Queen Mary
University of London

Main Examination period 2018

MTH5126: Statistics for Insurance

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks available are shown next to the questions.

Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

The New Cambridge Statistical Tables 2nd Edition are provided.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: D. Boland, L. Pettit

Question 1. [20 marks]

(a) Define **Insurable Interest**. [**2**]

(b) Give five criteria a risk needs to meet to be insurable. [**5**]

(c) Widgets Limited is analysing the number of accidents that occur each year on the factory floor. They believe that the number of accidents per year, N, has a geometric distribution with parameter 0.8 so that:

$$P(N = n) = 0.8 \times 0.2^n, \qquad n = 0, 1, 2, ..$$

For each accident, the number of employees injured is Y, where $Y = X + 1$ and X is believed to have a $Poisson(2.2)$ distribution.

The company has taken out an insurance policy, which provides a benefit of £1,000 to each injured employee, up to a maximum of three employees per accident, irrespective of the level of injury. There is no limit on the number of accidents that may be claimed for in a year.

(i) Show that $E(S) = 0.634$ and $var(S) = 2.125$, where S is the total number of employees claiming benefit in a year under this policy [**9**]

(ii) Hence find the mean and variance of the aggregate amount paid out under this policy in a year. [**4**]

Question 2. [18 marks] A random sample $x_1, x_2, \ldots, x_{20}$ is taken from a distribution having the density function:

$$f(x) = \frac{k}{5} x^{-\frac{4}{5}} e^{-kx^{\frac{1}{5}}}, \qquad x > 0$$

For this sample:

$$\sum_{i=1}^{20} x_i = 247,360$$

$$\sum_{i=1}^{20} x_i^{\frac{1}{5}} = 102.778$$

and the median is 10,000.

(a) Determine the maximum likelihood estimate of k. **[5]**

(b) Determine the method of moments estimate of k. **[5]**

(c) Determine the method of percentiles estimate of k. **[5]**

(d) Comment on your results. **[3]**

Question 3. [10 marks]
The table below shows aggregate annual claim statistics for three risks over a period of eight years. Annual aggregate claims for risk i in year j are denoted by X_{ij}.

Risk i	$\bar{X}_{i.} = \frac{1}{8}\sum_{j=1}^{8} X_{ij}$	$S_i^2 = \frac{1}{7}\sum_{j=1}^{8}(X_{ij} - \bar{X}_{i.})^2$
1	117.0	356.3
2	80.8	84.1
3	156.2	39.4

Calculate the credibility premium of each risk under the assumptions of EBCT Model 1. **[10]**

Question 4. [12 marks]
Two rival Marketing companies, Marketing4U and Marketing4ME share profits of £100,000 each year. Every year the companies need to choose between two distinct business approaches: Cautious and Aggressive.

If both companies adopt the same approach in a given year, Marketing4U captures 60% of the total profits. If they adopt different approaches then Marketing4ME captures 90% of the total profits if Marketing4U is cautious and 75% of the total profits if Marketing4U is aggressive.

Neither company knows what the other company's approach will be before adopting its own approach.

(a) Explain why the above can be thought of as a zero-sum two person game. [**3**]

Marketing4U decides to adopt a randomised strategy to setting its approach each year.

(b) Explain what is meant by a randomised strategy. [**3**]

(c) Determine Marketing4U's optimal randomised strategy. [**6**]

Question 5. [15 marks]
The run-off triangle below shows cumulative claims incurred on a portfolio of general insurance policies.

	Development Year			
Policy Year	0	1	2	3
2014	1,679	2,875	3,956	4,946
2015	4,123	5,422	5,676	
2016	5,399	6,312		
2017	7,899			

Earned premiums for the policy years given are as follows:

2014 £5,400
2015 £6,500
2016 £7,600
2017 £8,700

The ultimate loss ratio is 92% and the paid claims to date are £18,652.
Calculate the total ultimate liability and hence the total reserve required.

[**15**]

Question 6. [25 marks]
Claims on a portfolio of insurance policies arrive as a Poisson process with parameter 200. Individual claim amounts follow a normal distribution with mean 20 and variance 16. The insurer calculates premiums using a premium loading of 15% and has initial surplus of 200.

(a) Define the ruin probabilities $\psi(200), \psi(200,1)$ and $\psi_1(200,1)$. [**7**]

(b) Define the adjustment coefficient R. [**2**]

(c) Show that for this portfolio the value of R is 0.013 correct to 3 decimal places. [**7**]

(d) Calculate an upper bound for $\psi(200)$ and an estimate of $\psi_1(200,1)$. [**6**]

(e) Comment on the results in part (d). [**3**]

End of Paper – An appendix of 2 pages follows.

