

B. Sc. Examination by course unit 2015

MTH6100: Actuarial Mathematics

Duration: 2 hours

Date and time: 27th April 2015, 10:00–12:00

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators ARE permitted in this examination. The unauthorised use of material stored in pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and **cross through any work that is not to be assessed.**

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Exam papers must not be removed from the examination room.

Examiner(s): D. Stark, N. Rodosthenous

Question 1.

- (a) Amanda Walnut invests £3000 in a savings account. The effective interest rate is 3.2% per annum in each of the first two years and 2.7% in the third year. Find, to the nearest penny, the accumulation after three years. [3]
- (b) Joan Wilson invests £15,000 in a savings account for which the nominal rate of interest is constant at 5.1% per annum compounded monthly. Find, to the nearest penny, the accumulation after four years and nine months. [3]
- (c) (i) State the relationship between constant force of interest δ and the rate of discount d . [2]
- (ii) If the force of interest is 0.045 per annum, then how much interest (to the nearest penny) must be paid in advance to borrow £800 for one year? [3]
- (d) Sammy Seal currently is obligated to pay a bank £600 in exactly one year, £700 in exactly two years, and £800 in exactly three years. Assume an effective interest rate of 3.3%.
- (i) The bank to which he owes the money gives him an offer of instead paying £ P monthly in arrears over four years. Determine the value of P . [7]
- (ii) The bank to which he owes the money gives him an offer of instead paying £49.51 monthly in arrears. Determine how many payments Sammy will make if he takes this offer. [6]

Question 2.

- (a) Show that the expectation of the further life time of an individual age x is given by the formula

$$\dot{e}_x = \int_0^\infty \frac{s(x+t)}{s(x)} dt,$$

where $s(x)$ is the survival function of the individual. You may use the fact that the further life time of an individual age x has probability density function

$$f_{T(x)}(t) = -\frac{s'(x+t)}{s(x)} \quad \text{for } t \geq 0.$$

You may also assume that

$$\lim_{x \rightarrow \infty} xs(x) = 0.$$

[5]

- (b) Assuming the mortality given by table A1967-1970 select values, calculate the probability ${}_{2q}_{[53]}$ to 4 decimal places.

[4]

- (c) (i) Use linear interpolation on l_x to show that, for x a positive integer and $0 < t < 1$,

$${}_t p_x \approx 1 - t + t p_x.$$

[3]

- (ii) Hence, or otherwise, use ELT12–Males to calculate ${}_{0.8} p_{36}$ to 5 decimal places.

[3]

- (d) Suppose that a population is subject to mortality described by ELT12. A group of 6000 newborns were born in a specific year. Estimate, to 4 significant figures, the number of survivors to age 50 from the group.

[3]

Question 3.

Professor Fuzz proposes that for the race of Tyrannosaurs, the survival function at age x years is given by

$$s(x) = \frac{60 - x}{C} \quad \text{for } 0 \leq x \leq 60,$$

where C is a constant.

- (a) What must the value of C be? [3]

Use the value of C you found in part (a) in the remaining questions.

- (b) Find the probability that a Tyrannosaur of age 2 lives to be age 20 (leaving your answer as a fraction). [2]
- (c) Find the probability that a Tyrannosaur of age 30 lives to be age 40 but dies before age 47 (leaving your answer as a fraction). [3]
- (d) Find the force of mortality $\mu(x)$ for $0 \leq x \leq 60$. [3]
- (e) Suppose that Professor Fuzz uses his survival function to construct a life table for Tyrannosaurs with radix 12,000. What does he find for the life table value d_{55} ? [4]

Question 4. Give your answers to the following questions to the nearest penny. Assume an effective annual interest rate of 4% and the mortality given by table A1967–70 select values.

- (a) (i) At age 56, Aisha Hussain purchases in one payment a nine year endowment policy paying a death benefit of £35,000 if she dies within nine years and £35,000 if she survives nine years. What is the cost of her policy? [6]
- (ii) Suppose, instead, that she chose to pay for her policy with a nine year life annuity while she is alive, with yearly payments in advance. How much would her yearly payment be? [6]
- (b) At age 42, Linda O'Neill purchases in one payment a whole-life monthly annuity, with payments of £100 received at beginnings of months. How much does she pay for this annuity? [7]

Question 5.

- (a) A widget manufacturer is to develop a new model to be produced from 1 January 2018 to an indefinite time in the future.
- The development costs will be £12,000 on 1 January 2016 and £7,000 on 1 January 2017.
 - It is assumed that 100 widgets will be produced each year from 2018 onwards and that all will be sold.
 - The production cost per widget will be £100 during 2018 and every year thereafter. All production costs are assumed to be incurred at the beginning of each calendar year.
 - The sale price of each widget will be £200 during 2018 and every year thereafter. All revenue from sales is assumed to be received at the end of each calendar year.

