

Main Examination period 2019

MTH5124: Actuarial Mathematics I

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: A. Baule, C. Sutton

Question 1. [18 marks]

- (a) A lump sum of £5,000 is invested at a nominal interest rate of 6% p.a., compounded monthly. Determine the accumulation of the lump sum after 25 years. [4]
- (b) Determine the AER corresponding to the nominal interest rate $i^{(4)} = 4.5\%$ p.a.. [4]
- (c) You sign a mobile phone contract agreeing to pay £35 at the beginning of each month for the next two years. Assuming an APR of 2% p.a., how much money do you need to put into a bank account to cover all future payments? [5]
- (d) In return for a loan of £200, a loan shark charges £10 per week payable when the loan is paid back. What is the APR if you pay back the loan and fees after 1 year? [5]

Question 2. [18 marks]

John Smith is paying off a loan of £50,000 taken out exactly four years ago and being paid annually in arrears for 8 years. The fourth payment has just been made. The effective annual interest rate charged on the loan is 8.5%.

- (a) Show that the annual payment is approximately £8,866 and the capital outstanding after 4 years is approximately £29,043. [7]
- (b) Determine the schedule of payments for years 5-8. [7]
- (c) John decides to pay back the capital outstanding after 4 years by taking out a new loan. The new loan is paid in monthly instalments in advance over 3 years at an AER of 7.5%. What are the new monthly payments? [4]

Question 3. [15 marks]

An investor buys £10,000 nominal of a government bond on 1st April 2018. The bond is redeemable at 110% of its nominal value on 1st September 2030. Coupons are paid twice yearly in arrears on 1st April and 1st September at an annual rate of 1.5% of nominal.

- (a) Assuming that the gross redemption yield on the bond at the time of purchase is 2.7% per annum what is the price of the bond? [6]
- (b) Assuming the same gross redemption yield of 2.7% per annum what is the price of the bond on 1st January 2020? [5]
- (c) Show, with explanation, that the investor's net redemption yield will be greater than 2.0% per annum if the bond is held until maturity and the investor pays tax at the rate of 20% on all coupon payments. [4]

Question 4. [20 marks]

(a) Use mortality given by table AMC00 ultimate values. You can find the table in the appendix.

(i) Explain in words the meaning of the symbol ${}_2|_3q_{21}$ and calculate its value. [5]

(ii) Calculate the value of the symbol ${}_{2.5}p_{45}$. State any assumptions made. [5]

(b) Consider the survival function

$$s(x) = \frac{50 - 2x^2}{50}, \quad \text{for } 0 \leq x \leq 5.$$

(i) Show that $s(x)$ is a valid survival function. [6]

(ii) Calculate the complete expectation of life at age $x = 2$ using this survival function. [4]

Question 5. [15 marks] John Brown plans to retire at age 63 with a pension of £24,000 per year, paid annually in advance during his remaining lifetime. Assume an effective annual interest rate of 4% and the mortality given by table AMC00 select values.

(a) Show that the expected present value of the pension at retirement is approximately £331,561. [6]

(b) In order to cover the cost of the pension, John wants to make monthly payments in arrears starting at age 35 until retirement. What are the monthly premiums assuming he survives until retirement? [5]

(c) How does the expected present value in (a) change when he switches to a pension of £2,000 paid monthly in advance during his remaining lifetime? Give reasons. [4]

Question 6. [14 marks]

On his 40th birthday, Adam Smith takes out a 15-year endowment policy with £25,000 death benefit payable at the end of year of death and £25,000 survival benefit. The premium is payable annually in advance. Assume an effective annual interest rate of 4% and the mortality given by table AMC00 select values.

(a) Calculate the amount of the annual premium. [6]

(b) Calculate the expected present value of the survival benefit. [4]

(c) Calculate the expected present value of the death benefit. [4]

End of Paper – An appendix of 5 pages follows.

