Queen Mary University of London

Main Examination period 2023 – January – Semester A

MTH5124: Actuarial Mathematics I

Duration: 2 hours

The exam is intended to be completed within **2 hours**. However, you will have a period of **4 hours** to complete the exam and submit your solutions.

For actuarial students only: This module also counts towards IFoA exemptions. For your submission to be eligible, **you must submit within the first 3 hours.**

You should attempt ALL questions. Marks available are shown next to the questions.

All work should be **handwritten** and should **include your student number**. Only one attempt is allowed – **once you have submitted your work, it is final**.

In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;

Examiners: A. Baule, C. Sutton

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Question 1 [25 marks].

(a) A lecturer borrows £10,000 from a bank with an effective interest rate of 3.5% per annum. She pays back the loan after 5 years. How much does she pay back? [**5**]

(b) Determine the AER corresponding to the nominal rate of discount $d^{(12)} = 7\%$ per annum. [**5**]

(c) Suppose you are paying off your new fridge by monthly instalments of £25 paid in arrears over five years. Assuming an effective interest rate of 3% per annum, what was the price of the fridge? [**5**]

(d) Determine the value of the symbol $\bar{a}_{\overline{\infty}|}$ for an AER of 6%. [**5**]

(e) In return for a loan of £500, a loan shark charges £1 per day payable when the loan is paid back. What is the APR if you pay back the loan and fees after 155 days? [**5**]

Question 2 [18 marks].
A family has taken out a mortgage of £350,000 to purchase a house on 1.1.2022. The mortgage is repaid in monthly instalments at the beginning of each month over 20 years.

(a) Determine the monthly payment amount assuming an effective interest rate of 1.8% per annum. [**5**]

(b) After having made 5 years of payments the family takes out a new mortgage with an effective interest rate of 6% per annum for the remaining 15 years. Payments are made monthly in arrears. Determine the new monthly payment amount. [**6**]

(c) If the family would have originally agreed to make monthly instalments of £2,000 paid at the beginning of each month starting on 1.1.2022, when would they have paid off the loan? Assume an effective interest rate of 1.8% per annum throughout. [**7**]

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Question 3 [18 marks]. The UK government has issued an index-linked gilt on 1.2.2020. The gilt has a coupon rate of 1.0% of nominal paid twice yearly in arrears. Upon maturity on 1.2.2025 a redemption payment of 105% of nominal will be made. There is a no time lag in indexing payments. The bond is indexed by reference to the Retail Prices Index and the relevant values are shown below.

Year	**Feb**	**April**	**May**	**Aug**	**Sep**
2020	292.0	292.6	292.2	293.3	294.3
2021	296.0	301.1	301.9	307.4	308.6
2022	320.2	334.6	337.1	345.2	347.6

An investor purchases £10,000 nominal of the gilt on 1.2.2021 (after the coupon due on that date) and sells 15 months later.

(a) Calculate the coupon payments that the investor receives. [**6**]

(b) Calculate the sale price of the bond if the real rate of return is 1.5% per annum. [**6**]

(c) Calculate the investor's yield assuming she bought the bond for £11,200. [**6**]

Question 4 [21 marks].

(a) Use mortality given by table AMC00 ultimate values. You can find the table in the appendix. Determine the following quantities:

(i) The probability that a young man of age 18 survives to age 30. [**3**]

(ii) The probability that a man aged 30 survives to age 67, but dies within the next 10 years. [**3**]

(iii) Calculate the value of the symbol ${}_{1.25}p_{25}$. [**4**]

(b) Consider the survival function

$$s(x) = \frac{2}{1+e^x}, \quad \text{for} \quad 0 \leq x \leq 100.$$

(i) Show that $s(x)$ is a valid survival function. [**6**]

(ii) Calculate the force of mortality. [**5**]

Question 5 [18 marks].
On her 45th birthday, Lisa Martin takes out a 20-year endowment policy with a £50,000 death benefit payable at the end of the year of death and a £30,000 survival benefit. The premium is payable annually in arrears. Assume an effective annual interest rate of 4% and the mortality given by table AMC00 ultimate values.