Given this information

- (i) Assume an effective rate of interest of 1% per annum. Find the Discounted Payback Period determined by the end of year cumulative discounted cashflows by writing down a schedule of payments with four columns: year indexed by $t = 2016, 2017, 2018, \dots$; the total undiscounted payments in all of year t ; the total discounted payments in all of year t ; and the cumulative discounted cashflow up to and including all of year t . [8]
- (ii) Find the Payback Period. [3]
- (iii) Without presenting any further calculations, explain whether the discounted payback period would be greater than, equal to, or less than the discounted payback period calculated in part (i) if the effective rate of interest was substantially more than 1% per annum. [3]

- (b) Assume the force of interest is given by

$$\delta(t) = 0.1 + 0.05t.$$

What is the value at time $t = 0$ of £50 received at time $t = 1$? State your answer to the nearest penny. [5]

- (c) Show that

$$(\bar{I}\bar{a})_{\overline{n}|} = \frac{\bar{a}_{\overline{n}|} - nv^n}{\delta}. \quad [5]$$

End of Paper—An appendix of 8 pages follows.

List of selected formulae

Present values of annuities-certain:

$$\ddot{a}_{\overline{n}|} = \frac{1-v^n}{1-v} \qquad a_{\overline{n}|} = v\ddot{a}_{\overline{n}|}$$
$$\ddot{a}_{\overline{n}|}^{(p)} = \frac{1}{p} \left[\frac{1-v^n}{1-v^{\frac{1}{p}}} \right] \qquad a_{\overline{n}|}^{(p)} = v^{\frac{1}{p}} \ddot{a}_{\overline{n}|}^{(p)}$$

Expected present values of life annuities:

$$\ddot{a}_x = \frac{N_x}{D_x} \qquad \ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x} \qquad a_x = \ddot{a}_x - 1$$
$$\ddot{a}_x^{(p)} \approx \ddot{a}_x - \frac{p-1}{2p} \qquad a_x^{(p)} \approx a_x + \frac{p-1}{2p} \qquad a_x^{(p)} = \ddot{a}_x^{(p)} - \frac{1}{p}$$

Conversion relationships:

$$\bar{A}_x = 1 - \delta \bar{a}_x$$
$$A_x = 1 - d \ddot{a}_x$$
$$A_x^{(p)} = 1 - d^{(p)} \ddot{a}_x^{(p)}$$
$$A_{x:\overline{n}|}^{(p)} = 1 - d^{(p)} \ddot{a}_{x:\overline{n}|}^{(p)}$$

ENGLISH LIFE TABLES NO. 12 - MALES

| Age x | l_x | d_x | P_x | q_x | μ_x | 0e_x | Age x |
|-------|--------|-------|--------|--------|---------|-----------|-------|
| 0 | 100000 | 2449 | .97551 | .02449 | | 68.09 | 0 |
| 1 | 97551 | 153 | .99843 | .00157 | .00210 | 68.80 | 1 |
| 2 | 97398 | 96 | .99901 | .00099 | .00134 | 67.90 | 2 |
| 3 | 97302 | 67 | .99931 | .00069 | .00079 | 66.97 | 3 |
| 4 | 97235 | 60 | .99938 | .00062 | .00063 | 66.02 | 4 |
| 5 | 97175 | 55 | .99943 | .00057 | .00059 | 65.06 | 5 |
| 6 | 97120 | 51 | .99948 | .00052 | .00054 | 64.09 | 6 |
| 7 | 97069 | 47 | .99952 | .00048 | .00050 | 63.13 | 7 |
| 8 | 97022 | 43 | .99956 | .00044 | .00046 | 62.16 | 8 |
| 9 | 96979 | 40 | .99959 | .00041 | .00043 | 61.18 | 9 |
