Main Examination period 2020 – May/June – Semester B
Online Alternative Assessments

MTH5126: Statistics for Insurance

You should attempt ALL questions. Marks available are shown next to the questions.

Statistical tables
You may use any of the following: your own copy of statistical tables, online versions from the internet or appendices provided at the end of this assessment paper.

In completing this assessment, you may use books, notes, and the internet. You may use calculators and computers, but you should show your working for any calculations you do. You must not seek or obtain help from anyone else.

At the start of your work, please **copy out and sign** the following declaration:

> I declare that my submission is entirely my own, and I have not sought or obtained help from anyone else.

All work should be **handwritten**, and should **include your student number**.

You have **24 hours** in which to complete and submit this assessment. When you have finished your work:

- scan your work, convert it to a **single PDF file** and upload this using the upload tool on the QMplus page for the module;
- e-mail a copy to maths@qmul.ac.uk with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are not expected to spend a long time working on this assessment. We expect you to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the assessment period, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions
This module counts towards IFoA actuarial exemptions. For your submission to be eligible for IFoA exemptions, you must submit within the first **3 hours** of the assessment period. You may then submit a second version later in the assessment period if you wish, which will count only towards your degree. There are two separate upload tools on the QMplus page to enable you to submit a second version of your work.

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Question 1 [14 marks].
There are only two companies operating in the space tourism industry: Helios plc and Jupiter plc. They share profits of £1 billion each year. Every year the companies need to decide between two business approaches: cautious and aggressive.

If both companies adopt the same approach in a given year, Helios captures $x\%$ of the total profits, where x = 70 + the last digit of your student ID number. So for example:

If the last digit of your student ID number is 0, then x = 70;
If the last digit of your student ID number is 1, then x = 71;
...
If the last digit of your student ID number is 9, then x = 79.

If they adopt different approaches, then Jupiter captures 80% of the total profits if Helios is cautious and 60% of the total profits if Helios is aggressive. Neither company knows what the other company's approach will be before adopting its own approach.

(a) Explain why the above can be thought of as a zero-sum two-person game. [**2**]

(b) Helios plc decides to adopt a randomised strategy to setting its approach each year. Determine the optimal randomised strategy for Helios plc. [**8**]

(c) Under the strategy in (b), what is the expected profit for Helios plc? [**4**]

Question 2 [11 marks].

Suppose we have an exponentially distributed sampling distribution so that $X|\theta \sim exp(\theta)$. A random sample $x_1, x_2, \ldots, x_n$ is taken from this distribution.

(a) Write down the likelihood function for this sample. [**4**]

(b) The prior distribution for θ is $Gamma(\alpha, \beta)$ and the probability density function corresponding to this prior distribution for θ is

$$f(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta} \qquad \text{for } \theta > 0$$

Find the posterior distribution for θ. [**7**]

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Question 3 [13 marks].

A random sample $x_1, x_2, \ldots, x_n$ is taken from a distribution having the density function:

$$f(x) = \frac{k}{5}x^{-\frac{4}{5}}exp(-kx^{\frac{1}{5}}),\ x > 0.$$

(a) Find $L(k)$, the likelihood function for this sample. **[4]**

(b) A sample of n =20 values was collected and the following summary statistic was computed.

$$\sum_{i=1}^{20} x_i^{\frac{1}{5}} = 102.778$$

Determine the maximum likelihood estimate of k. **[9]**

Question 4 [17 marks].

The table below shows annual aggregate claim statistics for four risks over six years.

Annual aggregate claims for risk i, in year j are denoted by X_{ij}.

Risk, i	$\bar{X}_i = \frac{1}{6}\sum_{j=1}^{6} X_{ij}$	$s_i^2 = \frac{1}{5}\sum_{j=1}^{6}\left(X_{ij} - \bar{X}_i\right)^2$
1	46.8	1227.4
2	30.2	1161.4
3	74.5	1340.3
4	60.7	1414.7

(a) For the data above, calculate the value of Z, the credibility factor under the assumptions of Empirical Bayes Credibility Theory (EBCT) Model 1. **[9]**

(b) Comment on why the credibility factor is relatively low in this case. **[4]**

(c) Using your answer from part (a), calculate the credibility premium for each risk above. **[4]**

Question 5 [22 marks].
Individual claim amounts from a portfolio of general insurance policies have a uniform distribution over the range (0, 200). Excess of loss reinsurance is arranged so that the expected amount paid by the insurer on any claim is £50.

(a) Show that the retention limit, M, in the reinsurance arrangement above is £58.58. [**11**]

(b) Aggregate annual claims from this same portfolio have a compound Poisson distribution with Poisson parameter 20. Calculate the variance of the aggregate annual claims paid by the insurer. You may quote without proof the result for the variance of a compound Poisson distribution with Poisson parameter λ. [**11**]

Question 6 [13 marks].
Peace-Of-Mind Insurance Ltd assumes that individual claims arising during each year from a particular type of annual insurance policy follow a normal distribution. Claims are assumed to arise independently and this insurer assesses its solvency position at the end of each year.

Peace-Of-Mind has an initial surplus of £100,000 and expects to sell 100 policies at the beginning of the coming year in respect of identical risks. The insurer incurs expenses of $0.2P$ at the time of writing each policy, where P is the annual premium for an individual policy. In the coming year, it is expected that each policy will be sold for an annual premium of £5,000.

In your calculations in this question, you can ignore interest.

