Queen Mary University of London

Main Examination period 2020 – May/June – Semester B
Online Alternative Assessments

MTH6108 / MTH6108P: Coding Theory

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment, you may use books, notes, and the internet. You may use calculators and computers, but you should show your working for any calculations you do. You must not seek or obtain help from anyone else.

At the start of your work, please **copy out and sign** the following declaration:

> I declare that my submission is entirely my own, and I have not sought or obtained help from anyone else.

All work should be **handwritten**, and should **include your student number**.

You have **24 hours** in which to complete and submit this assessment. When you have finished your work:

- scan your work, convert it to a **single PDF file** and upload this using the upload tool on the QMplus page for the module;
- e-mail a copy to maths@qmul.ac.uk with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are not expected to spend a long time working on this assessment. We expect you to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the assessment period, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

Examiners: S. Sasaki, I. Tomašić

Question 1 [25 marks].

(a) What is the minimum distance of the code

$$\{\text{LONDON}, \text{BERLIN}, \text{DUBLIN}, \text{LISBON}\}$$

over the alphabet $\mathbb{A} = \{\text{A}, \text{B}, \ldots, \text{Z}\}$? **[3]**

(b) Find a 4-ary code that is 3-error-detecting but not 2-error-correcting. **[4]**

(c) Compute the following numbers and justify your answers. State clearly any results you use from the lectures without proofs.

(i) $A_2(4, 3)$ **[5]**

(ii) $A_3(4, 3)$ **[5]**

(d) Prove that $A_3(7, 4) \leq 57$. State clearly any results you use from the lectures without proofs. **[8]**

Question 2 [10 marks]. Let C be the code $\{000, 010, 101\}$ of length 3 over $\mathbb{A} = \{0, 1\}$.

(a) Find a nearest-neighbour decoding process for C. **[5]**

(b) If the symbol error probability is $\frac{1}{3}$, what is the word error probability for the word 010? Justify your answer. **[5]**

Question 3 [25 marks].

(a) Prove that
$$\{x_1x_2x_3x_4 \in \mathbb{F}_q^4 \mid x_1 + x_2 + x_3 = x_2 + x_3 + x_4 = 0\}$$
is a linear $[4, 2, 2]$-code over $\mathbb{F}_q$ (where q is a prime power). **[9]**

(b) Find a generator matrix and a parity-check matrix for the code
$$\{0000, 1010, 1001, 0011\}$$
over $\mathbb{F}_2 = \{0, 1\}$. **[8]**

(c) Let C be the parity-check code of length 3 over $\mathbb{F}_3 = \{0, 1, 2\}$.

(i) Find a generator matrix for C. **[4]**

(ii) Find a generator matrix for a code equivalent, but not equal, to C. **[4]**

Question 4 [20 marks]. Let C be the binary code
$$\{0000, 0110, 1101, 1011\}$$
of length 4 over $\mathbb{F}_2 = \{0, 1\}$.

(a) Write down a Slepian array for C. **[7]**

(b) Write down a syndrome look-up table for C. **[7]**

(c) Using the syndrome look-up table, find a nearest-neighbour decoding process. **[6]**

Question 5 [20 marks].

(a) Find a basis for the Reed-Muller code $\mathcal{R}(1,3)$ over $\mathbb{F}_2 = \{0,1\}$ and write down all the words of weight 1 in the code. **[8]**

(b) Write down all the words in the Hamming code $\mathrm{Ham}(2,3)$ over $\mathbb{F}_3 = \{0,1,2\}$. **[5]**

(c) In each of the following cases, give an example of an MDS code of length n and redundancy r over $\mathbb{F}_q$ that works for every prime power q. Justify your answer concisely.

(i) $n = 2020$ and $r = 1$, **[3]**

(ii) $n = q + 1$ and $r = 2$. **[4]**

End of Paper.