| 10 | 96939 | 38 | .99961 | .00039 | .00040 | 60.21 | 10 |
| 11 | 96901 | 37 | .99962 | .00038 | .00039 | 59.23 | 11 |
| 12 | 96864 | 37 | .99962 | .00038 | .00038 | 58.25 | 12 |
| 13 | 96827 | 40 | .99959 | .00041 | .00039 | 57.28 | 13 |
| 14 | 96787 | 45 | .99953 | .00047 | .00043 | 56.30 | 14 |
| 15 | 96742 | 57 | .99941 | .00059 | .00052 | 55.33 | 15 |
| 16 | 96685 | 75 | .99922 | .00078 | .00067 | 54.36 | 16 |
| 17 | 96610 | 96 | .99901 | .00099 | .00089 | 53.40 | 17 |
| 18 | 96514 | 108 | .99888 | .00112 | .00107 | 52.45 | 18 |
| 19 | 96406 | 113 | .99883 | .00117 | .00115 | 51.51 | 19 |
| 20 | 96293 | 115 | .99881 | .00119 | .00119 | 50.57 | 20 |
| 21 | 96178 | 113 | .99882 | .00118 | .00119 | 49.63 | 21 |
| 22 | 96065 | 110 | .99886 | .00114 | .00116 | 48.69 | 22 |
| 23 | 95955 | 104 | .99892 | .00108 | .00112 | 47.74 | 23 |
| 24 | 95851 | 98 | .99898 | .00102 | .00105 | 46.80 | 24 |
| 25 | 95753 | 95 | .99901 | .00099 | .00100 | 45.84 | 25 |
| 26 | 95658 | 94 | .99902 | .00098 | .00098 | 44.89 | 26 |
| 27 | 95564 | 96 | .99900 | .00100 | .00099 | 43.93 | 27 |
| 28 | 95468 | 99 | .99896 | .00104 | .00102 | 42.98 | 28 |
| 29 | 95369 | 104 | .99891 | .00109 | .00106 | 42.02 | 29 |
| 30 | 95265 | 110 | .99885 | .00115 | .00112 | 41.06 | 30 |
| 31 | 95155 | 115 | .99879 | .00121 | .00118 | 40.11 | 31 |
| 32 | 95040 | 122 | .99872 | .00128 | .00125 | 39.16 | 32 |
| 33 | 94918 | 129 | .99864 | .00136 | .00132 | 38.21 | 33 |
| 34 | 94789 | 137 | .99855 | .00145 | .00140 | 37.26 | 34 |
| 35 | 94652 | 147 | .99845 | .00155 | .00150 | 36.31 | 35 |
| 36 | 94505 | 158 | .99833 | .00167 | .00161 | 35.37 | 36 |
| 37 | 94347 | 171 | .99819 | .00181 | .00174 | 34.43 | 37 |
| 38 | 94176 | 185 | .99804 | .00196 | .00189 | 33.49 | 38 |
| 39 | 93991 | 201 | .99786 | .00214 | .00205 | 32.55 | 39 |
| 40 | 93790 | 220 | .99765 | .00235 | .00224 | 31.62 | 40 |
| 41 | 93570 | 242 | .99741 | .00259 | .00246 | 30.70 | 41 |
| 42 | 93328 | 268 | .99713 | .00287 | .00273 | 29.77 | 42 |
| 43 | 93060 | 297 | .99681 | .00319 | .00303 | 28.86 | 43 |
| 44 | 92763 | 330 | .99644 | .00356 | .00337 | 27.95 | 44 |
| 45 | 92433 | 369 | .99601 | .00399 | .00377 | 27.05 | 45 |
| 46 | 92064 | 412 | .99552 | .00448 | .00423 | 26.15 | 46 |
| 47 | 91652 | 463 | .99495 | .00505 | .00476 | 25.27 | 47 |
| 48 | 91189 | 520 | .99430 | .00570 | .00538 | 24.40 | 48 |
| 49 | 90669 | 584 | .99356 | .00644 | .00607 | 23.53 | 49 |
| 50 | 90085 | 656 | .99272 | .00728 | .00687 | 22.68 | 50 |
| 51 | 89429 | 736 | .99177 | .00823 | .00777 | 21.84 | 51 |
| 52 | 88693 | 825 | .99070 | .00930 | .00878 | 21.02 | 52 |
| 53 | 87868 | 923 | .98949 | .01051 | .00993 | 20.21 | 53 |
| 54 | 86945 | 1029 | .98816 | .01184 | .01121 | 19.42 | 54 |