AMC00 Male Life Table

Age x	$I_{[x]}$	$I_{[x-1]+1}$	I_x	Age x	Age x	$I_{[x]}$	$I_{[x-1]+1}$	I_x	Age x
17	9997.5094		10000.0000	17	59	9464.7096	9478.8531	9480.7305	59
18	9992.9305	9994.6901	9995.4200	18	60	9410.6849	9427.0212	9429.5820	60
19	9988.3337	9990.1025	9990.8321	19	61	9350.1805	9369.0144	9372.4010	61
20	9983.6991	9985.4871	9986.2163	20	62	9282.4873	9304.1589	9308.5187	62
21	9979.0567	9980.8438	9981.5827	21	63	9206.8320	9231.7028	9237.1968	63
22	9974.3666	9976.1827	9976.9213	22	64	9122.3956	9150.8360	9157.6369	64
23	9969.6487	9971.4740	9972.2222	23	65	9028.3257	9060.7100	9069.0001	65
24	9964.8832	9966.7276	9967.4854	24	66	8923.7129	8960.4327	8970.3747	66
25	9960.0701	9961.9336	9962.7010	25	67	8807.6361	8849.0571	8860.8106	67
26	9955.1996	9957.0921	9957.8691	26	68	8679.1539	8725.6105	8739.3111	68
27	9950.2618	9952.1832	9952.9698	27	69	8537.3365	8589.1250	8604.8568	69
28	9945.2469	9947.1972	9947.9933	28	70	8381.2654	8438.6022	8456.4058	70
29	9940.1847	9942.1340	9942.9398	29	71	8210.1056	8273.0800	8292.9181	71
30	9935.0655	9936.9939	9937.7794	30	72	8023.0608	8091.6666	8113.3848	72
31	9929.8794	9931.7671	9932.5024	31	73	7819.4936	7893.5124	7916.8461	73
32	9924.5767	9926.4238	9927.0892	32	74	7598.8879	7677.9138	7702.4263	74
33	9919.1182	9920.9344	9921.5201	33	75	7360.9680	7444.3113	7469.3894	75
34	9913.4841	9915.2596	9915.7755	34	76	7105.6585	7192.3651	7217.1705	76
35	9907.6152	9909.3799	9909.8162	35	77	6833.2053	6921.9772	6945.4296	77
36	9901.4922	9903.2460	9903.6126	36	78	6543.2750	6633.3614	6654.1244	78
37	9895.0760	9896.8385	9897.1357	37	79	6234.6920	6326.2215	6343.5631	79
38	9888.3275	9890.1087	9890.3363	38	80	5905.8869	6000.3050	6014.4463	80
39	9881.1880	9882.9977	9883.1559	39	81	5557.1270	5655.4241	5667.9541	81
40	9873.5893	9875.4570	9875.5558	40	82	5191.0860	5293.5691	5305.7888	82
41	9865.4534	9867.4085	9867.4578	41	83	4811.5773	4917.5469	4930.2080	83
42	9856.7223	9858.7745	9858.7942	42	84	4422.4381	4531.0383	4544.0494	84
43	9847.3090	9849.4875	9849.4875	43	85	4027.2597	4137.4915	4150.7347	85
44	9837.1169	9839.4312	9839.4312	44	86	3630.1730	3740.8772	3754.2191	86
45	9826.0499	9828.5291	9828.5291	45	87	3235.7358	3345.6473	3358.9374	87
46	9813.9735	9816.6562	9816.6562	46	88	2848.8406	2956.5986	2969.6769	88
47	9800.7439	9803.6786	9803.6786	47	89	2474.5254	2578.7306	2591.4291	89
48	9786.2090	9789.4437	9789.4437	48	90	2117.7709	2217.0436	2229.1940	90
49	9770.1885	9773.7708	9773.7708	49	91		1876.3069	1887.7528	91
50	9752.4550	9756.4712	9756.4712	50	92			1571.4202	92
51	9732.8031	9737.3192	9737.3192	51	93			1283.8047	93
52	9710.9228	9716.0627	9716.0627	52	94			1027.5920	94
53	9686.5083	9692.4138	9692.4332	53	95			804.3661	95
54	9659.2100	9666.0213	9666.1182	54	96			614.5164	96
55	9628.6170	9636.5205	9636.7719	55	97			457.2229	97
56	9594.3554	9603.4960	9604.0069	56	98			330.5502	98
57	9555.9547	9566.5317	9567.3964	57	99			231.6268	99
58	9512.9284	9525.1558	9526.4767	58	100			156.9026	100