(a) Calculate the expected present value of the survival benefit. [**6**]

(b) Calculate the expected present value of the death benefit. [**6**]

(c) What is the annual premium for the endowment policy? [**6**]

End of Paper – An appendix of 3 pages follows.

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AMC00 Male Life Table

Age x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	Age x
17	9997.5094		10000.0000	17
18	9992.9305	9994.6901	9995.4200	18
19	9988.3337	9990.1025	9990.8321	19
20	9983.6991	9985.4871	9986.2163	20
21	9979.0567	9980.8438	9981.5827	21
22	9974.3666	9976.1827	9976.9213	22
23	9969.6487	9971.4740	9972.2222	23
24	9964.8832	9966.7276	9967.4854	24
25	9960.0701	9961.9336	9962.7010	25
26	9955.1996	9957.0921	9957.8691	26
27	9950.2618	9952.1832	9952.9698	27
28	9945.2469	9947.1972	9947.9933	28
29	9940.1847	9942.1340	9942.9398	29
30	9935.0655	9936.9939	9937.7794	30
31	9929.8794	9931.7671	9932.5024	31
32	9924.5767	9926.4238	9927.0892	32
33	9919.1182	9920.9344	9921.5201	33
34	9913.4841	9915.2596	9915.7755	34
35	9907.6152	9909.3799	9909.8162	35
36	9901.4922	9903.2460	9903.6126	36
37	9895.0760	9896.8385	9897.1357	37
38	9888.3275	9890.1087	9890.3363	38
39	9881.1880	9882.9977	9883.1559	39
40	9873.5893	9875.4570	9875.5558	40
41	9865.4534	9867.4085	9867.4578	41
42	9856.7223	9858.7745	9858.7942	42
43	9847.3090	9849.4875	9849.4875	43
44	9837.1169	9839.4312	9839.4312	44
45	9826.0499	9828.5291	9828.5291	45
46	9813.9735	9816.6562	9816.6562	46
47	9800.7439	9803.6786	9803.6786	47
48	9786.2090	9789.4437	9789.4437	48
49	9770.1885	9773.7708	9773.7708	49
50	9752.4550	9756.4712	9756.4712	50
51	9732.8031	9737.3192	9737.3192	51
52	9710.9228	9716.0627	9716.0627	52
53	9686.5083	9692.4138	9692.4332	53
54	9659.2100	9666.0213	9666.1182	54
55	9628.6170	9636.5205	9636.7719	55
56	9594.3554	9603.4960	9604.0069	56
57	9555.9547	9566.5317	9567.3964	57
58	9512.9284	9525.1558	9526.4767	58

Age x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	Age x
59	9464.7096	9478.8531	9480.7305	59
60	9410.6849	9427.0212	9429.5820	60
61	9350.1805	9369.0144	9372.4010	61
62	9282.4873	9304.1589	9308.5187	62
63	9206.8320	9231.7028	9237.1968	63
64	9122.3956	9150.8360	9157.6369	64
65	9028.3257	9060.7100	9069.0001	65
66	8923.7129	8960.4327	8970.3747	66
67	8807.6361	8849.0571	8860.8106	67
68	8679.1539	8725.6105	8739.3111	68
69	8537.3365	8589.1250	8604.8568	69
70	8381.2654	8438.6022	8456.4058	70
71	8210.1056	8273.0800	8292.9181	71
72	8023.0608	8091.6666	8113.3848	72
73	7819.4936	7893.5124	7916.8461	73
74	7598.8879	7677.9138	7702.4263	74
75	7360.9680	7444.3113	7469.3894	75
76	7105.6585	7192.3651	7217.1705	76
77	6833.2053	6921.9772	6945.4296	77
78	6543.2750	6633.3614	6654.1244	78
79	6234.6920	6326.2215	6343.5631	79
80	5905.8869	6000.3050	6014.4463	80
81	5557.1270	5655.4241	5667.9541	81
82	5191.0860	5293.5691	5305.7888	82
83	4811.5773	4917.5469	4930.2080	83
84	4422.4381	4531.0383	4544.0494	84
85	4027.2597	4137.4915	4150.7347	85
86	3630.1730	3740.8772	3754.2191	86
87	3235.7358	3345.6473	3358.9374	87
88	2848.8406	2956.5986	2969.6769	88
89	2474.5254	2578.7306	2591.4291	89
90	2117.7709	2217.0436	2229.1940	90
91		1876.3069	1887.7528	91
92			1571.4202	92
93			1283.8047	93
94			1027.5920	94
95			804.3661	95
96			614.5164	96
97			457.2229	97
98			330.5502	98
99			231.6268	99
100			156.9026	100