| Age x | l_x | d_x | P_x | q_x | μ_x | 0e_x | Age x |
|-------|--------|--------|--------|--------|---------|-----------|-------|
| 55 | 85916 | 1144 | .98669 | .01331 | .01263 | 18.65 | 55 |
| 56 | 84772 | 1265 | .98508 | .01492 | .01420 | 17.89 | 56 |
| 57 | 83507 | 1393 | .98332 | .01668 | .01590 | 17.16 | 57 |
| 58 | 82114 | 1526 | .98141 | .01859 | .01776 | 16.44 | 58 |
| 59 | 80588 | 1664 | .97935 | .02065 | .01978 | 15.74 | 59 |
| 60 | 78924 | 1805 | .97713 | .02287 | .02197 | 15.06 | 60 |
| 61 | 77119 | 1947 | .97475 | .02525 | .02433 | 14.40 | 61 |
| 62 | 75172 | 2088 | .97222 | .02778 | .02684 | 13.76 | 62 |
| 63 | 73084 | 2228 | .96951 | .03049 | .02953 | 13.14 | 63 |
| 64 | 70856 | 2366 | .96661 | .03339 | .03243 | 12.54 | 64 |
| 65 | 68490 | 2499 | .96352 | .03648 | .03553 | 11.95 | 65 |
| 66 | 65991 | 2625 | .96022 | .03978 | .03884 | 11.39 | 66 |
| 67 | 63366 | 2745 | .95668 | .04332 | .04239 | 10.84 | 67 |
| 68 | 60621 | 2856 | .95288 | .04712 | .04622 | 10.31 | 68 |
| 69 | 57765 | 2959 | .94878 | .05122 | .05036 | 9.79 | 69 |
| 70 | 54806 | 3051 | .94434 | .05566 | .05487 | 9.29 | 70 |
| 71 | 51755 | 3130 | .93953 | .06047 | .05976 | 8.81 | 71 |
| 72 | 48625 | 3195 | .93430 | .06570 | .06509 | 8.35 | 72 |
| 73 | 45430 | 3243 | .92861 | .07139 | .07092 | 7.90 | 73 |
| 74 | 42187 | 3273 | .92241 | .07759 | .07730 | 7.47 | 74 |
| 75 | 38914 | 3282 | .91566 | .08434 | .08432 | 7.05 | 75 |
| 76 | 35632 | 3266 | .90833 | .09167 | .09200 | 6.66 | 76 |
| 77 | 32366 | 3225 | .90037 | .09963 | .10042 | 6.28 | 77 |
| 78 | 29141 | 3154 | .89176 | .10824 | .10962 | 5.92 | 78 |
| 79 | 25987 | 3054 | .88248 | .11752 | .11964 | 5.57 | 79 |
| 80 | 22933 | 2923 | .87253 | .12747 | .13053 | 5.25 | 80 |
| 81 | 20010 | 2763 | .86192 | .13808 | .14231 | 4.94 | 81 |
| 82 | 17247 | 2576 | .85066 | .14934 | .15503 | 4.66 | 82 |
| 83 | 14671 | 2365 | .83878 | .16122 | .16863 | 4.39 | 83 |
| 84 | 12306 | 2137 | .82634 | .17366 | .18311 | 4.14 | 84 |
| 85 | 10169 | 1897.4 | .81341 | .18659 | .19849 | 3.90 | 85 |
| 86 | 8271.6 | 1654.1 | .80003 | .19997 | .21468 | 3.68 | 86 |
| 87 | 6617.5 | 1414.1 | .78631 | .21369 | .23165 | 3.48 | 87 |
| 88 | 5203.4 | 1184.6 | .77235 | .22765 | .24928 | 3.30 | 88 |
| 89 | 4018.8 | 971.6 | .75823 | .24177 | .26748 | 3.13 | 89 |
| 90 | 3047.2 | 779.9 | .74407 | .25593 | .28616 | 2.97 | 90 |
| 91 | 2267.3 | 612.2 | .72997 | .27003 | .30518 | 2.83 | 91 |
| 92 | 1655.1 | 470.0 | .71604 | .28396 | .32439 | 2.70 | 92 |
| 93 | 1185.1 | 352.73 | .70236 | .29764 | .34372 | 2.58 | 93 |
| 94 | 832.3 | 258.83 | .68904 | .31096 | .36294 | 2.47 | 94 |
| 95 | 573.5 | 185.74 | .67615 | .32385 | .38197 | 2.38 | 95 |
| 96 | 387.8 | 130.39 | .66377 | .33623 | .40066 | 2.29 | 96 |
| 97 | 257.4 | 89.59 | .65194 | .34806 | .41886 | 2.21 | 97 |
| 98 | 167.8 | 60.30 | .64071 | .35929 | .43651 | 2.14 | 98 |
| 99 | 107.5 | 39.771 | .63011 | .36989 | .45354 | 2.07 | 99 |
| 100 | 67.7 | 25.733 | .62017 | .37983 | .46972 | 2.00 | 100 |
| 101 | 42.0 | 16.349 | .61088 | .38912 | .48512 | | 101 |
| 102 | 25.6 | 10.209 | .60224 | .39776 | .49967 | | 102 |
| 103 | 15.4 | 6.272 | .59425 | .40575 | .51335 | | 103 |
| 104 | 9.18 | 3.794 | .58688 | .41312 | | | 104 |
| 105 | 5.39 | | | | | | 105 |