Statistics – Common Distributions

Discrete Distributions

Distribution	Density	Range of Variates	Mean	Variance
Uniform	$\frac{1}{N}$	$N = 1, 2, \ldots$ $x = 1, 2, \ldots, N$	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$
Bernoulli	$p^x(1-p)^{1-x}$	$0 \leq p \leq 1, x = 0, 1$	p	$p(1-p)$
Binomial	$\binom{n}{x} p^x(1-p)^{n-x}$	$0 \leq p \leq 1, n = 1, 2, \ldots$ $x = 0, 1, \ldots n$	np	$np(1-p)$
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$	$\lambda > 0, x = 0, 1, 2, \ldots$	λ	λ
Geometric	$p(1-p)^x$	$0 < p \leq 1, x = 0, 1, 2, \ldots$	$\frac{(1-p)}{p}$	$\frac{(1-p)}{p^2}$

Continuous Distributions

Distribution	Density	Range of Variates	Mean	Variance
Uniform	$\frac{1}{b-a}$	$-\infty < a < b < \infty$ $a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp[\frac{-(x-\mu)^2}{2\sigma^2}]$	$-\infty < \mu < \infty$ $\sigma > 0, -\infty < x < \infty$	μ	σ^2
Lognormal (μ, σ^2)	$\frac{1}{x\sqrt{2\pi\sigma^2}} \exp[\frac{-(logx-\mu)^2}{2\sigma^2}]$	$-\infty < \mu < \infty$ $\sigma > 0, -\infty < x < \infty$	$e^{(\mu+\frac{1}{2}\sigma^2)}$	$e^{(2\mu+\sigma^2)}(e^{\sigma^2} - 1)$
Exponential	$\lambda e^{-\lambda x}$	$\lambda > 0, x \geq 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma (α, λ)	$\frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$	$\lambda > 0, \alpha > 0, x > 0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
Weibull (c, γ)	$c\gamma x^{\gamma-1} e^{-cx^\gamma}$	$c > 0, \gamma > 0, x > 0$	$c^{-\frac{1}{\gamma}}\Gamma(1+\gamma^{-1})$	$c^{-\frac{2}{\gamma}}[\Gamma(1+2\frac{1}{\gamma})$ $-\Gamma^2(1+\frac{1}{\gamma})]$
Pareto(α, λ)	$\frac{\alpha\lambda^\alpha}{(\lambda+x)^{\alpha+1}}$	$\alpha > 0, \lambda > 0, x > 0$	$\frac{\lambda}{(\alpha-1)}$	$\frac{\lambda^2\alpha}{(\alpha-1)^2(\alpha-2)}$
Burr$(\alpha, \lambda, \gamma)$	$\frac{\alpha\gamma\lambda^\alpha x^{\gamma-1}}{(\lambda+x^\gamma)^{\alpha+1}}$	$\alpha > 0, \lambda > 0, \gamma > 0, x > 0$	Not required	Not required

Useful Formulae

EBCT Model 1

$$E[m(\theta)] = \bar{X}$$

$$E[s^2(\theta)] = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2$$

$$var[m(\theta)] = \frac{1}{(N-1)}\sum_{i=1}^{N}(\bar{X}_i-\bar{X})^2 - \frac{1}{Nn}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2$$

EBCT Model 2

$$E[m(\theta)] = \bar{X}$$

$$E[s^2(\theta)] = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X}_i)^2$$

$$var[m(\theta)] = \frac{1}{P^*}\left[\frac{1}{Nn-1}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X})^2 - \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(n-1)}\sum_{j=1}^{n}P_{ij}(X_{ij}-\bar{X}_i)^2\right]$$

Intermediate calculations

$\sum_{j=1}^{n} P_{ij} = \bar{P}_i$ $\quad\sum_{i=1}^{N} \bar{P}_i = \bar{P}$ $\quad\frac{1}{(Nn-1)}\sum_{i=1}^{N}\bar{P}_i\left(1-\frac{\bar{P}_i}{\bar{P}}\right) = P^*$

$\sum_{j=1}^{n}\frac{P_{ij}X_{ij}}{\bar{P}_i} = \bar{X}_i$ $\quad\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{P_{ij}X_{ij}}{\bar{P}} = \bar{X}$

End of Appendix.