AMC00 Male Life Functions (4%)

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x	Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
17	5132.45		5133.73	17	59	935.71	937.11	937.29	59
18	4932.79	4933.66	4934.02	18	60	894.58	896.14	896.38	60
19	4740.89	4741.73	4742.07	19	61	854.65	856.37	856.68	61
20	4556.43	4557.25	4557.58	20	62	815.83	817.73	818.11	62
21	4379.15	4379.93	4380.25	21	63	778.05	780.16	780.62	63
22	4208.74	4209.50	4209.82	22	64	741.27	743.58	744.13	64
23	4044.95	4045.69	4045.99	23	65	705.41	707.94	708.59	65
24	3887.51	3888.23	3888.53	24	66	670.42	673.18	673.92	66
25	3736.19	3736.89	3737.18	25	67	636.25	639.24	640.09	67
26	3590.73	3591.42	3591.70	26	68	602.85	606.08	607.03	68
27	3450.92	3451.58	3451.85	27	69	570.19	573.65	574.70	69
28	3316.52	3317.17	3317.43	28	70	538.24	541.92	543.07	70
29	3187.33	3187.96	3188.22	29	71	506.97	510.86	512.08	71
30	3063.17	3063.76	3064.00	30	72	476.36	480.44	481.73	72
31	2943.81	2944.37	2944.59	31	73	446.42	450.65	451.98	73
32	2829.08	2829.61	2829.80	32	74	417.14	421.48	422.82	74
33	2718.77	2719.27	2719.43	33	75	388.54	392.94	394.26	75
34	2612.72	2613.19	2613.32	34	76	360.64	365.04	366.30	76
35	2510.74	2511.19	2511.30	35	77	333.47	337.80	338.95	77
36	2412.68	2413.11	2413.20	36	78	307.04	311.27	312.24	78
37	2318.39	2318.80	2318.87	37	79	281.31	285.44	286.22	79
38	2227.70	2228.10	2228.15	38	80	256.22	260.32	260.93	80
39	2140.47	2140.86	2140.90	39	81	231.82	235.92	236.44	81
40	2056.56	2056.95	2056.97	40	82	208.22	212.33	212.82	82
41	1975.83	1976.22	1976.23	41	83	185.58	189.66	190.15	83
42	1898.16	1898.55	1898.56	42	84	164.01	168.03	168.52	84
43	1823.41	1823.81	1823.81	43	85	143.61	147.54	148.01	85
44	1751.46	1751.87	1751.87	44	86	124.47	128.26	128.72	86
45	1682.20	1682.63	1682.63	45	87	106.68	110.30	110.74	87
46	1615.52	1615.96	1615.96	46	88	90.31	93.73	94.14	88
47	1551.29	1551.75	1551.75	47	89	75.43	78.60	78.99	89
48	1489.41	1489.90	1489.90	48	90	62.07	64.98	65.34	90
49	1429.78	1430.30	1430.30	49	91		52.88	53.20	91
50	1372.29	1372.86	1372.86	50	92			42.58	92
51	1316.85	1317.47	1317.47	51	93			33.45	93
52	1263.36	1264.03	1264.03	52	94			25.74	94
53	1211.71	1212.45	1212.46	53	95			19.38	95
54	1161.83	1162.65	1162.66	54	96			14.23	96
55	1113.60	1114.52	1114.55	55	97			10.18	97
56	1066.96	1067.98	1068.03	56	98			7.08	98
57	1021.82	1022.95	1023.04	57	99			4.77	99
58	978.09	979.35	979.49	58	100			3.11	100