A1967/70 SELECT MORTALITY TABLE
Mortality function l

| Age [x] | $l_{[x]}$ | $l_{[x]+1}$ | l_{x+2} | Age x+2 | Age [x] | $l_{[x]}$ | $l_{[x]+1}$ | l_{x+2} | Age x+2 |
|---------|-----------|-------------|-----------|---------|---------|-----------|-------------|-----------|---------|
| | | | 34489.000 | 0 | 53 | 31970.942 | 31850.639 | 31685.203 | 55 |
| | | | 34463.823 | 1 | 54 | 31728.226 | 31597.933 | 31417.739 | 56 |
| 0 | 34481.408 | 34461.409 | 34440.388 | 2 | 55 | 31458.342 | 31317.610 | 31121.815 | 57 |
| 1 | 34456.927 | 34438.320 | 34418.690 | 3 | 56 | 31158.931 | 31007.338 | 30795.116 | 58 |
| 2 | 34433.841 | 34416.624 | 34398.727 | 4 | 57 | 30827.543 | 30664.702 | 30435.255 | 59 |
| 3 | 34412.836 | 34397.007 | 34380.496 | 5 | 58 | 30461.645 | 30287.215 | 30039.787 | 60 |
| 4 | 34393.221 | 34378.776 | 34363.650 | 6 | 59 | 30058.648 | 29872.344 | 29606.239 | 61 |
| 5 | 34375.681 | 34362.274 | 34348.186 | 7 | 60 | 29615.936 | 29417.538 | 29132.138 | 62 |
| 6 | 34359.181 | 34346.811 | 34333.760 | 8 | 61 | 29130.898 | 28920.265 | 28615.051 | 63 |
| 7 | 34344.063 | 34332.386 | 34320.026 | 9 | 62 | 28600.975 | 28378.059 | 28052.632 | 64 |
| 8 | 34329.638 | 34318.653 | 34306.985 | 10 | 63 | 28023.708 | 27788.571 | 27442.681 | 65 |
| 9 | 34315.907 | 34305.612 | 34294.291 | 11 | 64 | 27396.808 | 27149.632 | 26783.206 | 66 |
| 10 | 34303.210 | 34292.919 | 34281.602 | 12 | 65 | 26718.225 | 26459.331 | 26072.500 | 67 |
| 11 | 34290.518 | 34280.230 | 34268.918 | 13 | 66 | 26986.236 | 25716.097 | 25309.230 | 68 |
| 12 | 34277.830 | 34267.547 | 34255.210 | 14 | 67 | 25199.536 | 24918.797 | 24492.529 | 69 |
| 13 | 34264.461 | 34253.497 | 34239.110 | 15 | 68 | 24357.348 | 24066.835 | 23622.102 | 70 |
| 14 | 34250.070 | 34237.055 | 34218.225 | 16 | 69 | 23459.538 | 23160.273 | 22698.338 | 71 |
| 15 | 34232.259 | 34215.485 | 34190.508 | 17 | 70 | 22506.732 | 22199.940 | 21722.421 | 72 |
| 16 | 34209.439 | 34187.202 | 34154.424 | 18 | 71 | 21500.445 | 21187.559 | 20696.450 | 73 |
| 17 | 34179.680 | 34151.017 | 34120.378 | 19 | 72 | 20443.198 | 20125.863 | 19623.545 | 74 |
| 18 | 34143.368 | 34116.899 | 34088.257 | 20 | 73 | 19338.635 | 19018.696 | 18507.942 | 75 |
| 19 | 34109.166 | 34084.731 | 34057.937 | 21 | 74 | 18191.617 | 17871.109 | 17355.074 | 76 |
| 20 | 34076.957 | 34054.389 | 34029.283 | 22 | 75 | 17008.294 | 16689.418 | 16171.618 | 77 |
| 21 | 34046.610 | 34025.734 | 34002.148 | 23 | 76 | 15796.140 | 15481.232 | 14965.496 | 78 |
| 22 | 34017.983 | 33998.619 | 33976.374 | 24 | 77 | 14563.940 | 14255.427 | 13745.841 | 79 |
| 23 | 33990.921 | 33972.879 | 33951.787 | 25 | 78 | 13321.717 | 13022.064 | 12522.890 | 80 |
| 24 | 33965.254 | 33948.338 | 33928.197 | 26 | 79 | 12080.592 | 11792.241 | 11307.812 | 81 |
| 25 | 33940.795 | 33924.799 | 33905.397 | 27 | 80 | 10852.568 | 10577.865 | 10112.467 | 82 |
| 26 | 33917.341 | 33902.051 | 33883.161 | 28 | | | | 8949.0836 | 83 |
| 27 | 33894.668 | 33879.860 | 33861.242 | 29 | | | | 7829.8752 | 84 |
| 28 | 33872.531 | 33857.972 | 33839.370 | 30 | | | | 6766.5922 | 85 |
| 29 | 33850.662 | 33836.106 | 33817.250 | 31 | | | | 5770.0459 | 86 |
| 30 | 33829.764 | 33813.958 | 33794.559 | 32 | | | | 4849.6219 | 87 |
| 31 | 33806.514 | 33791.191 | 33770.942 | 33 | | | | 4012.8253 | 88 |
| 32 | 33783.557 | 33767.439 | 33746.015 | 34 | | | | 3264.8949 | 89 |
| 33 | 33759.503 | 33742.299 | 33719.354 | 35 | | | | 2608.5274 | 90 |
| 34 | 33733.924 | 33715.331 | 33690.498 | 36 | | | | 2043.7464 | 91 |
| 35 | 33706.352 | 33686.054 | 33658.943 | 37 | | | | 1567.9405 | 92 |
| 36 | 33676.272 | 33653.938 | 33624.136 | 38 | | | | 1176.0783 | 93 |
| 37 | 33643.122 | 33618.409 | 33585.478 | 39 | | | | 861.08935 | 94 |
| 38 | 33606.286 | 33578.835 | 33542.311 | 40 | | | | 614.37801 | 95 |
| 39 | 33565.089 | 33534.529 | 33493.920 | 41 | | | | 426.42117 | 96 |
| 40 | 33518.794 | 33484.739 | 33439.528 | 42 | | | | 287.38847 | 97 |
| 41 | 33466.599 | 33428.646 | 33378.285 | 43 | | | | 187.72094 | 98 |
| 42 | 33407.624 | 33365.360 | 33309.271 | 44 | | | | 118.61237 | 99 |
| 43 | 33340.915 | 33293.909 | 33231.486 | 45 | | | | 72.353686 | 100 |
| 44 | 33265.431 | 33213.241 | 33143.847 | 46 | | | | 42.523157 | 101 |
| 45 | 33180.042 | 33122.213 | 33045.181 | 47 | | | | 24.028676 | 102 |
| 46 | 33083.523 | 33019.589 | 32934.221 | 48 | | | | 13.027677 | 103 |
| 47 | 32974.549 | 32904.032 | 32809.601 | 49 | | | | 6.7627284 | 104 |
| 48 | 32851.686 | 32774.102 | 32669.855 | 50 | | | | 3.3540934 | 105 |
| 49 | 32713.392 | 32628.250 | 32513.405 | 51 | | | | 1.5860118 | 106 |
| 50 | 32558.008 | 32464.813 | 32338.568 | 52 | | | | .71350781 | 107 |
| 51 | 32383.756 | 32282.013 | 32143.546 | 53 | | | | .30474896 | 108 |
| 52 | 32188.740 | 32077.958 | 31926.430 | 54 | | | | .12332121 | 109 |