(a) Show that:

$$U(1) = 500{,}000 - S(1)$$

where
$U(1)$ is the insurer's surplus at the end of the coming year, and
$S(1)$ is the aggregate claims at the end of the coming year. **[5]**

(b) The distribution of $S(1)$ is given as $N(\mu, \sigma^2)$, where $\mu = 350{,}000$ and $\sigma = 100{,}000$. Hence, calculate the probability that Peace-Of-Mind will prove to be insolvent at the end of the coming year. **[8]**

Question 7 [10 marks].
A speed camera used on a London motorway incorrectly gives a positive reading for drivers who are not over the legal limit one time in one hundred and an incorrect negative reading for drivers who are over the limit one time in ten. A positive reading means the driver will receive a ticket for speeding.

If one driver in ten is actually over the limit on a particular day, what is the probability that a driver who receives a ticket for speeding is in fact over the legal limit? **[10]**

End of Paper – An appendix of 3 pages follows.

Statistics – Common Distributions

Discrete Distributions

Distribution	Density	Range of Variates	Mean	Variance
Uniform	$\frac{1}{N}$	$N = 1, 2, \ldots$ $x = 1, 2, \ldots, N$	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$
Bernoulli	$p^x(1-p)^{1-x}$	$0 \leq p \leq 1, x = 0, 1$	p	$p(1-p)$
Binomial	$\binom{n}{x}p^x(1-p)^{n-x}$	$0 \leq p \leq 1, n = 1, 2, \ldots$ $x = 0, 1, \ldots n$	np	$np(1-p)$
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$	$\lambda > 0, x = 0, 1, 2, \ldots$	λ	λ
Geometric	$p(1-p)^x$	$0 < p \leq 1, x = 0, 1, 2, \ldots$	$\frac{(1-p)}{p}$	$\frac{(1-p)}{p^2}$

Continuous Distributions

Distribution	Density	Range of Variates	Mean	Variance
Uniform	$\frac{1}{b-a}$	$-\infty < a < b < \infty$ $a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}}\exp[\frac{-(x-\mu)^2}{2\sigma^2}]$	$-\infty < \mu < \infty$ $\sigma > 0, -\infty < x < \infty$	μ	σ^2
Lognormal (μ, σ^2)	$\frac{1}{x\sqrt{2\pi\sigma^2}}\exp[\frac{-(logx-\mu)^2}{2\sigma^2}]$	$-\infty < \mu < \infty$ $\sigma > 0, -\infty < x < \infty$	$e^{(\mu+\frac{1}{2}\sigma^2)}$	$e^{(2\mu+\sigma^2)}(e^{\sigma^2}-1)$
Exponential	$\lambda e^{-\lambda x}$	$\lambda > 0, x \geq 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma (α, λ)	$\frac{\lambda^\alpha x^{\alpha-1}e^{-\lambda x}}{\Gamma(\alpha)}$	$\lambda > 0, \alpha > 0, x > 0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
Weibull (c, γ)	$c\gamma x^{\gamma-1}e^{-cx^\gamma}$	$c > 0, \gamma > 0, x > 0$	$c^{-\frac{1}{\gamma}}\Gamma(1+\gamma^{-1})$	$c^{-\frac{2}{\gamma}}[\Gamma(1+2\frac{1}{\gamma})$ $-\Gamma^2(1+\frac{1}{\gamma})]$
Pareto(α, λ)	$\frac{\alpha\lambda^\alpha}{(\lambda+x)^{\alpha+1}}$	$\alpha > 0, \lambda > 0, x > 0$	$\frac{\lambda}{(\alpha-1)}$	$\frac{\lambda^2\alpha}{(\alpha-1)^2(\alpha-2)}$
Burr$(\alpha, \lambda, \gamma)$	$\frac{\alpha\gamma\lambda^\alpha x^{\gamma-1}}{(\lambda+x^\gamma)^{\alpha+1}}$	$\alpha > 0, \lambda > 0, \gamma > 0, x > 0$	Not required	Not required

Useful Formulae

EBCT Model 1

$$E[m(\theta)] = \bar{X}$$

$$E[s^2(\theta)] = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2$$

$$var[m(\theta)] = \frac{1}{(N-1)}\sum_{i=1}^{N}(\bar{X}_i-\bar{X})^2 - \frac{1}{Nn}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2$$

EBCT Model 2

$$E[m(\theta)] = \bar{X}$$

$$E[s^2(\theta)] = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X}_i)^2$$

$$var[m(\theta)] = \frac{1}{P^*}\left[\frac{1}{Nn-1}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X})^2 - \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X}_i)^2\right]$$

Intermediate calculations

$\sum_{j=1}^{n} P_{ij} = \bar{P}_i$ $\quad \sum_{i=1}^{N} \bar{P}_i = \bar{P}$ $\quad \frac{1}{(Nn-1)}\sum_{i=1}^{N}\bar{P}_i\left(1-\frac{\bar{P}_i}{\bar{P}}\right) = P^*$

$\sum_{j=1}^{n}\frac{P_{ij}X_{ij}}{\bar{P}_i} = \bar{X}_i$ $\quad \sum_{i=1}^{N}\sum_{j=1}^{n}\frac{P_{ij}X_{ij}}{\bar{P}} = \bar{X}$

Standard normal distribution: values of the cumulative distribution function

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \mathrm{e}^{-t^2/2}\mathrm{d}t, \quad \Phi(-x) = 1 - \Phi(x)$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

End of Appendix.