Queen Mary
University of London

B. Sc. Examination by course unit 2015

MTH5122: Statistical Methods

Duration: 2 hours

Date and time: 15 May 2015, 10.00-12.00

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators ARE permitted in this examination. Please state on your answer book the name and type of machine used.
Statistical functions provided by the calculator may be used provided that you state clearly where you have used them.
The New Cambridge Statistical Tables are provided.

Complete all rough workings in the answer book and **cross through any work that is not to be assessed**.

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Examiner(s): L I Pettit

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Question 1. [18 marks]
The random variables X and Y have joint probability density function

$$f(x,y) = Ce^{-(2x+y)} \qquad 0 < x < \infty, 0 < y < \infty$$

and zero otherwise.

(a) Find the value of C. **[3]**

(b) Are X and Y independent? Justify your answer. **[3]**

(c) Let $W = X + Y$ and $V = X$. Find the joint probability density function of W and V. **[5]**

(d) Find the marginal probability density function of W and hence its mean. **[7]**

Question 2. [15 marks]
A random sample of size n is assumed to be normally distributed with unknown variance σ^2. The sample variance is S^2. The distribution of the statistic $W = \frac{(n-1)S^2}{\sigma^2}$ is chi-squared with $n-1$ degrees of freedom (χ^2_{n-1}).

(a) Derive the form of a $100(1-\alpha)\%$ confidence interval for σ^2. **[5]**

(b) If $n = 9$ and the observed sample variance is $s^2 = 16.3$ find a 90% confidence interval for σ^2. **[4]**

(c) Find the P value for a two sided test of the hypothesis that $\sigma^2 = 10.0$ and comment on what it shows. **[6]**

Question 3. [13 marks]
A company recorded the number of sales made by 30 of their employees in the previous month. The sample mean was found to be 71.0 and the sample standard deviation 18.50. The company wishes to find out if a normal distribution gives a reasonable fit to the data. The following table shows part of the analysis the company statistician carried out. For the first class, for example, the statistician found $P(X < 45.5)$ and when calculating the standardised normal class boundaries rounded to 2 decimal places.

Class	Observed Frequency	Probability	Expected Frequency
45 or less	4	0.0838	2.514
46-55	2	0.1167	3.501
56-65	6	0.1816	5.448
66-75	8	0.2127	6.381
76-85	3	0.1875	5.625
86-95	4	0.1243	3.729
96 or more	3		

(a) Show how the values 0.1167 and 3.501 in the second row of the table were calculated and confirm they are correct. [**4**]

(b) Find the missing values in the last row of the table. [**2**]

(c) Carry out a goodness of fit test of the hypothesis that the data follow a normal distribution using a significance level of $\alpha = 0.05$. [**7**]

Question 4. [22 marks]
A researcher tests two types of pain relieving pill on two groups of randomly selected patients. One set of patients is given pills made by company A and a second independent set of patients is given pills made by company B. The sample mean and sample standard deviation of the time before a patient reported relief from pain (in minutes) are given below.

Company	Sample Size	Sample Mean	Sample Standard Deviation
A	10	90.0	27.2
B	9	98.4	22.3

(a) Show that you cannot reject the null hypothesis that the population variances are the same for the two companies using a significance level of $\alpha = 0.05$ and a two-sided test. [**8**]

(b) Test that there is no difference in the mean times to relief from pain of the two types of pill against an alternative that for brand A the mean time to relief from pain is shorter at the 1% significance level. [**10**]

(c) Explain briefly how you would have proceeded if you had been able to reject the null hypothesis in part (a). [**4**]

Question 5. [22 marks]

(a) If W has a binomial distribution with parameters n and θ, show the moment generating function of W is

$$M_W(t) = (\theta e^t + 1 - \theta)^n.$$

[3]

(b) Define the joint moment generating function of random variables X and Y. **[2]**

(c) X and Y are discrete random variables with joint moment generating function

$$M_{X,Y}(t_1, t_2) = (0.5e^{t_1} + 0.3e^{t_2} + 0.2)^{10}.$$

Find the following:

(i) $\mathrm{E}[X]$, **[2]**

(ii) $\mathrm{E}[Y]$, **[2]**

(iii) $\mathrm{E}[XY]$, **[3]**

(iv) $\mathrm{Cov}[X, Y]$. **[2]**

(d) Deduce the marginal distribution of X. **[4]**

(e) Find the marginal distribution of $X + Y$. **[4]**

Question 6. [10 marks]

Two different varieties of wheat are grown on the same set of 51 experimental sites. For one variety the sample correlation coefficient between mean soil temperature and time to germination is -0.45 and for the other variety it is -0.59. Using a 1% significance level test whether the correlation coefficients for the two varieties are different.

End of Paper.