A1967/70 SELECT MORTALITY TABLES 4% interest

| Age [x] | $D_{[x]}$ | $D_{[x]+1}$ | D_{x+2} | Age x+2 |
|---------|-----------|-------------|-----------|---------|
| | | | 34489.000 | 0 |
| | | | 33138.291 | 1 |
| 0 | 34481.408 | 33135.970 | 31842.074 | 2 |
| 1 | 33131.660 | 31840.163 | 30598.090 | 3 |
| 2 | 31836.022 | 30596.253 | 29404.176 | 4 |
| 3 | 30592.886 | 29402.705 | 28258.262 | 5 |
| 4 | 29399.470 | 28256.848 | 27158.091 | 6 |
| 5 | 28254.304 | 27157.005 | 26101.798 | 7 |
| 6 | 27154.560 | 26100.764 | 25087.342 | 8 |
| 7 | 26098.665 | 25086.338 | 24112.795 | 9 |
| 8 | 25084.331 | 24111.830 | 23176.570 | 10 |
| 9 | 24109.901 | 23175.642 | 22276.917 | 11 |
| 10 | 23174.019 | 22276.026 | 21412.188 | 12 |
| 11 | 22274.466 | 21411.331 | 20581.024 | 13 |
| 12 | 21409.831 | 20580.201 | 19781.530 | 14 |
| 13 | 20578.348 | 19780.541 | 19011.763 | 15 |
| 14 | 19778.562 | 19010.621 | 18269.390 | 16 |
| 15 | 19007.958 | 18267.927 | 17552.492 | 17 |
| 16 | 18264.699 | 17550.795 | 16859.584 | 18 |
| 17 | 17546.933 | 16857.902 | 16194.979 | 19 |
| 18 | 16854.126 | 16193.328 | 15557.436 | 20 |
| 19 | 16189.657 | 15555.826 | 14945.767 | 21 |
| 20 | 15552.279 | 14944.210 | 14358.839 | 22 |
| 21 | 14940.797 | 14357.342 | 13795.567 | 23 |
| 22 | 14354.071 | 13794.135 | 13254.913 | 24 |
| 23 | 13791.012 | 13253.550 | 12735.886 | 25 |
| 24 | 13250.575 | 12734.592 | 12237.535 | 26 |
| 25 | 12731.763 | 12236.310 | 11758.953 | 27 |
| 26 | 12233.620 | 11757.793 | 11299.271 | 28 |
| 27 | 11755.233 | 11298.170 | 10857.655 | 29 |
| 28 | 11295.726 | 10856.607 | 10433.310 | 30 |
| 29 | 10854.263 | 10432.303 | 10025.471 | 31 |
| 30 | 10430.039 | 10024.495 | 9633.4073 | 32 |
| 31 | 10022.288 | 9632.4474 | 9256.4185 | 33 |
| 32 | 9630.2713 | 9255.4583 | 8893.8327 | 34 |
| 33 | 9253.2832 | 8892.8535 | 8545.0060 | 35 |
| 34 | 8890.6463 | 8543.9865 | 8209.3206 | 36 |
| 35 | 8541.7111 | 8208.2376 | 7886.1842 | 37 |
| 36 | 8205.8542 | 7885.0116 | 7575.0280 | 38 |
| 37 | 7882.4775 | 7573.7377 | 7275.3065 | 39 |
| 38 | 7571.0065 | 7273.8677 | 6986.4959 | 40 |
| 39 | 7270.8899 | 6984.8750 | 6708.0930 | 41 |
| 40 | 6981.5977 | 6706.2541 | 6439.6147 | 42 |
| 41 | 6702.6211 | 6437.5192 | 6180.5970 | 43 |
| 42 | 6433.4709 | 6178.2037 | 5930.5940 | 44 |
| 43 | 6173.6773 | 5927.8589 | 5689.1776 | 45 |
| 44 | 5922.7885 | 5686.0541 | 5455.9365 | 46 |
| 45 | 5680.3705 | 5452.3753 | 5230.4756 | 47 |
| 46 | 5446.0064 | 5226.4249 | 5012.4160 | 48 |
| 47 | 5219.2958 | 5007.8215 | 4801.3938 | 49 |
| 48 | 4999.8546 | 4796.1989 | 4597.0607 | 50 |
| 49 | 4787.3144 | 4591.2064 | 4399.0830 | 51 |
| 50 | 4581.3224 | 4392.5083 | 4207.1417 | 52 |
| 51 | 4381.5413 | 4199.7841 | 4020.9326 | 53 |
| 52 | 4187.6496 | 4012.7281 | 3840.1664 | 54 |

