Queen Mary
University of London

Main Examination period 2023 – May/June – Semester B

MTH5126: Statistics for Insurance

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

The exam is intended to be completed within **2 hours**. However, you will have a period of **4 hours** to complete the exam and submit your solutions.

For actuarial students only: This module also counts towards IFoA exemptions. For your submission to be eligible, **you must submit within the first 3 hours.**

You should attempt ALL questions. Marks available are shown next to the questions.

You are allowed to bring **three A4 sheets of paper** as notes for the exam.

Only approved non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any unauthorised notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately.

It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms, it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiners: L. Fang, M. Nica

Question 1 [15 marks]. Let the random variable S denote the aggregate claims paid by the insurer, the random variable X_i denote the amount of the i-th claim, and $\{X_i\}_{i=1}^n$ are independent and identical random variables. $S = \sum_{i=1}^N X_i$, where the discrete random variable N represents the number of claims.
Suppose N follows a Poisson(λ) distribution, whose MGF is given by:

$$M_N(t) = e^{\lambda(e^t-1)}.$$

The mean and variance of the Poisson distribution are both λ.
All $\{X_i\}_{i=1}^n$ follow Normal(μ, σ^2), whose MGF is given by:

$$M_X(t) = e^{\mu t+\frac{1}{2}\sigma^2 t^2}.$$

You can use the following formulae without proof:
The moment generating functions (MGF) of S, X_i, N satisfy the equation $M_S(t) = M_N[logM_X(t)]$.

(a) Find an expression for the MGF of the aggregate claim amount if the number of claims has a Poission(100) distribution and individual claim sizes are Normal(20, 6^2). **[8]**

(b) Find the mean and variance of the aggregate claim amount. **[7]**

Question 2 [30 marks]. An insurer has effected excess of loss reinsurance with retention level 700. The annual aggregate claim amount from the insurer's risk has a compound Poisson distribution with Poisson parameter 20. Individual claim amounts are **uniformly** distributed on [0, 1000].

(a) Calculate the mean and variance of the **insurer's** aggregate claims under this reinsurance arrangement, S_I. **[10]**

(b) Calculate the mean and variance of the **reinsurer's** aggregate claims under this reinsurance arrangement S_R. **[10]**

(c) What is the variance of S, the aggregate claim amount **before reinsurance**? **[10]**

Question 3 [30 marks].

(a) Explain, in words, the meaning of the following copula expression: $C(u, v)$. [**2**]

(b) A **Clayton** copula is defined in the bivariate case as:

$$C[u, v] = (u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1}{\alpha}} \ for \ \alpha > 0.$$

Derive the coefficient of lower tail dependence for the Claydon copula. [**10**]

(c) Let X and Y be two random variables representing the future lifetimes of two 30-year old individuals, who are married and live together. You are given that:

$$P(X \leq 40) = 0.16, P(Y \leq 40) = 0.25.$$

Calculate the joint probability that both lives will die by the age of 70 using the Clayton copula with $\alpha = 0.5$. [**5**]

(d) Is it appropriate if we use the Clayton copula with a new α value, $\alpha = -0.3$, to calculate the joint probability that the two lives in (c) will die by a certain age, with all else being the same? [**3**]

(e) Now consider the **Frank** copula. The **Frank** copula is an example of Archimedean copulas. For the case where there are 3 variables, Archimedean copulas take the form:

$$C[u, v, w] = \Psi^{[-1]}(\Psi(u) + \Psi(v) + \Psi(w))$$

Derive an expression for the **Frank** copula for the case where the parameter $\alpha \neq 0$ and there are 3 variables.
The Frank copula has a generator function:

$$\Psi(t) = -ln \left(\frac{e^{-\alpha t} - 1}{e^{-\alpha} - 1} \right)$$

[**10**]

Question 4 [25 marks]. Assume individual claim amounts, X, follow a distribution with density function

$$f(x) = 0.02xe^{-x}, \quad x > 0.$$

Claim events on a portfolio of insurance policies follow a Poisson process with parameter λ.
The insurance company calculates premiums using a premium loading of 30%.

(a) Derive the moment generating function, $M_X(t)$, given that $t < 1$. **[10]**

(b) The adjustment coefficient, R, is the unique positive root of the following equation, where $c = (1+\theta)\lambda E(X)$ is the rate of premium income and λ is the Poisson parameter:

$$M_X(R) = 1 + \frac{cR}{\lambda}.$$

Determine the adjustment coefficient R. (Showing the equation in the simplest form, with the only unknown parameter being R, is enough to get full marks. You don't have to solve for all the Rs.) **[10]**

(c) Suppose instead that individual claims are now for a fixed amount of $E(X)$. Will the probability of ultimate ruin for this business increase, decrease or remain unchanged? Explain your answer. **[5]**

End of Paper.