Queen Mary
University of London

Main Examination period 2019

MTH6157: Survival Models

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks available are shown next to the questions.

Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

The New Cambridge Statistical Tables are provided.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any unauthorised notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately.

It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms, it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiners: C. Sutton, D. Boland

Question 1. [**23 marks**] A busy mobile phone store has introduced a scheme whereby customers take a numbered ticket on entering the store and wait for their number to be shown on a screen before being served. After a few days the store manager decided to collect data on the number of minutes customers had to wait under the new system. The table below records time of entry, time served in the store and, where applicable, the time customers left the store before being served.

Time of entry into the store	Time customer was served	Time of leaving without being served
09:00	09:03	
09:02	09:11	
09:06		09:07
09:06	09:08	
09:08	09:16	
09:10	09:20	
09:20	09:25	
09:21		09:29
09:25	09:30	
09:26	09:33	
09:34	09:38	
09:37	09:39	
09:40	09:43	
09:40		09:49
09:43	09:51	

(a) Write down the formula for the Kaplan-Meier survival function and explain what each term means for this mobile phone store study. [**6**]

(b) Calculate the Kaplan-Meier estimate of the survival function for the length of time between entering the store and being served. [**10**]

(c) The store is considering giving any customer who has to wait 7 minutes or more to be served, a £5 voucher to spend in store. If 1,000 customers are expected in a week, estimate the total value of vouchers given per week. [**3**]

(d) Comment on the assumptions made in using the estimate from (b) in the calculation of the cost to the store of the vouchers. [**4**]

Question 2. [**22 marks**] A new drug for people who have been bitten by poisonous snakes is being trialled in Australia by two teams of paramedics operating in areas where tourists have been bitten in previous years: one in Queensland and one in Tasmania. The drug is intended to neutralise the poison for a few hours to give time for transportation to hospital. Results are analysed using a Cox′s Proportional Hazard Model and the model fitted is

$$h_i(t) = h_0(t)exp(\beta\ z^T)$$

where
$h_i(t)$ is the hazard at time t hours after administering the drug, and
$h_0(t)$ is the baseline hazard.
$z = (z_1, z_2, z_3)$ is a vector of covariates where z_1 is 1 for patients in Tasmania and 0 for those in Queensland; z_2 is the period in hours from being bitten by the snake to taking the drug; and z_3 is 1 if the patient is age 40 or over and 0 otherwise.
$\beta = (\beta_1, \beta_2, \beta_3)$ is a vector of parameters where $\beta_1 = 0.1$; $\beta_2 = 0.01$; $\beta_3 =$ -0.2

(a) State the group of lives to whom the baseline hazard applies. [**5**]

(b) For a 28-year-old tourist given the drug 2 hours after being bitten by a snake in Queensland:
(i) Write down the hazard function in terms of $h_0(t)$
(ii) Express the survival function in terms of $h_0(t)$ [**6**]

(c) A 50-year-old tourist given the drug immediately after being bitten in Tasmania has a 90% probability of surviving for 5 hours. Calculate the probability that the tourist in (b) above will survive 5 hours. [**11**]

Question 3. [**23 marks**] An investigation is to be carried out into the relationship between employment status and mortality using a Markov multi-state model with three states: Employed, Unemployed and Dead.

(a) Draw a diagram for this model labelling the transition intensities between states. [**10**]

(b) Write down an expression for the likelihood of the data in terms of transition intensities and waiting times, defining all the terms you use. [**6**]

(c) The investigation observes a total of: 1,564 years in employment, 165 years of unemployment, 18 transitions from employed to unemployed, 8 transitions from unemployed to employed, and 7 deaths in total, six of employed people and one of an unemployed person. Calculate the maximum likelihood estimator of the transition intensity from employed to dead. [**5**]

(d) Why should the investigators be careful interpreting the results from these observations? [**2**]

Question 4. [24 marks] A life assurance company that has a large pensions annuity business wishes to compare its own mortality experience amongst people who have recently retired with rates in a standard mortality table. The relevant data is shown below:

Age	Exposed to risk	Observed deaths	Standard table mortality rate
65	1,651	20	0.012
66	1,766	24	0.014
67	1,656	34	0.016
68	1,493	25	0.018
69	1,256	29	0.020
70	1,169	20	0.023
71	966	29	0.027
72	846	19	0.030

(a) Perform a test of the overall adherence of the standard table mortality rates to the observed data at the 95% confidence level, stating any assumptions you make. [**14**]

(b) List three limitations of this test and for each one, recommend a further statistical test which the company could carry out. [**6**]

(c) Explain why this company should be particularly concerned about using a standard table in which mortality rates are too high. [**4**]

Question 5. [8 marks] To reach the top of a certain mountain a group of 50 tourists need to take a bus and then transfer to a ski-lift for the rest of the journey. On arrival at the ski-lift centre, people wait in a line for the next chair up the mountain. Each chair carries one person and ski-lift chairs depart continuously at a rate of one chair every 12 seconds. Whilst people wait in line they are offered a hot drink. If 28 of the tourists choose to have a hot drink, calculate the rate of accepting drinks per person-hour stating any assumptions you make. [**8**]

End of Paper.