A1967/70 SELECT MORTALITY TABLES 4% interest

| Age [x] | $D_{[x]}$ | $D_{[x]+1}$ | D_{x+2} | Age x+2 |
|---------|-----------|-------------|-----------|---------|
| 53 | 3999.3411 | 3831.0501 | 3664.5684 | 55 |
| 54 | 3816.3261 | 3654.4752 | 3493.8796 | 56 |
| 55 | 3638.3307 | 3482.7444 | 3327.8564 | 57 |
| 56 | 3465.0983 | 3315.6153 | 3166.2716 | 58 |
| 57 | 3296.3898 | 3152.8628 | 3008.9150 | 59 |
| 58 | 3131.9850 | 2994.2794 | 2855.5942 | 60 |
| 59 | 2971.6826 | 2839.6770 | 2706.1356 | 61 |
| 60 | 2815.3028 | 2688.8874 | 2560.3853 | 62 |
| 61 | 2662.6874 | 2541.7641 | 2418.2107 | 63 |
| 62 | 2513.7020 | 2398.1830 | 2279.5016 | 64 |
| 63 | 2368.2373 | 2258.0445 | 2144.1713 | 65 |
| 64 | 2226.2106 | 2121.2746 | 2012.1584 | 66 |
| 65 | 2087.5676 | 1987.8264 | 1883.4277 | 67 |
| 66 | 1952.2839 | 1857.6818 | 1757.9716 | 68 |
| 67 | 1820.3664 | 1730.8523 | 1635.8114 | 69 |
| 68 | 1691.8542 | 1607.3801 | 1516.9972 | 70 |
| 69 | 1566.8197 | 1487.3388 | 1401.6093 | 71 |
| 70 | 1445.3689 | 1370.8335 | 1289.7567 | 72 |
| 71 | 1327.6401 | 1257.9996 | 1181.5772 | 73 |
| 72 | 1213.8036 | 1149.0019 | 1077.2347 | 74 |
| 73 | 1104.0584 | 1044.0315 | 976.91702 | 75 |
| 74 | 998.62904 | 943.30265 | 880.83121 | 76 |
| 75 | 897.76009 | 847.04681 | 789.19865 | 77 |
| 76 | 801.70978 | 755.50684 | 702.24821 | 78 |
| 77 | 710.74161 | 668.92859 | 620.20821 | 79 |
| 78 | 625.11472 | 587.55160 | 543.29713 | 80 |
| 79 | 545.07270 | 511.59842 | 471.71326 | 81 |
| 80 | 470.83137 | 441.26304 | 405.62366 | 82 |
| | | | 345.15280 | 83 |
| | | | 290.37173 | 84 |
| | | | 241.28824 | 85 |
| | | | 197.83908 | 86 |
| | | | 159.88487 | 87 |
| | | | 127.20858 | 88 |
| | | | 99.518086 | 89 |
| | | | 76.453060 | 90 |
| | | | 57.596108 | 91 |
| | | | 42.487615 | 92 |
| | | | 30.643310 | 93 |
| | | | 21.573188 | 94 |
| | | | 14.800229 | 95 |
| | | | 9.8772987 | 96 |
| | | | 6.4008172 | 97 |
| | | | 4.0201797 | 98 |
| | | | 2.4424709 | 99 |
| | | | 1.4326059 | 100 |
| | | | .80957713 | 101 |
| | | | .43987495 | 102 |
| | | | .22931531 | 103 |
| | | | .11446024 | 104 |
| | | | .05458515 | 105 |
| | | | .02481832 | 106 |
| | | | .01073573 | 107 |
| | | | .00440902 | 108 |
| | | | .00171555 | 109 |

A 1967/70SELECT MORTALITY TABLE 4% interest

| Age [x] | $N_{[x]}$ | $N_{[x]+1}$ | N_{x+2} | Age x+2 |
|---------|-----------|-------------|-----------|---------|
| | | | 835843.39 | 0 |
| | | | 801354.39 | 1 |
| 0 | 835833.48 | 801352.07 | 768216.10 | 2 |
| 1 | 801345.85 | 768214.19 | 736374.03 | 3 |
| 2 | 768208.21 | 736372.19 | 705775.94 | 4 |
| 3 | 736367.35 | 705774.47 | 676371.76 | 5 |
| 4 | 705769.82 | 676370.35 | 648113.50 | 6 |
| 5 | 676366.72 | 648112.41 | 620955.41 | 7 |
| 6 | 648108.92 | 620954.36 | 594853.61 | 8 |
| 7 | 620951.27 | 594852.61 | 569766.27 | 9 |
| 8 | 594849.63 | 569765.30 | 545653.47 | 10 |
| 9 | 569762.45 | 545652.55 | 522476.90 | 11 |
| 10 | 545650.03 | 522476.01 | 500199.99 | 12 |
| 11 | 522473.60 | 500199.13 | 478787.80 | 13 |
| 12 | 500196.81 | 478786.98 | 458206.77 | 14 |
| 13 | 478784.13 | 458205.79 | 438425.24 | 15 |
| 14 | 458202.67 | 438424.10 | 419413.48 | 16 |
| 15 | 438419.98 | 419412.02 | 401144.09 | 17 |
| 16 | 419407.09 | 401142.40 | 383591.60 | 18 |
| 17 | 401136.85 | 383589.92 | 366732.02 | 19 |
| 18 | 383584.49 | 366730.36 | 350537.04 | 20 |
| 19 | 366725.08 | 350535.43 | 334979.60 | 21 |
| 20 | 350530.32 | 334978.04 | 320033.83 | 22 |
| 21 | 334973.13 | 320032.34 | 305674.99 | 23 |
| 22 | 320027.63 | 305673.56 | 291879.43 | 24 |
| 23 | 305669.08 | 291878.06 | 278624.51 | 25 |
| 24 | 291873.80 | 278623.22 | 265888.63 | 26 |
| 25 | 278619.17 | 265887.40 | 253651.09 | 27 |
| 26 | 265883.55 | 253649.93 | 241892.14 | 28 |
| 27 | 253646.27 | 241891.04 | 230592.87 | 29 |
| 28 | 241887.55 | 230591.82 | 219735.21 | 30 |
| 29 | 230588.47 | 219734.21 | 209301.91 | 31 |
| 30 | 219730.97 | 209300.93 | 199276.43 | 32 |
| 31 | 209297.76 | 199275.47 | 189643.03 | 33 |
| 32 | 199272.34 | 189642.07 | 180386.61 | 34 |
| 33 | 189638.91 | 180385.63 | 171492.78 | 35 |
| 34 | 180382.40 | 171491.76 | 162947.77 | 36 |
| 35 | 171488.40 | 162946.69 | 154738.45 | 37 |
| 36 | 162943.13 | 154737.28 | 146852.27 | 38 |
| 37 | 154733.45 | 146850.97 | 139277.24 | 39 |
| 38 | 146846.80 | 139275.80 | 132001.93 | 40 |
| 39 | 139271.20 | 132000.31 | 125015.43 | 41 |
| 40 | 131995.19 | 125013.60 | 118307.34 | 42 |
| 41 | 125007.87 | 118305.25 | 111867.73 | 43 |
| 42 | 118298.80 | 111865.33 | 105687.13 | 44 |
| 43 | 111858.07 | 105684.39 | 99756.536 | 45 |
| 44 | 105676.20 | 99753.413 | 94067.358 | 46 |
| 45 | 99744.168 | 94063.797 | 88611.422 | 47 |
| 46 | 94053.378 | 88607.371 | 83380.946 | 48 |
| 47 | 88595.648 | 83376.352 | 78368.530 | 49 |
| 48 | 83363.190 | 78363.335 | 73567.136 | 50 |
| 49 | 78348.597 | 73561.282 | 68970.076 | 51 |
| 50 | 73544.823 | 68963.501 | 64570.993 | 52 |
| 51 | 68945.176 | 64563.635 | 60363.851 | 53 |
| 52 | 64543.296 | 60355.647 | 56342.918 | 54 |

