Queen Mary
University of London

Main Examination period 2018

MTH6150: Numerical Computing in C and C++

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Log into your computer and start Visual Studio before the exam begins.

You should attempt ALL questions. Marks available are shown next to the questions.

Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination.

This is an OPEN BOOK exam.
Permitted:

- any printed material, e.g. books;
- any handwritten notes or photocopies;
- use of a computer and the internet (QMPlus, google, wikipedia,...);
- access to your university network folder.

Prohibited:

- electronic communication devices (e.g. mobile phones, smart watches);
- any use of email;
- communicating via the internet (Facebook, Twitter, Stack Exchange,...);
- file sharing websites such as Dropbox;
- sharing material with other students.

Details of all internet activity on your computer may be logged.

To submit your solution there is a link on the module's QMPlus web page named "Exam submission". Put the entire Visual Studio solution in a zip file and upload it at this link.
As a backup, there are links in the same place for each individual question's code file. Upload each .cpp file here.

Exam papers must not be removed from the examination room.

Examiners: J. Griffin, V. Nicosia

All answers are to be written in C++ code files which will compile when included in a project. Use one file for each question. If a question asks for any statement about the results, include the answer as a comment in the code.

Question 1. [23 marks]

(a) The lower incomplete gamma function can be expressed as an infinite series

$$\gamma(\alpha, x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{\alpha+k}}{k!(\alpha+k)}.$$

Write a function that calculates the sum of the first n terms of the series. The input arguments should be α, x and n. **[12]**

(b) Write a modified version of the function from part (a) that calculates the sum up to the first value of k where

$$\left| \frac{(-1)^k x^{\alpha+k}}{k!(\alpha+k)} \right| < \epsilon$$

for some $\epsilon > 0$. The input arguments should be α, x and ϵ. **[7]**

(c) Approximate $\gamma(\alpha, x)$ with $\alpha = 4.2, x = 1.5$ using both versions of the function, with $n = 12$ and with $\epsilon = 1 \times 10^{-4}$. **[4]**

Question 2. [25 marks]

(a) The Weibull distribution with parameters $\eta, \kappa > 0$ has cumulative distribution function (cdf)

$$F(x) = 1 - e^{-(\eta x)^{\kappa}}, \ x \geq 0.$$

Write code that calculates the inverse function of $F(x)$, i.e. the function $G(u)$ such that if $u = F(x)$ then $x = G(u)$.

Hence, using the inverse cdf method, write code to generate random variables from the Weibull distribution. Fill in a vector of size 10,000 with Weibull random variables, using parameters $\eta = 0.5, \kappa = 2$. **[17]**

(b) The theoretical median of the Weibull distribution is

$$a = \frac{1}{\eta} \left(1 - log(0.5)\right)^{1/\kappa}.$$

Count what proportion of your random sample is less than a. **[8]**

Question 3. [30 marks]

(a) Suppose that we want to approximate the following integral

$$I = \int_0^{\pi/4} cos(x)^4 \, dx.$$

Let I_n be the estimate of I obtained by using the trapezium method with n sub-intervals. Define

$$r_n = \left| \frac{I_{2n} - I_n}{I_n} \right|,$$

the relative change in the result when n is doubled. Compute values of I_n starting from $n = 5$, doubling n until $r_n < 1 \times 10^{-4}$. **[18]**

(b) Write code for a function object that has one member variable m of type `int`, a suitable constructor, and a member function of the form

```
double operator()(const double x) const {
```

which returns $cos(x)^m$.

Use this function object with the trapezium method to approximate I, using the final value of n from part (a).

Use another instance of this function object to approximate

$$J = \int_0^{\pi/2} cos(x)^6 \, dx$$

using the same value of n. **[12]**

Question 4. [22 marks]

(a) A simple model for an infectious disease outbreak in a population is the following

$$\begin{aligned}\frac{dS}{dt} &= -\beta S I \\ \frac{dE}{dt} &= \beta S I - \gamma E \\ \frac{dI}{dt} &= \gamma E - \alpha I \\ \frac{dR}{dt} &= \alpha I\end{aligned}$$

where β, γ and α are constants. The model states S, E, I and R are the number of people who are susceptible, infected, infectious and recovered, respectively.

Use the modified Euler method to numerically solve these equation from $t = 0$ to 1000, with $\gamma = 0.15$, $\alpha = 0.25$ and $\beta = 0.0005$ (all time units are in days), using $n = 5000$ steps. As initial values, take $S(0) = 999, E(0) = 1, I(0) = 0, R(0) = 0$. **[16]**

(b) Create a vector of size 1000, and fill it in with the numerical solution of the value of $S(t)$ for $t = 1, 2, \ldots, 1000$. Also, output the final values of S, E, I and R at $t = 1000$ to the screen. **[6]**

End of Paper.