Main Examination period 2019

MTH6139 / MTH6139P: Time Series

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: W. W. Yoo and L. Pettit

Question 1. [26 marks]
Let $\{X_t\}$ be a time series such that

$$X_t = m_t + Z_t$$

where m_t denotes a polynomial trend and Z_t is WN$(0, \sigma^2)$ which is white noise with mean 0 and variance σ^2.

(a) Suppose $m_t = \beta_0 + \beta_1 t$ is a linear trend.

(i) Define what it means for a process to be **weakly stationary**. **[2]**

(ii) Is $\{X_t\}$ weakly stationary? Justify your answer. **[3]**

(iii) Show that the mean of the moving average filter

$$W_t = \frac{1}{7} \sum_{j=-3}^{3} X_{t-j}$$

is $\beta_0 + \beta_1 t$. **[3]**

(iv) Define the operator ∇ and show that ∇X_t is weakly stationary. **[7]**

(b) Suppose now $m_t = \beta_0 + \beta_1 t + \beta_2 t^2$ is a quadratic trend.

(i) Show that ∇m_t is a polynomial of degree 1, i.e., a straight line. **[3]**

(ii) Compute $\mathrm{Cov}(W_{t+7}, W_t)$ and explain your answer. **[8]**

Question 2. [12 marks] For each of the following processes, describe the expected behavior of the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots:

(a) Gaussian white noise **[3]**

(b) MA(2) **[3]**

(c) AR(3) **[3]**

(d) ARMA(3,12). **[3]**

Question 3. [19 marks]
Let $\{X_t\}$ be the moving average process of order 1 given by

$$X_t = Z_t + 0.8 Z_{t-1},$$

where $\{Z_t\}$ is $\mathrm{WN}(0, 1)$.

(a) Define what it means for a general moving average process to be **invertible**, and state whether the MA(1) process above is invertible or not. **[4]**

(b) Compute the autocovariance and autocorrelation functions for this process. **[6]**

(c) Using the result from (b), compute the variance of $(X_1 + X_2)/2$. **[9]**

Question 4. [27 marks]

(a) A time series with a periodic component can be constructed from

$$X_t = U_1 \sin(2\pi t) + U_2 \cos(2\pi t),$$

where U_1 and U_2 are independent random variables with zero means and $\mathrm{E}(U_1^2) = \mathrm{E}(U_2^2) = \sigma^2$. Show that this series is weakly stationary with autocovariance function

$$\gamma(h) = \sigma^2 \cos(2\pi h).$$

(Hint: use trigonometric identities such as
$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$,
$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ and $\sin^2(A) + \cos^2(A) = 1$.) **[8]**

(b) Consider the time series

$$X_t = 0.5X_{t-1} + 0.5X_{t-2} + Z_t - 1.7Z_{t-1} + 0.7Z_{t-2},$$

where Z_t is $\mathrm{WN}(0, 3)$.

(i) What model is this? Beware of parameter redundancy! **[6]**

(ii) Check whether this process is causal and invertible. **[4]**

(iii) Express this time series in the form of a linear process.
Hint: Taylor series

$$\frac{1}{1+x} = \sum_{j=0}^{\infty}(-x)^j = 1 - x + x^2 - x^3 + x^4 - \cdots .$$ **[9]**

Question 5. [16 marks] Figure 1 below shows quarterly UK gas consumption from the first quarter of 1960 to the last quarter of 1986, in millions of therms.

(a) Describe the main features of the time series shown by the plot in Figure 1. **[6]**

(b) Describe the steps needed in order to identify the best model and do forecasting for our time series data. Include brief discussions on:

(i) methods to remove trend and seasonality,

(ii) model diagnostics,

(iii) method and algorithm to obtain best linear predictors.

(Note: you are not required to discuss `R` functions or output here, just describe the steps in plain English.) **[10]**

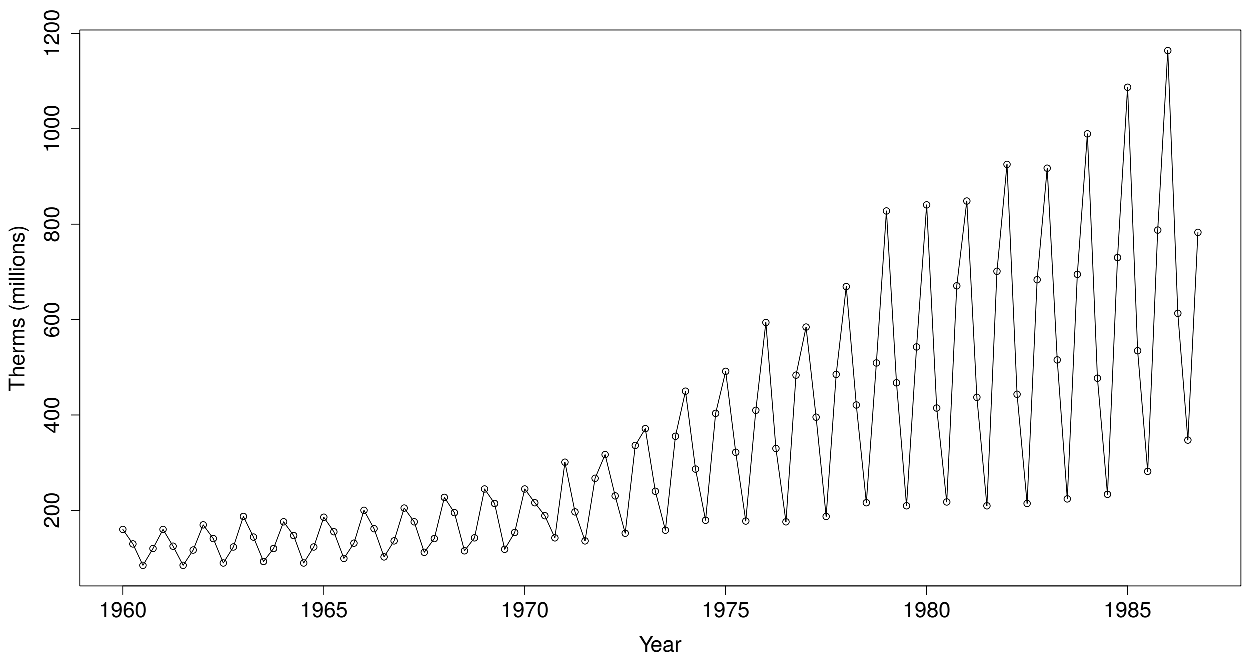

Figure 1: UK quarterly gas consumption.

End of Paper.