A 1967/70SELECT MORTALITY TABLE 4% interest

| Age [x] | $N_{[x]}$ | $N_{[x]+1}$ | N_{x+2} | Age x+2 |
|---------|-----------|-------------|-----------|---------|
| 53 | 60333.143 | 56333.802 | 52502.752 | 55 |
| 54 | 56308.985 | 52492.659 | 48838.184 | 56 |
| 55 | 52465.379 | 48827.049 | 45344.304 | 57 |
| 56 | 48797.161 | 45332.063 | 42016.448 | 58 |
| 57 | 45299.429 | 42003.039 | 38850.176 | 59 |
| 58 | 41967.525 | 38835.540 | 35841.261 | 60 |
| 59 | 38797.026 | 35825.344 | 32985.667 | 61 |
| 60 | 35783.721 | 32968.419 | 30279.531 | 62 |
| 61 | 32923.597 | 30260.910 | 27719.146 | 63 |
| 62 | 30212.820 | 27699.118 | 25300.935 | 64 |
| 63 | 27647.715 | 25279.478 | 23021.434 | 65 |
| 64 | 25224.747 | 22998.537 | 20877.262 | 66 |
| 65 | 22940.498 | 20852.930 | 18865.104 | 67 |
| 66 | 20791.642 | 18839.358 | 16981.68 | 68 |
| 67 | 18774.923 | 16954.557 | 15223.705 | 69 |
| 68 | 16887.127 | 15195.273 | 13587.893 | 70 |
| 69 | 15125.055 | 13558.235 | 12070.896 | 71 |
| 70 | 13485.489 | 12040.120 | 10669.287 | 72 |
| 71 | 11965.170 | 10637.530 | 9379.5300 | 73 |
| 72 | 10560.758 | 9346.9547 | 8197.9528 | 74 |
| 73 | 9268.8080 | 8164.7496 | 7120.7181 | 75 |
| 74 | 8085.7327 | 7087.1037 | 6143.8011 | 76 |
| 75 | 7007.7767 | 6110.0167 | 5262.9698 | 77 |
| 76 | 6030.9878 | 5229.2780 | 4473.7712 | 78 |
| 77 | 5151.1932 | 4440.4516 | 3771.5230 | 79 |
| 78 | 4363.9811 | 3738.8664 | 3151.3148 | 80 |
| 79 | 3664.6888 | 3119.6161 | 2608.0176 | 81 |
| 80 | 3048.3988 | 2577.5674 | 2136.3044 | 82 |
| | | | 1730.6807 | 83 |
| | | | 1385.5279 | 84 |
| | | | 1095.1562 | 85 |
| | | | 853.86794 | 86 |
| | | | 656.02886 | 87 |
| | | | 496.14399 | 88 |
| | | | 368.93541 | 89 |
| | | | 269.41733 | 90 |
| | | | 192.96427 | 91 |
| | | | 135.36816 | 92 |
| | | | 92.880543 | 93 |
| | | | 62.237233 | 94 |
| | | | 40.664045 | 95 |
| | | | 25.863816 | 96 |
| | | | 15.986518 | 97 |
| | | | 9.5857004 | 98 |
| | | | 5.5655208 | 99 |
| | | | 3.1230498 | 100 |
| | | | 1.6904439 | 101 |
| | | | .88086681 | 102 |
| | | | .44099186 | 103 |
| | | | .21167655 | 104 |
| | | | .09721631 | 105 |
| | | | .04263116 | 106 |
| | | | .01781284 | 107 |
| | | | .00707711 | 108 |
| | | | .00266810 | 109 |

End of Appendix.