Queen Mary
University of London

Main Examination period 2021 – May/June – Semester B
Online Alternative Assessments

MTH6139 / MTH6139P: Time Series

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment:

- **You may use books and notes.**
- **You may use calculators and computers, but you must show your working for any calculations you do.**
- **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
- **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

You have **24 hours** to complete and submit this assessment. When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are expected to spend about 2 hours to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. You are allowed two submissions for this exam—the first for your IFoA mark, and the second for your module mark. To be eligible for IFoA exemptions, **your IFoA submission must be within the first 3 hours of the assessment period**.

Examiners: W. W. Yoo, D. Arrowsmith

Question 1 [26 marks]. Consider the following MA(2) process

$$X_t = Z_t + \theta_1 Z_{t-1} + \frac{1}{8} Z_{t-2},$$

where $\theta_1 \neq 0$ is a constant and $\{Z_t\}$ is a Gaussian white noise process with mean 0 and variance 1.

(a) Why do we require our weakly stationary models to be invertible? Explain the reason. **[2]**

(b) Let $\rho(\cdot)$ be the autocorrelation function (ACF) for the MA(2) process above. Suppose $\rho(1) = -\frac{54}{101}$ and $\rho(2) = \frac{8}{101}$. Find the value of θ_1. **[7]**

(c) Based on the θ_1 value computed in (b), determine whether the MA(2) process above is invertible. **[4]**

(d) Using the value of θ_1 computed in (b), calculate the variance of the sample mean $(X_1 + X_2 + X_3 + X_4)/4$. **[13]**

Question 2 [12 marks]. Let $\{Z_t\}$ be a Gaussian white noise process with mean 0 and variance σ^2. Consider the time series

$$X_t = Z_t Z_{t-1}.$$

Is $\{X_t\}$ weakly stationary? **[12]**

Question 3 [30 marks]. Consider the following model

$$X_t - 0.5X_{t-1} - 0.2X_{t-2} = Z_t + 0.4Z_{t-1}$$

where $\{Z_t\}$ is a Gaussian white noise process with mean 0 and variance σ^2.

(a) What SARIMA model is this? **[6]**

(b) Is this model causal? **[3]**

(c) Use the method of coefficients matching and express this model in the form of a linear process $X_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j}$ with $\psi_0 = 1$. Find an explicit formula for ψ_j without any recursions. **[17]**

(d) Note that in order for $X_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j}$ to hold in terms of mean square convergence, we need $\sum_{j=0}^{\infty} |\psi_j| < \infty$. For the ψ_j's you found in (c), do they satisfy $\sum_{j=0}^{\infty} |\psi_j| < \infty$? Explain your reasoning. **[4]**

Question 4 [12 marks]. Let $\{Z_t\}$ be a white noise process with mean 0 and variance 1. For the following processes, compute the partial autocorrelation function (PACF) at lag 3:

(a) $X_t = 0.45X_{t-1} + 0.05X_{t-2} - 0.35X_{t-3} + Z_t$. **[2]**

(b) $X_t = 0.7X_{t-1} - 0.4X_{t-2} + 0.5X_{t-3} + Z_t - 0.7Z_{t-1} + 0.4Z_{t-2} - 0.5Z_{t-3}$. **[3]**

(c) $X_t = Z_t - 0.75Z_{t-1}$. **[3]**

(d) $X_t = \dfrac{41}{36}X_{t-1} - \dfrac{2}{3}X_{t-2} + \dfrac{1}{9}X_{t-3} + Z_t - \dfrac{1}{4}Z_{t-1}$. **[4]**

Question 5 [20 marks]. Figure 1 below shows quarterly earnings per share for Johnson & Johnson (J&J) from the first quarter of 1960 to the last quarter of 1980.

