarf problem even worse: it reduces the quantity of enriched gas to make stars

- **Allow for inflow of gas into the volume**

– this gas could be unprocessed, primordial gas

– usually set the inflow rate proportional to star formation rate

– so inflow of gas in time δt is $c\,\delta M_{\rm stars}$ where c is a constant

– can produce a better fit between models and observations provided the yield p is well-chosen

Element Abundance Ratios

So far we've considered changes in overall metallicity Z, but what about abundances of individual elements?

– provides important additional information

Ratios of the abundances of individual elements e.g. [O/Fe] or [C/O] have been predetermined by nucleosynthesis and enrichment of ISM over time

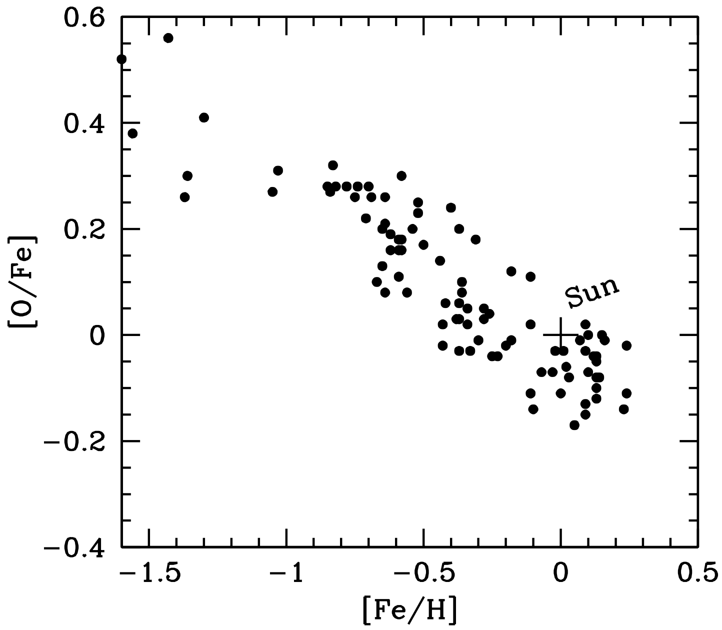
Using observations of stellar spectra :

– we often see correlations between abundance ratios

– an important example is how [O/Fe] varies with [Fe/H]

– stars with metallicities similar to Sun have, unsurprisingly, similar [O/Fe] values to Sun

– metal-poor stars have [O/Fe] values larger than Sun with [O/Fe] increasing with decreasing [Fe/H] until a near-constant value is reached (see next figure)



The [O/Fe] element abundance ratio plotted as a function of [Fe/H]. [Based on data from Edvardsson et al., Astron. Astrophys., 275, 101, 1993, supplemented with data from Zhang & Zhao, M.N.R.A.S., 364, 712, 2005.]

The conventional interpretation of this is that the heavy metal enrichment that produced the material that went into very metal-poor stars was mainly caused by type II supernovae

– Type II supernovae occur soon (typically $\sim 10^7$ yr) after a burst of star formation

– they produce large quantities of O relative to Fe, so the material in very metal-poor stars had high values of [O/Fe]

– later ($\gtrsim 10^8$ yr after the burst), Type Ia supernovae produced larger quantities of Fe compared with O, reducing the [O/Fe] ratio

in the ISM and increasing the [Fe/H] ratio

– **later stars were therefore less metal-poor and had [O/Fe] values closer to the Sun**