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AMC00 Male Life Functions (4%)

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
17	5132.45		5133.73	17
18	4932.79	4933.66	4934.02	18
19	4740.89	4741.73	4742.07	19
20	4556.43	4557.25	4557.58	20
21	4379.15	4379.93	4380.25	21
22	4208.74	4209.50	4209.82	22
23	4044.95	4045.69	4045.99	23
24	3887.51	3888.23	3888.53	24
25	3736.19	3736.89	3737.18	25
26	3590.73	3591.42	3591.70	26
27	3450.92	3451.58	3451.85	27
28	3316.52	3317.17	3317.43	28
29	3187.33	3187.96	3188.22	29
30	3063.17	3063.76	3064.00	30
31	2943.81	2944.37	2944.59	31
32	2829.08	2829.61	2829.80	32
33	2718.77	2719.27	2719.43	33
34	2612.72	2613.19	2613.32	34
35	2510.74	2511.19	2511.30	35
36	2412.68	2413.11	2413.20	36
37	2318.39	2318.80	2318.87	37
38	2227.70	2228.10	2228.15	38
39	2140.47	2140.86	2140.90	39
40	2056.56	2056.95	2056.97	40
41	1975.83	1976.22	1976.23	41
42	1898.16	1898.55	1898.56	42
43	1823.41	1823.81	1823.81	43
44	1751.46	1751.87	1751.87	44
45	1682.20	1682.63	1682.63	45
46	1615.52	1615.96	1615.96	46
47	1551.29	1551.75	1551.75	47
48	1489.41	1489.90	1489.90	48
49	1429.78	1430.30	1430.30	49
50	1372.29	1372.86	1372.86	50
51	1316.85	1317.47	1317.47	51
52	1263.36	1264.03	1264.03	52
53	1211.71	1212.45	1212.46	53
54	1161.83	1162.65	1162.66	54
55	1113.60	1114.52	1114.55	55
56	1066.96	1067.98	1068.03	56
57	1021.82	1022.95	1023.04	57
58	978.09	979.35	979.49	58

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
59	935.71	937.11	937.29	59
60	894.58	896.14	896.38	60
61	854.65	856.37	856.68	61
62	815.83	817.73	818.11	62
63	778.05	780.16	780.62	63
64	741.27	743.58	744.13	64
65	705.41	707.94	708.59	65
66	670.42	673.18	673.92	66
67	636.25	639.24	640.09	67
68	602.85	606.08	607.03	68
69	570.19	573.65	574.70	69
70	538.24	541.92	543.07	70
71	506.97	510.86	512.08	71
72	476.36	480.44	481.73	72
73	446.42	450.65	451.98	73
74	417.14	421.48	422.82	74
75	388.54	392.94	394.26	75
76	360.64	365.04	366.30	76
77	333.47	337.80	338.95	77
78	307.04	311.27	312.24	78
79	281.31	285.44	286.22	79
80	256.22	260.32	260.93	80
81	231.82	235.92	236.44	81
82	208.22	212.33	212.82	82
83	185.58	189.66	190.15	83
84	164.01	168.03	168.52	84
85	143.61	147.54	148.01	85
86	124.47	128.26	128.72	86
87	106.68	110.30	110.74	87
88	90.31	93.73	94.14	88
89	75.43	78.60	78.99	89
90	62.07	64.98	65.34	90
91		52.88	53.20	91
92			42.58	92
93			33.45	93
94			25.74	94
95			19.38	95
96			14.23	96
97			10.18	97
98			7.08	98
99			4.77	99
100			3.11	100

