Queen Mary
University of London

Main Examination period 2018

MTH5122: Statistical Methods

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Statistical functions provided by the calculator may be used provided that you state clearly where you have used them.
The New Cambridge Statistical Tables are provided.
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Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: A. Gnedin

 Turn Over

Question 1. [25 marks] Consider a random variable X with probability density function

$$f_X(x) = \begin{cases} cx^{-3}\,, \text{ if } x \geq 1, \\ 0\,, \quad \text{otherwise.} \end{cases}$$

(a) Find the constant c. **[5]**

(b) Find the cumulative distribution function $F_X(x)$ for all $x \in \mathbb{R}$. **[5]**

(c) Calculate $\mathbb{P}(2 < X < 3)$. **[5]**

(d) Calculate $\mathbb{E}(X)$. **[5]**

(e) For $Y = \log_e X$, determine the probability density function $f_Y(y)$. What is the name of the distribution of Y? **[5]**

Question 2. [15 marks] A manufacturer of detergents conducted a regular check of the net weight of boxes filled with washing powder. For a sample of 25 boxes the standard deviation of the net weight was $s = 20$ gram.

(a) Find a 95% confidence interval for the variance of the net weight of a filled box. State explicitly any assumptions you make. **[10]**

(b) Hence test at the 5% significance level that the variance of the net weight of a filled box is 200. **[5]**

Question 3. [25 marks] Let $X \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(0,1)$ be independent normal random variables. To answer the following questions you may quote without proof any results from the course.

(a) Determine the moment generating function $M_{2X+Y}(t)$ of the random variable $2X + Y$. **[8]**

(b) Apply Chebyshev's inequality to obtain an upper bound of the probability $\mathbb{P}(|3X + 4Y| > 10)$. **[7]**

(c) For random variables

$$U = 2X - 2Y, \;\; V = 2X + 2Y$$

derive the joint probability density function. **[10]**

Question 4. [20 marks] The times it takes a packing machine to pack cartons with jars were recorded before and after maintenance work. Before the maintenance, for a sample of eleven cartons the mean was 44.3 seconds and the standard deviation was 2.2 seconds. After the maintenance, for a sample of eleven cartons the mean was 41.6 seconds and the standard deviation was 1.4 seconds.

(a) Test the hypothesis that the mean time to pack a carton is the same before and after maintenance against the alternative that it is not the same. Assume the population variances are equal and use a significance level of $\alpha = 0.05$. **[7]**

(b) Test the hypothesis that the mean time to pack a carton is the same before and after the maintenance against the alternative that after the maintenance it is smaller. Assume the population variances are equal and use a significance level of $\alpha = 0.05$. **[3]**

(c) Test at the 10% significance level that the variance of the packing time is the same before and after the maintenance. **[10]**

Question 5. [15 marks] In a sample of 408 people the frequencies of births across the days of the week were as follows:

day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
frequency	49	57	71	60	63	65	43

(a) Test at the 5% significance level that the distribution of birthdays throughout the days of the week is uniform. State explicitly the hypothesis and the alternative. **[10]**

(b) Give the definition of the P value of the test in (a), and estimate the P value numerically. **[5]**

End of Paper.