Queen Mary
University of London

Main Examination period 2022 – May/June – Semester B

MTH5126: Statistics for Insurance

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment:

- **You may use books and notes.**
- **You may use calculators and computers, but you must show your working for any calculations you do.**
- **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
- **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. To be eligible for IFoA exemption, **your must submit your exam within the first 3 hours of the assessment period**.

Examiners: F. Parsa, J. Griffin

Question 1 [31 marks]. A home insurance company's total monthly claim amounts have a mean of 250 and a standard deviation of 300. The company has estimated that it will face insolvency if the total monthly claim amounts exceed 1,000 in any given month.

(a) Determine the probability that the company faces insolvency in any given month if the company assumes that total monthly claim amounts follow the Normal distribution. [**4**]

(b) Determine the revised value of the probability in part (a) if the company assumes that total monthly claim amounts follow the two-parameter Pareto distribution. [**6**]

(c) An Analyst has determined that the two-parameter Pareto distribution is the best fit for the distribution of the total monthly claim amounts for this company. Outline, using the results from parts (a) and (b), the potential consequences of the company assuming that the total monthly claim amounts follow the Normal distribution rather than the two-parameter Pareto distribution.
Explain why the Normal distribution is unlikely to be a good fit for the distribution of the total monthly claim amounts for this company. [**4**]

Suppose that the aggregate claims from a risk have a compound Poisson distribution with parameter μ, and individual claim amounts have a two-parameter Pareto distribution with a mean of 250 and a standard deviation of 300. The insurer of this risk is considering effecting proportional reinsurance with a retention level of 0.75, and calculates the premium using a premium loading factor of 0.1 (this means they charge 10% in excess of the risk premium). The reinsurance premium would be calculated using a premium loading factor of 0.2.
(The insurer's profit is defined to be the premium charged by the insurer less the reinsurance premium and less the claims paid by the insurer, net of reinsurance.)

(d) Calculate the insurer's expected profit before effecting the reinsurance. [**4**]

(e) Calculate the insurer's expected profit after effecting the reinsurance and hence find the percentage reduction in the insurer's expected profit. [**13**]

Question 2 [29 marks]. Claims on a portfolio of insurance policies arrive as a Poisson process with parameter 200. Individual claim amounts follow a normal distribution with the mean 20 and variance 16. The insurer calculates premiums using a premium loading of 15% and has initial surplus of 200.

(a) Define the ruin probabilities $\Psi(200)$,$\Psi(200,1)$ and $\Psi_1(200,1)$. **[5]**

(b) Define the adjustment coefficient R. **[2]**

(c) Show that for this portfolio the value of R is 0.013 correct to 3 decimal places. **[7]**

(d) Calculate an upper bound for $\Psi(200)$ and an estimate of $\Psi_1(200,1)$. **[8]**

(e) Comment on the results in part (d). **[2]**

(f) If the parameter of Poisson process changes to 100 for the above portfolio, explain how this will affect the probability of ruin in finite and in infinite time. **[5]**

Question 3 [20 marks].

(a) Demonstrate that the coefficient of lower tail dependence is $2^{-\frac{1}{\alpha}}$ where α is the Clayton copula parameter and $\alpha > 0$. **[7]**

(b) Comment on how the value of the parameter α affects the degree of upper tail dependence in the case of the Clayton copula. **[3]**

(c) Derive an expression for the Clayton copula for the case where the parameter $\alpha > 0$ and there are 3 variables. The Clayton copula has a generator function:

$$\Psi(t) = \frac{1}{\alpha}(t^{-\alpha} - 1).$$

[10]

Question 4 [20 marks]. An actuary assumes that the underlying gross claims follow an exponential distribution of some unknown rate λ. The payments of n claims are to be split between an insurance company and its reinsurer under an Excess of Loss reinsurance arrangement with a retention level M. The actuary needs to find the maximum likelihood estimate of λ using only the claims amount paid by the insurer. If the amount of only r claims are above the retention level, find the maximum likelihood estimate of λ. Assume that all claims are independent. **[20]**

End of Paper.