(a) The time series in Figure 1 shows an increasing variance. Describe how you would transform the data to solve the problem of increasing variance. [**4**]

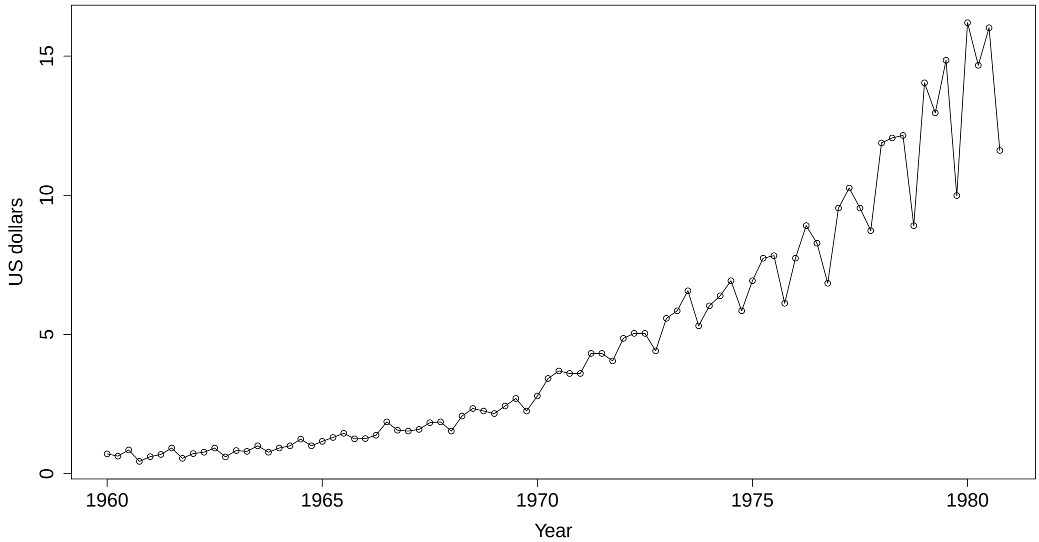

Figure 1: Johnson & Johnson quarterly earnings per share in US dollars.

First-order differencing and then lag-4 differencing were performed on the transformed J&J data (as detailed in (a)). Figure 2 shows the sample ACF and PACF plots after applying both differencing operators, i.e., $\nabla_4 \nabla$ on the transformed J&J data.

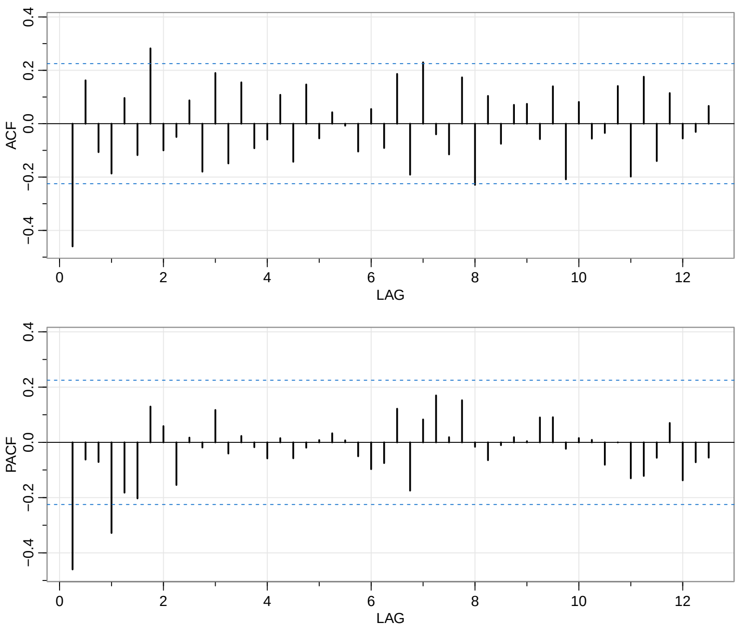

Figure 2: (Top) Sample ACF, (Bottom) Sample PACF of $\nabla_4 \nabla$ on transformed J&J data.

