

Main Examination period 2022 – January – Semester A

MTH6157 / MTH6157P: Survival Models

You should attempt **ALL** questions. Marks available are shown next to the questions.

In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final.**

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. To be eligible for IFoA exemption, **you must submit your exam within the first 3 hours of the assessment period.**

Examiners: C.Sutton, A.Baule

Question 1 [14 marks]. The government of a country is considering reducing its social security expenditure by changing the age at which citizens receive a national pension from age 65 to age 68.

- (a) Explain why this change will result in fewer people receiving a pension. [1]
- (b) If there are 100,000 citizens age 65, write down an expression for the number that would be expected to start receiving a pension at age 68 in terms of a survival function. [2]
- (c) You have a survival model that produces estimates of the force of mortality by age. Re-write your answer to (b) in terms of these model outputs. [3]
- (d) The government statistical service has occurrence exposure rate data for the population. Explain how you can use this data to estimate forces of mortality and what assumptions you need to make. [6]
- (e) What forms of selection should the government be concerned about here? [2]

Question 2 [22 marks]. Athletes who suffer knee injury can select whether to take a drug or to follow a particular set of daily exercises for their rehab. 20 injured athletes were divided into two groups, half taking the drug each day and the other half following the exercise plan. The number of days before each athlete was able to start running again was recorded and the results are given below. It was determined before the trial that the drug group would stop being assessed when 70% were back running and the exercise group would be assessed for a maximum of 7 days.

Drug 2, 4, 5, 2*, 4, 5, 6*, 4, 5, 6*

Exercise 2, 3, 4, 2, 3, 2, 5, 6, 7*, 7*

where * indicates the athlete stopped being part of the trial before they were back running again.

- (a) Calculate a Kaplan Meier estimate of the survival function for knee injury for each of the two trial groups. [10]
- (b) What types of censoring are present here? [4]
- (c) What conclusions can you make about which treatment is more effective? [8]

Question 3 [19 marks]. A pizza delivery company changed from petrol to electric vehicles (EVs) one year ago. Because the company was worried about vehicle battery failure affecting their business, they agreed a contract with the EV supplier who pays £1000 each time a vehicle battery fails to compensate for any lost pizza business. The vehicles are at all times either out delivering pizzas or are having their battery recharged, unless either the vehicle suffers a battery failure or is involved in a crash.

- (a) Draw a multi-state model diagram which could be used to analyse EV batteries and the compensation arrangement clearly labelling and defining all terms. [6]
- (b) Over a week the fleet of EV vehicles were found to be out delivering pizza for a total of 1136 hours and having their batteries recharged for 544 hours. Vehicles were recharged 112 times in total. There were 4 battery failures of which 3 occurred whilst vehicles were being driven and one during a recharge. One vehicle was involved in a crash. Develop a formula and then find the maximum likelihood estimate for the transition intensity for battery failure whilst out delivering pizza. [8]
- (c) How useful is this estimate in (b) for assessing value of the £1000 compensation terms? [5]

Question 4 [15 marks]. A performance of an opera is live-streamed online. The opera has four acts which last 34, 35, 26 and 36 minutes. Between each act the streaming service shows 5 minutes of adverts. All the time that adverts are being shown there is a number displayed on screen for viewers to make a £2 donation to a charity for retired opera singers. The opera begins at 7.00pm when there are 214,353 people watching online. The number of online viewers is then recorded again at half-hour intervals:

Time	Viewers
7.30	256,743
8.00	269,459
8.30	238,850
9.00	234,653

At the end of the opera there are 202,464 people viewing.

- (a) If donations to the charity made during the opera total £1,286 calculate the rate of texting the charity number per person hour of online viewing stating clearly any assumptions you make. [12]
- (b) Explain the Principle of Correspondence as it relates to this opera charity study. [3]

Question 5 [30 marks]. An actuary produces a new standard mortality table for a country using a Poisson model to obtain estimates for the force of mortality and then graduates these model outputs by reference to a parametric formula based on Gompertz Law to produce the final table.

- (a) What are the advantages and disadvantages of this method of graduation in these circumstances? [6]
- (b) The quality of the graduation is examined using a series of statistical tests with each test completed separately for ages 0-18, 19-45, 46-65 and 66-110. The table below shows for each test whether the relevant null hypothesis was accepted or rejected at the 95% significance level.

Test	Age 0-18	Age 19-45	Age 46-65	Age 66-110
Chi Squared	accept	accept	reject	accept
Standard Deviations	reject	accept	accept	accept
Signs	accept	accept	reject	reject
Grouping of Signs	accept	accept	accept	reject

What does this set of test results tell you about the graduation? [16]

- (c) What adjustments would you recommend to the method of graduation in the light of your answer to (b) above? [8]

End of Paper.