Main Examination period 2023 – May/June – Semester B

MTH5125: Actuarial Mathematics II

Duration: 2 hours

The exam is intended to be completed within **2 hours**. However, you will have a period of **4 hours** to complete the exam and submit your solutions.

For actuarial students only: This module also counts towards IFoA exemptions. For your submission to be eligible, **you must submit within the first 3 hours.**

You should attempt ALL questions. Marks available are shown next to the questions.

All work should be **handwritten** and should **include your student number**. Only one attempt is allowed – **once you have submitted your work, it is final**.

In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;

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Question 1 [26 marks]. A life assurance company sells a life annuity which pays out £1,000 per year to one individual aged 60 in return for £11,280 (one payment at the signing of the contract). The annuity is paid annually in arrears. Mortality follows AM92 ultimate life table, interest is 6% per annum.

(a) Write down an expression for the profit random variable at the issuance of this policy. **[3]**

(b) Calculate the expected present value of the life insurer's profit. **[5]**

(c) Calculate the standard deviation of the present value of the life insurer's profit. **[5]**

(d) Calculate the probability that the present value of the life insurer's profit on this contract will be positive. **[5]**

(e) Do you have any concerns about issuing this policy, from a profit viewpoint? Briefly explain your answer. **[5]**

(f) Suppose the life annuity was paid annually in advance. Without doing any more calculations, would the standard deviation of the present value of the profit be less than, equal to or greater than the standard deviation calculated in (b)? Explain your answer. **[3]**

Question 2 [19 marks]. A life assurance company issues a 20-year term, insurance policy to a life aged 40, with sum insured £100,000 payable immediately on death. Level premiums are payable monthly in advance throughout the term. The commissions are 10% of each premium payment (incurred at the premium payment times). Mortality follows AM92 ultimate life table with uniform distribution of deaths (UDD) assumption between integer years. Interest is $i = 6\%$ per annum.

(a) Write down an expression for the gross loss at issue random variable. **[7]**

(b) Write down the equation of value for this policy. **[5]**

(c) Calculate the gross monthly premium. **[7]**

Question 3 [21 marks]. On 1 January 2017, an insurer issued whole life assurances to lives then aged exactly 65. The number of policies in force on 1 January 2022 was 1,900, the number in force on 1 January 2023 was 1,867. The sum insured was £100,000 payable at the end of the year of death. The level premiums are payable annually in advance for the whole of life. Assume that death is the only cause of policy termination, and that the insurer holds net premium reserves for these contracts. Mortality follows AM92 ultimate life table, interest is 4% per annum.

(a) Calculate the net premium for these policies. **[7]**

(b) Calculate the death strain at risk for the policy in the year commencing on 1 January 2022. **[7]**

(c) Calculate the mortality profit for the policy in the year commencing on 1 January 2022. **[7]**

Question 4 [18 marks]. Shaun and Riley are independent lives, both aged 60. They purchase an insurance policy which provides £200,000 payable at the end of the year of Shaun's death, provided that Shaun dies after Riley. Annual premiums are payable in advance throughout Shaun's lifetime. You are given that $\ddot{a}_{60} = 15.632$, $\ddot{a}_{70} = 11.762$ and $\ddot{a}_{60:60} = 14.090$, $\ddot{a}_{70:70} = 9.766$. Interest rate is $i = 4\%$ per annum.

(a) Calculate the net annual premium of this policy. **[5]**

(b) Calculate the net premium policy value after 10 years if only Shaun is alive. **[5]**

(c) Calculate the net premium policy value after 10 years if both Shaun and Riley are alive. **[5]**

(d) Explain why the insurer should differentiate between the cases where both Shaun and Riley are alive or only Shaun is alive. **[3]**

Question 5 [16 marks]. A population of healthy people over the year of age 50 to 51 is subject to a constant force of decrement due to sickness of 0.08 per annum, and a constant force of mortality of 0.002 per annum. Assume that a double decrement model is used.

(a) Calculate the probability that a healthy person aged exactly 50 will still be healthy at exact age 51. **[3]**

(b) Calculate the probability that a healthy person aged exactly 50 will die due to any cause other than from sickness before exact age 51. **[3]**

(c) Calculate the independent probability of a life aged exactly 50 dying before exact age 51. **[3]**

(d) Explain why the probability found at c) is unlikely to be a realistic estimate of the probability of a healthy life aged exactly 50 dying before exact age 51. **[3]**

(e) Propose a more suitable model to estimate the probability required in part d). **[4]**

End of Paper.