AMC00 Male Life Functions (4%)

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x	Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
17	121013.22		121014.86	17	59	14258.53	14260.18	14260.36	59
18	115879.55	115880.77	115881.13	18	60	13320.96	13322.83	13323.07	60
19	110945.59	110946.76	110947.11	19	61	12424.28	12426.38	12426.69	61
20	106203.56	106204.70	106205.03	20	62	11567.26	11569.63	11570.01	62
21	101646.03	101647.13	101647.45	21	63	10748.78	10751.44	10751.90	63
22	97265.82	97266.89	97267.20	22	64	9967.77	9970.73	9971.28	64
23	93056.05	93057.08	93057.39	23	65	9223.22	9226.50	9227.15	65
24	89010.09	89011.10	89011.39	24	66	8514.21	8517.82	8518.56	66
25	85121.59	85122.57	85122.86	25	67	7839.85	7843.79	7844.64	67
26	81384.45	81385.41	81385.69	26	68	7199.32	7203.60	7204.55	68
27	77792.78	77793.72	77793.99	27	69	6591.87	6596.47	6597.52	69
28	74340.96	74341.87	74342.13	28	70	6016.77	6021.68	6022.82	70
29	71023.58	71024.44	71024.70	29	71	5473.35	5478.53	5479.75	71
30	67835.43	67836.24	67836.49	30	72	4960.98	4966.38	4967.67	72
31	64771.51	64772.26	64772.48	31	73	4479.04	4484.61	4485.94	73
32	61827.01	61827.70	61827.89	32	74	4026.96	4032.62	4033.96	74
33	58997.30	58997.93	58998.09	33	75	3604.16	3609.81	3611.14	75
34	56277.95	56278.53	56278.66	34	76	3210.07	3215.62	3216.88	76
35	53664.69	53665.23	53665.34	35	77	2844.13	2849.44	2850.58	77
36	51153.45	51153.95	51154.04	36	78	2505.65	2510.66	2511.63	78
37	48740.30	48740.77	48740.84	37	79	2193.87	2198.61	2199.39	79
38	46421.48	46421.92	46421.97	38	80	1907.94	1912.56	1913.17	80
39	44193.38	44193.79	44193.82	39	81	1647.13	1651.72	1652.24	81
40	42052.51	42052.91	42052.93	40	82	1410.71	1415.31	1415.80	82
41	39995.55	39995.95	39995.96	41	83	1197.92	1202.49	1202.97	83
42	38019.32	38019.72	38019.72	42	84	1007.84	1012.34	1012.82	84
43	36120.76	36121.17	36121.17	43	85	839.45	843.83	844.31	85
44	34296.94	34297.35	34297.35	44	86	691.61	695.84	696.30	86
45	32545.06	32545.48	32545.48	45	87	563.10	567.14	567.58	87
46	30862.41	30862.85	30862.85	46	88	452.62	456.42	456.84	88
47	29246.43	29246.89	29246.89	47	89	358.78	362.31	362.70	89
48	27694.65	27695.14	27695.14	48	90	280.12	283.35	283.71	90
49	26204.72	26205.24	26205.24	49	91		218.05	218.37	91
50	24774.37	24774.94	24774.94	50	92			165.17	92
51	23401.47	23402.08	23402.08	51	93			122.59	93
52	22083.94	22084.61	22084.61	52	94			89.14	94
53	20819.83	20820.58	20820.58	53	95			63.39	95
54	19607.27	19608.12	19608.13	54	96			44.02	96
55	18444.47	18445.44	18445.47	55	97			29.78	97
56	17329.76	17330.87	17330.93	56	98			19.60	98
57	16261.53	16262.80	16262.89	57	99			12.52	99
58	15238.27	15239.71	15239.85	58	100			7.75	100

AMC00 Male Life Functions (4%)