(b) State SARIMA model or models indicated by the sample ACF and PACF plots in Figure 2. Explain how you arrived at your conclusion. **[8]**

(c) Suppose that an ARIMA$(0, 1, 0) \times (2, 1, 0)_4$ model is fitted to the transformed J&J data. Figure 3 below shows the resulting model diagnostics for the standardized residuals. Moreover, performing the Box-Ljung Q test statistic on these residuals yielded

```
        Box-Ljung test

data:  resid(model$fit)
X-squared = 42.161, df = 18, p-value = 0.00105
```

Would you recommend this model to financial analysts working to understand the time dependence of these data? If no, give two suggestions on how to improve this model. Explain your reasoning. Use $\alpha = 0.05$. **[8]**

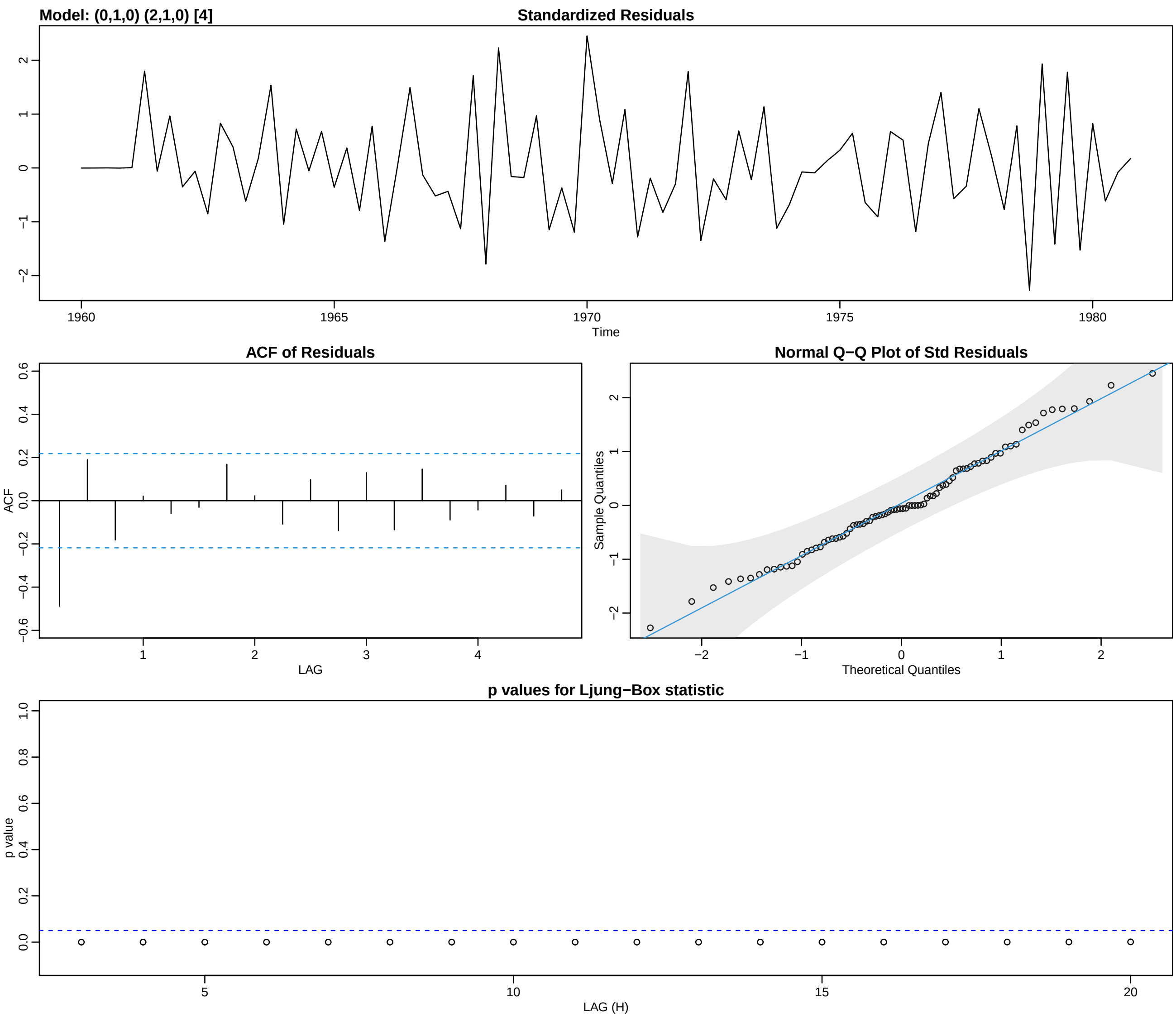

Figure 3: Model diagnostics for fitting ARIMA$(0, 1, 0) \times (2, 1, 0)_4$ to the transformed J&J data.

End of Paper.