# M. Sc. Examination by course unit 2015

# MTH731U: Computational Statistics

## Duration: 3 hours

## Date and time: 12th May 2015, 14:30–17:30

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**Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.**

**You should attempt ALL questions. Marks awarded are shown next to the questions.**

**Calculators ARE permitted in this examination. Please state on your answer book the name and type of machine used.**
**The New Cambridge Statistical Tables are provided.**

Complete all rough workings in the answer book and **cross through any work that is not to be assessed**.

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**Examiner(s): B. M. Parker**

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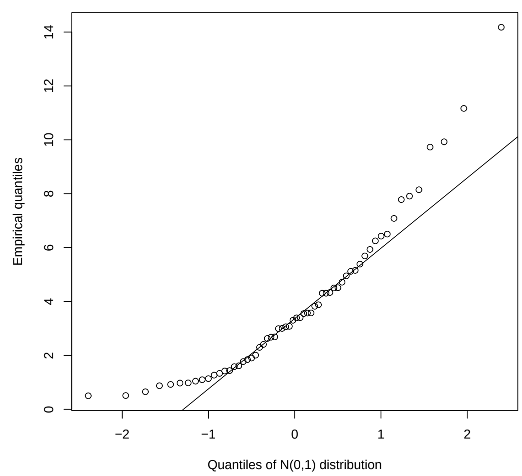
**Question 1.**

(a) Consider the following quantile-quantile plot. Explain how this plot helps us to assess whether a sample comes from a particular distribution. **[5]**




(b) Consider the following distributions:

(i) The normal distribution with mean $\mu = 1$ and variance $\sigma^2 = 2.5$.

(ii) The chi-squared distribution with 4 degrees of freedom.

(iii) Student's-t distribution with 4 degrees of freedom.

For a sample of size 60, which of these distributions would you expect to give a quantile-quantile plot similar to that shown above? Justify your answer. **[5]**

**Question 2.** We wish to test at the 10% level of significance if the computer generated sample

$$1.85, \quad 2.03, \quad 1.00, \quad 7.40, \quad 1.45$$

of size $n = 5$ comes from an exponential distribution with parameter $\lambda = 0.5$, using the Kolmogorov-Smirnov one sample test.

(a) Carry out this test, stating clearly your hypothesis and conclusions. **[12]**

(b) Why might it be preferable to carry out this test at the 10% level of significance rather than using, for example, the 1% level of significance? **[3]**

(c) If $Y$ is uniformly distributed on the interval (0,1), show that the distribution of $X = -2\log(1-Y)$ is exponentially distributed with parameter $\lambda = 0.5$. **[5]**

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**Question 3.** Consider the following data:

$$1.4, \quad 2.7, \quad 2.8, \quad 3.2, \quad 5.9, \quad 6.8.$$

(a) Find the histogram estimator $\hat{f}_H$ and sketch its graph. Assume that the intervals are of equal width 1 with integer endpoints. [**8**]

(b) Comment on the bias and variance of the histogram estimator. How does this vary with the interval width? [**3**]

(c) Rosenblatt's estimator is given by

$$\hat{f}_{n,h}(y) = \frac{\#\{i : y_i - h < y < y_i + h\}}{2nh}$$

For the same data, compute Rosenblatt's estimator for a bandwidth of $h = 1$ and sketch its graph. [**9**]

(d) Comment on how changing the bandwidth would affect the graph. [**3**]

(e) Define the rectangular kernel, and prove that it is a kernel estimator for any given sample $y_1, \ldots, y_n$. [**7**]

**Question 4.** For $n = 4$ psychology students, the level of aptitude for a particular task was measured before and after lunch. A continuous variable was measured. The results are shown below, where the first value in each pair was measured before, and the second value after lunch:

$$(1.76, 3.68), \quad (3.95, 3.90), \quad (4.52, 4.56), \quad (3.50, 4.10).$$

(a) Perform a permutation test for matched pairs to test at the 10% level of significance whether there is a treatment effect. [**8**]

(b) Name a parametric test which could generally be used for this problem. State why a non-parametric test might be more appropriate here. [**3**]

(c) Describe briefly how one would carry out a permutation test using a computer. You do not have to write any code, but suggest the form of an algorithm that a program would carry out. [**4**]

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**Question 5.** Let $y_1, \ldots, y_n$ be realizations of independent and identically distributed random variables $Y_1, \ldots, Y_n$ with variance $\sigma^2$. Show that when using

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

to estimate $\sigma^2$, the jackknife estimate of bias for $\hat{\sigma}^2$ is equal to

$$\widehat{bias}_{\text{jack}} = -\frac{1}{n} s^2,$$

where $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$. [**10**]

**Question 6.**

(a) The coefficient of skewness of a random variable $Y$ with mean $\mu = E(Y)$ and standard deviation $\sigma = \sqrt{Var(Y)}$ is defined by

$$\gamma = \frac{E[(Y - \mu)^3]}{\sigma^3}.$$

For a random sample $y_1, \ldots, y_n$ of realizations of $Y$, find the plug-in estimate of $\gamma$. [**4**]

(b) For a random sample $y_1, \ldots, y_n$ of realizations of $Y$, describe briefly how we might use the plug-in estimate with a bootstrap procedure to estimate the bias of the estimator for $\gamma$. [**5**]

(c) Given a data set $y_1, \ldots, y_n$ of $n$ distinct values, show that the number of distinct nonparametric bootstrap samples is ${}^{2n-1}C_n = \binom{2n-1}{n}$. [**6**]

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**End of Paper.**

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