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AMC00 Male Life Functions (4%)

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
17	121013.22		121014.86	17
18	115879.55	115880.77	115881.13	18
19	110945.59	110946.76	110947.11	19
20	106203.56	106204.70	106205.03	20
21	101646.03	101647.13	101647.45	21
22	97265.82	97266.89	97267.20	22
23	93056.05	93057.08	93057.39	23
24	89010.09	89011.10	89011.39	24
25	85121.59	85122.57	85122.86	25
26	81384.45	81385.41	81385.69	26
27	77792.78	77793.72	77793.99	27
28	74340.96	74341.87	74342.13	28
29	71023.58	71024.44	71024.70	29
30	67835.43	67836.24	67836.49	30
31	64771.51	64772.26	64772.48	31
32	61827.01	61827.70	61827.89	32
33	58997.30	58997.93	58998.09	33
34	56277.95	56278.53	56278.66	34
35	53664.69	53665.23	53665.34	35
36	51153.45	51153.95	51154.04	36
37	48740.30	48740.77	48740.84	37
38	46421.48	46421.92	46421.97	38
39	44193.38	44193.79	44193.82	39
40	42052.51	42052.91	42052.93	40
41	39995.55	39995.95	39995.96	41
42	38019.32	38019.72	38019.72	42
43	36120.76	36121.17	36121.17	43
44	34296.94	34297.35	34297.35	44
45	32545.06	32545.48	32545.48	45
46	30862.41	30862.85	30862.85	46
47	29246.43	29246.89	29246.89	47
48	27694.65	27695.14	27695.14	48
49	26204.72	26205.24	26205.24	49
50	24774.37	24774.94	24774.94	50
51	23401.47	23402.08	23402.08	51
52	22083.94	22084.61	22084.61	52
53	20819.83	20820.58	20820.58	53
54	19607.27	19608.12	19608.13	54
55	18444.47	18445.44	18445.47	55
56	17329.76	17330.87	17330.93	56
57	16261.53	16262.80	16262.89	57
58	15238.27	15239.71	15239.85	58

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
59	14258.53	14260.18	14260.36	59
60	13320.96	13322.83	13323.07	60
61	12424.28	12426.38	12426.69	61
62	11567.26	11569.63	11570.01	62
63	10748.78	10751.44	10751.90	63
64	9967.77	9970.73	9971.28	64
65	9223.22	9226.50	9227.15	65
66	8514.21	8517.82	8518.56	66
67	7839.85	7843.79	7844.64	67
68	7199.32	7203.60	7204.55	68
69	6591.87	6596.47	6597.52	69
70	6016.77	6021.68	6022.82	70
71	5473.35	5478.53	5479.75	71
72	4960.98	4966.38	4967.67	72
73	4479.04	4484.61	4485.94	73
74	4026.96	4032.62	4033.96	74
75	3604.16	3609.81	3611.14	75
76	3210.07	3215.62	3216.88	76
77	2844.13	2849.44	2850.58	77
78	2505.65	2510.66	2511.63	78
79	2193.87	2198.61	2199.39	79
80	1907.94	1912.56	1913.17	80
81	1647.13	1651.72	1652.24	81
82	1410.71	1415.31	1415.80	82
83	1197.92	1202.49	1202.97	83
84	1007.84	1012.34	1012.82	84
85	839.45	843.83	844.31	85
86	691.61	695.84	696.30	86
87	563.10	567.14	567.58	87
88	452.62	456.42	456.84	88
89	358.78	362.31	362.70	89
90	280.12	283.35	283.71	90
91		218.05	218.37	91
92			165.17	92
93			122.59	93
94			89.14	94
95			63.39	95
96			44.02	96
97			29.78	97
98			19.60	98
99			12.52	99
100			7.75	100

End of Appendix.