Age x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	Age x	Age x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	Age x
17	478.10		479.31	17	59	387.30	388.64	388.82	59
18	475.89	476.71	477.05	18	60	382.24	383.72	383.95	60
19	473.75	474.54	474.88	19	61	376.79	378.43	378.73	61
20	471.68	472.45	472.77	20	62	370.93	372.74	373.11	62
21	469.68	470.42	470.74	21	63	364.64	366.64	367.09	63
22	467.74	468.47	468.77	22	64	357.89	360.09	360.62	64
23	465.87	466.57	466.86	23	65	350.67	353.07	353.70	65
24	464.05	464.73	465.02	24	66	342.95	345.57	346.29	66
25	462.28	462.94	463.22	25	67	334.71	337.55	338.37	67
26	460.56	461.21	461.48	26	68	325.95	329.02	329.93	68
27	458.89	459.52	459.78	27	69	316.66	319.94	320.95	69
28	457.25	457.86	458.12	28	70	306.83	310.32	311.42	70
29	455.66	456.25	456.50	29	71	296.46	300.15	301.32	71
30	454.11	454.67	454.91	30	72	285.56	289.42	290.66	72
31	452.60	453.13	453.34	31	73	274.15	278.16	279.44	73
32	451.12	451.62	451.80	32	74	262.26	266.38	267.67	74
33	449.65	450.12	450.27	33	75	249.92	254.10	255.37	75
34	448.18	448.63	448.76	34	76	237.17	241.36	242.57	76
35	446.72	447.14	447.25	35	77	224.08	228.21	229.31	77
36	445.24	445.65	445.74	36	78	210.67	214.70	215.64	78
37	443.76	444.15	444.22	37	79	196.93	200.88	201.63	79
38	442.25	442.64	442.69	38	80	182.84	186.76	187.35	80
39	440.72	441.10	441.13	39	81	168.47	172.39	172.90	81
40	439.16	439.53	439.55	40	82	153.96	157.90	158.37	82
41	437.54	437.92	437.93	41	83	139.50	143.41	143.88	83
42	435.88	436.26	436.26	42	84	125.24	129.10	129.56	84
43	434.15	434.54	434.54	43	85	111.32	115.08	115.54	85
44	432.35	432.75	432.75	44	86	97.87	101.50	101.94	86
45	430.47	430.88	430.88	45	87	85.02	88.49	88.91	87
46	428.50	428.92	428.92	46	88	72.90	76.17	76.57	88
47	426.42	426.87	426.87	47	89	61.63	64.67	65.04	89
48	424.23	424.70	424.70	48	90	51.30	54.08	54.42	90
49	421.91	422.41	422.41	49	91		44.49	44.80	91
50	419.43	419.98	419.98	50	92			36.23	92
51	416.80	417.39	417.39	51	93			28.74	93
52	413.98	414.62	414.62	52	94			22.32	94
53	410.95	411.66	411.66	53	95			16.94	95
54	407.70	408.49	408.50	54	96			12.54	96
55	404.20	405.08	405.10	55	97			9.04	97
56	400.43	401.41	401.46	56	98			6.33	98
57	396.37	397.46	397.55	57	99			4.29	99
58	392.01	393.21	393.34	58	100			2.81	100

AMC00 Male Life Functions (4%)

Age x	$A_{[x]}$	A_x	$\dot{a}_{[x]}$	\ddot{a}_x	Age x	Age x	$A_{[x]}$	A_x	$\dot{a}_{[x]}$	\ddot{a}_x	Age x
17	0.09315	0.09337	23.578	23.572	17	59	0.41391	0.41483	15.238	15.214	59
18	0.09647	0.09669	23.492	23.486	18	60	0.42728	0.42834	14.891	14.863	60
19	0.09993	0.10014	23.402	23.396	19	61	0.44087	0.44209	14.537	14.506	61
20	0.10352	0.10373	23.309	23.303	20	62	0.45467	0.45607	14.179	14.142	62
21	0.10725	0.10747	23.211	23.206	21	63	0.46866	0.47025	13.815	13.774	63
22	0.11114	0.11135	23.110	23.105	22	64	0.48281	0.48462	13.447	13.400	64
23	0.11517	0.11539	23.005	23.000	23	65	0.49711	0.49916	13.075	13.022	65
24	0.11937	0.11959	22.896	22.891	24	66	0.51154	0.51384	12.700	12.640	66
25	0.12373	0.12395	22.783	22.777	25	67	0.52608	0.52863	12.322	12.256	67
26	0.12826	0.12848	22.665	22.659	26	68	0.54069	0.54352	11.942	11.869	68
27	0.13297	0.13320	22.543	22.537	27	69	0.55536	0.55847	11.561	11.480	69
28	0.13787	0.13809	22.415	22.410	28	70	0.57005	0.57345	11.179	11.090	70
29	0.14296	0.14318	22.283	22.277	29	71	0.58476	0.58843	10.796	10.701	71
30	0.14825	0.14847	22.146	22.140	30	72	0.59945	0.60338	10.414	10.312	72
31	0.15375	0.15396	22.003	21.997	31	73	0.61411	0.61827	10.033	9.925	73
32	0.15946	0.15966	21.854	21.849	32	74	0.62870	0.63306	9.654	9.541	74
33	0.16539	0.16558	21.700	21.695	33	75	0.64322	0.64772	9.276	9.159	75
34	0.17154	0.17172	21.540	21.535	34	76	0.65765	0.66222	8.901	8.782	76
35	0.17792	0.17809	21.374	21.370	35	77	0.67197	0.67653	8.529	8.410	77
36	0.18454	0.18471	21.202	21.198	36	78	0.68613	0.69062	8.161	8.044	78
37	0.19141	0.19157	21.023	21.019	37	79	0.70005	0.70445	7.799	7.684	79
38	0.19853	0.19868	20.838	20.834	38	80	0.71360	0.71800	7.446	7.332	80
39	0.20590	0.20605	20.647	20.643	39	81	0.72672	0.73123	7.105	6.988	81
40	0.21354	0.21369	20.448	20.444	40	82	0.73942	0.74413	6.775	6.652	82
41	0.22145	0.22160	20.242	20.238	41	83	0.75173	0.75668	6.455	6.326	83
42	0.22963	0.22978	20.030	20.026	42	84	0.76365	0.76884	6.145	6.010	84
43	0.23810	0.23826	19.809	19.805	43	85	0.77518	0.78060	5.845	5.704	85
44	0.24685	0.24702	19.582	19.578	44	86	0.78629	0.79195	5.556	5.409	86
45	0.25590	0.25608	19.347	19.342	45	87	0.79698	0.80287	5.279	5.125	87
46	0.26524	0.26543	19.104	19.099	46	88	0.80724	0.81336	5.012	4.853	88
47	0.27488	0.27509	18.853	18.848	47	89	0.81705	0.82340	4.757	4.592	89
48	0.28483	0.28506	18.594	18.589	48	90	0.82642	0.83299	4.513	4.342	90
49	0.29508	0.29533	18.328	18.321	49	91		0.84213		4.105	91
50	0.30564	0.30591	18.053	18.046	50	92		0.85081		3.879	92
51	0.31651	0.31681	17.771	17.763	51	93		0.85905		3.665	93
52	0.32768	0.32801	17.480	17.472	52	94		0.86683		3.462	94
53	0.33915	0.33953	17.182	17.172	53	95		0.87417		3.272	95
54	0.35091	0.35135	16.876	16.865	54	96		0.88106		3.092	96
55	0.36297	0.36347	16.563	16.550	55	97		0.88751		2.925	97
56	0.37530	0.37589	16.242	16.227	56	98		0.89351		2.769	98
57	0.38791	0.38859	15.914	15.897	57	99		0.89904		2.625	99
58	0.40079	0.40158	15.580	15.559	58	100		0.90404		2.495	100

End of Appendix.